ELEMENTS OF THEORY TO SOLVE THE PROBLEM OF MANAGEMENT OF TRASPORT SAFETY

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Authors' original problem-solution-approach concerning aviation security management in civil aviation applying parallel calculation processes' method and neural computers' usage is considered in this paper. Problem statement by setting secure environment simulation tasks for grid models, and neural networks' usage is presented. The research subject area of this paper is airport services in civil aviation, considered from the point of view of aviation security, defined as the state of aviation security against unlawful interference into the aviation field. The key issue in this subject area is aviation safety provision at an acceptable level. In this case, airport security level management becomes one of the main objectives of aviation security. Aviation security management is the organizational regulations in modern systems that can no longer correspond to changing and increasingly complex requirements determined by factors of external and internal environment, associated with a set of potential threats to airport activity. Optimal control requires the most accurate identification of management parameters and their quantitative assessment. The authors examine the possibility of applying mathematical methods for processes and procedures' security management modeling in their latest works. Parallel computing methods and network neurocomputing for modeling control processes of airport security are examined in this paper. It is shown that the methods' practical application is most effective in the decision support system, where the decision maker plays a leading role. Decision support system on the aviation safety management should include risk assessment subsystem of adverse events.

Keywords*: aviation security, boundary value problem, differential equations in partial derivatives, grid model, neural network.*

1. The linguistic problem statement.

Aviation security is considered as the object's security state [1,2]. Such the state occurs as a result of the confrontation of two systems that are physically implemented: threats' systems and protection systems. This protection status is not physically implemented concept; it means that it is an imaginary concept. Lacan [3] proposed a fairly clear way for the study of such concepts: imaginary and symbolic - real. Security state is estimated with the help of vulnerability concept that answers to the following question: to what extent the subject meets the safety requirements. In this case, it is entitled to introduce the concept of "protection quality", that is understood as degree of conformity to characteristics and requirements. It has the symbolic representation and, moreover, the quality can be measured [4]. Security is provided by a set of tools (mainly technical), each of which creates a separate object protection fragment, and together form the security environment. It can be represented as a security field, which is characterized by such a parameter as quality. In this case, the field is a real concept that can be studied with the help of a specific mathematical apparatus technique. Then target security management functionality includes the parameters of the object protection field, measured as the quality of the technical means to ensure aviation security. The analysis shows that a formal description of the object protection field can be represented in the format of the boundary value problem at a first approximation.

2. Mathematic interpretation of the problem.

In the paper [2] the authors showed the fundamental possibility of solving aviation security management tasks as the solution of the boundary value problems described by the system of differential equations in partial derivatives. The statement of such the problem in the modeling of neural networks is presented below. It requires a finite-difference approximation of the original equation; it means replacing the field of continuous variation of the argument to its discrete area (*grid*) and replacing the differential operator to some difference ones, as well as replacement of difference analogues to boundary conditions, resulting in a system of algebraic equations.

Let us consider the finite-difference approximation of the Dirichlet problem for two-dimensional equation of elliptic type

$$\frac{\partial}{\partial x}\left(\sigma(x,y)\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\sigma(x,y)\frac{\partial u}{\partial y}\right)=f(x,y),\quad (x,y)\in S, \qquad (1)$$

$$u\big|_{\Gamma}=\varphi(x,y),\quad (x,y)\in\Gamma,$$

where Γ– a rectangle [5].

Let us consider a uniform grid on the plane *XOY*. For this purpose, we will construct two classes of straight equidistant lines

$$x_i=x_0+ih,\quad i=0,1,2,..., \qquad (2)$$

$$y_j=y_0+jh,\quad j=0,1,2,...,$$

where *h* – a grid step.

In this case, on the subspace *XOY* we will receive *plane frame with nodes* $(ih,jh),\quad i=0,1,2,...,\quad j=0,1,2,...$. Grid is uniform with respect to the axes *X* and *Y*. Let us consider the nodes that belong to such a zone as $\overline{S}=S+\Gamma$. Those nodes that are inside *S zone* are called *internal* (o), and their multitude is called *net domain* S_h. The points of straight lines' intersection with border line Γ is called *boundary nodes*, and their multitude is called *net domain* Γ_h. Nodes that are nearest to the borderline are called *border-straddling nodes*.

Instead of functions $u(x,y)$ of continuous arguments $x,y\in S$ let us consider *grid functions* $u_h(x_i,y_j)$ of grid points. Given a linear differential operator *L*, related to function *u*, then, changing included into *Lu* derivatives differential relationships, we will receive difference expression $L_h u_h$ that is a linear combination of meanings of grid functions u_h on some multitude of grid steps that is called *stencil*. That is the template specifies a set of grid points included into the differential expression

$$(L_h u_h)_{ij}=\sum_{x_{kl}\in N(x_{ij})} A_h(x_{ij},x_{kl})u_h(x_{kl}),$$

where $A_h(x_{ij},x_{kl})$ – coefficients, *h* – a grid step, $N(x_{ij})$ – pattern in the grid x_{ij}. Such an interchange is called *difference approximation of operator L*.

Using the integro-interpolation method and the five-point stencil, let us write difference approximation of the equation (1) for an internal node

$$-\left[\frac{\sigma(x_i+h/2,y_j)}{h^2}+\frac{\sigma(x_i-h/2,y_j)}{h^2}+\frac{\sigma(x_i,y_j+h/2)}{h^2}+\frac{\sigma(x_i,y_j-h/2)}{h^2}\right]u_{i,j}+$$
$$+\frac{\sigma(x_i+h/2,y_j)}{h^2}u_{i+1,j}+\frac{\sigma(x_i-h/2,y_j)}{h^2}u_{i-1,j}+\frac{\sigma(x_i,y_j+h/2)}{h^2}u_{i,j+1}+$$
$$+\frac{\sigma(x_i,y_j-h/2)}{h^2}u_{i,j-1}=f(x_i,y_j),\quad (x_i,y_i)\in S_h$$

or

$$a_{i,j}u_{i,j}-a_{i+1,j}u_{i+1,j}-a_{i-1,j}u_{i-1,j}-a_{i,j+1}u_{i,j+1}-a_{i,j-1}u_{i,j-1}=-f_{i,j}, \tag{3}$$

where

$$a_{i,j}=\frac{\sigma(x_i+h/2,y_j)}{h^2}+\frac{\sigma(x_i-h/2,y_j)}{h^2}+\frac{\sigma(x_i,y_j+h/2)}{h^2}+\frac{\sigma(x_i,y_j-h/2)}{h^2},$$

$$a_{i+1,j}=\frac{\sigma(x_i+h/2,y_j)}{h^2},\quad a_{i-1,j}=\frac{\sigma(x_i-h/2,y_j)}{h^2},\quad a_{i,j+1}=\frac{\sigma(x_i,y_j+h/2)}{h^2},$$
$$a_{i,j-1}=\frac{\sigma(x_i,y_j-h/2)}{h^2},\quad f_{i,j}=f(x_i,y_j),\quad (x_i,y_i)\in S_h.$$

Dirichlet boundary conditions are approximated as follows:

$$u_{i,j}=\varphi(x_i,y_j),\quad (x_i,y_j)\in\Gamma_h. \tag{4}$$

Writing down the equation (3) for each node, where grid function is unknown, taking into account boundary conditions and moving all the known terms into the right side, we obtain *a system of algebraic differential equations*. In the case of the Dirichlet problem, the solution is sought only in internal nodes, the conditions (4) are taken into account in the differential equations of the form (3). In order to write down a system of difference equations in matrix form let us number nodes of the grid area, where the solution is sought. In the case of *lexicographical ordering* nodes are numbered after-successively along the lines of the grid area. In accordance with the ordering, grid functions can be combined into a vector **X**. Writing equation (3) in the order of the nodes and moving the known boundary conditions of the members to the right side, we find a record of the system of difference equations in the matrix form

$$\mathbf{AX}=\mathbf{F} \tag{5}$$

The relationship between the structure of the matrix of the system (5) and the coefficients of the equations (3) is obvious enough. For a one-dimensional form of the equation (1) matrix **A** is *band tridiagonal*

$$\mathbf{A}=\begin{bmatrix} a_{11} & a_{12} & 0 & . & 0 & 0\\ a_{21} & a_{22} & a_{23} & . & 0 & 0\\ 0 & a_{32} & a_{33} & . & 0 & 0\\ . & . & . & . & . & .\\ 0 & 0 & 0 & . & a_{n-1,n-1} & a_{n-1,n}\\ 0 & 0 & 0 & . & a_{n,n-1} & a_{nn}\end{bmatrix}. \tag{6}$$

In the case of two-dimensional equations the matrix **A** has a *block-tridiagonal structure*

$$\mathbf{A}=\begin{bmatrix} \mathbf{T}_1 & \mathbf{D}_1 & 0 & . & 0 & 0\\ \mathbf{D}_1 & \mathbf{T}_2 & \mathbf{D}_2 & . & 0 & 0\\ 0 & \mathbf{D}_2 & \mathbf{T}_3 & . & 0 & 0\\ . & . & . & . & . & .\\ 0 & 0 & 0 & . & \mathbf{T}_{n-1} & \mathbf{D}_{n-1}\\ 0 & 0 & 0 & . & \mathbf{D}_{n-1} & \mathbf{T}_n\end{bmatrix}, \tag{7}$$

where $\mathbf{T_i}$ ($i=1,\ 2,\dots,\ n$) – square three-diagonal matrix, structures (6) and describing a two-dimensional grid line, $\mathbf{D_i}$ – diagonal matrix.

In the case of three-dimensional equation and numbering nodes on layers of grid we obtain block tridiagonal matrix structure (7), wherein the diagonal blocks are of the form (7) and describe the single layer grid.

2. Final element analysis (FEA)

Final element analysis (FEA) consists in solution domain partition Ω into set of nonintersecting subfields Ω^e – *finite elements* (FE). Within each FE desired solution is approximated by a piecewise continuous function, usually a polynomial. The coefficients of this polynomial are expressed through advance unknown meanings of the required function at certain points of FE, called *grid nodes* of finite elements. The unknown nodal parameters are found, using an integrated problem formulation. Let us consider the basic steps of solving boundary value problems by finite element analysis [6].

Solution region is divided into *finite elements* as a preliminary. Triangular or rectangular finite elements are commonly used for two-dimensional problems, tetrahedrons, parallelepipeds and straight triangular prisms - for three-dimensional problems. Automatic generation of *finite element grids* and dividing into finite elements is an actual and extensive area [7,8].

Approximating elements' function is determined. It is necessary to solve the problem of the approximate representation of the desired decision function through the meanings of the function at the nodes in each individual finite element. Lagrange finite elements are the most commonly used, using only the values of the function at the nodes. The approximated function is a polynomial interpolation of Lagrange in Lagrange elements. For Lagrange finite element meaning of function of decision $\varphi^{(e)}$ in arbitrary point of *e*-th of finite element is approximated by a polynomial

$$\varphi^{(e)} = \mathbf{A}^{(e)}\mathbf{R} + a_0,$$

where $\mathbf{A}^{(e)}$ – vector line polynomial coefficients; a_0 – constant term; $\mathbf{R} = (x, y, z)$ – vector of coordinates of the considered point of the finite element.

In order to determine vector $\mathbf{A}^{(e)}$ and a constant term a_0 the condition of continuity of the desired function in the element nodes is used. Substituting the coordinates of element nodes and unknown meanings of the function at the nodes, we obtain the equations' system

$$\mathbf{X}^{(e)}\mathbf{A}^{(e)\mathrm{T}} + \mathbf{A}_0 = \mathbf{\Phi}^{(e)}, \quad (8)$$

where $\mathbf{\Phi}^{(e)}$ – vector of nodal values of the function for *e*-th of finite element, $\mathbf{X}^{(e)}$ – matrix of coordinates of element nodes, for example, for two-dimensional triangular elements Sim-Plex

$$\mathbf{X}^{(e)} = \begin{bmatrix} x_i & y_i \\ x_j & y_j \\ x_k & y_k \end{bmatrix},$$

$i,\ j$ и k – number of nodes (nodes are numbered, starting with an arbitrary node, moving counter-clockwise); $\mathbf{A}^{(e)\mathrm{T}}$ – conjugate vector $\mathbf{A}^{(e)}$; $\mathbf{A}_0$ – vector whose elements are all equal to a_0.

Solving systems (8), let us find vector $\mathbf{A}^{(e)}$, it means expressing the coefficients of the coordinates of the element nodes and unknown values of the function at the nodes (vector $\mathbf{\Phi}^{(e)}$). Substituting vector $\mathbf{A}^{(e)}$, we receive

$$\varphi^{(e)} = \mathbf{N}^{(e)\mathrm{T}}\mathbf{\Phi}^{(e)},$$

where $\mathbf{N}^{(e)\mathrm{T}}$ – row vector, elements of which are called functions of forms of finite elements.

The vector of nodal meanings of the function is determined. Two main methods are used to solve this problem: a method based on the variational formulation of the problem, and Galerkin method [8].

Thus, the method of finite differences using regular grids is the easiest one to be implemented in a neural network basis. Cellular neural networks are perspective for solving the system-difference-equations approximating the differential equations in partial derivatives. More complex, but also very promising is the neural network implementation of the finite element method and an important part of this method - the problem of constructing an optimal finite difference grids.

When solving systems of equations that approximate the direct problems, the parameters of such neuro-networks are known in advance and must comply with the parameters of approximation. In the paper [9] such a network is called *formed neural networks*. Such networks can be called *networks of direct analogy* by analogy to models of direct analogy. It is necessary to respect the basic provisions of the theory of similarity when calculating parameters of such networks.

3. Solution of boundary value problems on neural networks

Solution of boundary value problems in the neural networks are built in accordance with the *general method of solving mathematical problems in neural network logical basis* [5,10], which comprises the following steps: a mathematical statement of the problem; geometric statement of the problem; neural network statement of the problem.

When solving systems of linear equations the realization is reduced to the implementation of instructions of the neural network structure defined by a mathematical formulation of the problem. Formed in such a way networks are called *formed neural networks*. Such networks can be called *networks of direct analogy* as well by analogy to models of direct analogy.

The structure of the neural networks for solving systems of linear algebraic equations is built on the basis of the chosen energy function (optimization of the functional or functional errors). The energy function must be selected in such a way that its minimum is reached on the exact solution of X* system of linear algebraic equations. Differentiation of the energy function makes it possible to convert the problem of minimization to the system of ordinary differential equations. Analog neural network with continuous presentation time should be described by the resulting system of ordinary differential equations; it means neural network is an analog circuit for solving the resulting method described a system of ordinary differential equations. It is necessary to replace the differential equations to difference ones to construct a network operating in discrete time.

Let us consider the analog neural networks for solving systems of differential equations of the form based on the model of *continuous Hopfield network* [10]. Every *i*-th neuron is described by the ordinary differential equation

$$c_i \frac{du_i}{dt} = -g_{ii}u_i + \sum_{\substack{j \\ j\neq i}} g_{ij} f(u_j) + I_i, \quad (9)$$

Where u_i – input (status) of *i*-th neuron, c_i – input capacitance of neuron, g_{ij} – elements of the matrix connections, or synaptic weight (g_{ij} –conductivity connecting the output of *j*-th neuron with input of *i*-th), $f(u_j)$ – activation function (continuous monotonically increasing linear function of the input u_j), I_i – input current (bias neuron) $g_{ii} = \rho_i^{-1} + \sum_j g_{ij}$, ρ_i – input impedance of *i*-th amplifier (for modern amplifiers can be taken $\rho_i = \infty$). Classically understood fully connected network, it means each neuron is associated with each other (in the expression (9) $i, j = 1,\ 2,\ \ldots,\ n$, where *n* – the number of neurons in the network).

Let us write the system (9) in a matrix form

$$\mathbf{C}\frac{d}{dt}\mathbf{U} = -\mathbf{D}\mathbf{U} + \mathbf{T} f(\mathbf{U}) + \mathbf{I}, \quad (10)$$

where $\mathbf{U}$ – vector network status, $\mathbf{D} = \mathrm{diag}(g_{11}, g_{22}, g_{33}, \ldots, g_{nn})$ – diagonal matrix, $\mathbf{T}$ – matrix of links of the outputs and inputs of neurons (

$T_{ii}=0, \quad T_{ij}=g_{ij}$), $f(\mathbf{U})$ – vector activation function of neurons, $\mathbf{I}$ – vector of external neurons' inputs (displacement), $\mathbf{C}$ – diagonal matrix of input neurons' capacitances.

The solution of differential equations of the form will be sought in the state class-asymptotic *asymptotically stable equilibrium neural network*, described by a system of ordinary differential equations, (10). Neural network parameters must be set so that the point of asymptotically stable equilibrium of the network coincides with the solution system.

Attention is necessary to be drawn to usage of *the theory of similarity*, which serves as the basis for the choice of the scale. Scaling is a mandatory step when under-preparation of the problem to the solution of an analog computer, the correct choice of scale does not only establishes the correspondence between the mathematical model and analog circuitry, but makes it possible to reduce the modeling error as well.

In the paper [10] Hopfield network is proposed and studied for solving linear algebraic equations of a standard form. A linear function of activation is used $f(u_j)=-u_j$, $\mathbf{C}=\mathbf{E}$, where $\mathbf{E}$ – identity matrix. Neural network parameters are selected in the following way:

$$D_{ii}=a_{ii}^2, \quad T_{ij}=\sum_{k=1}^{n} a_{ki}a_{kj}, T_{ii}=0, \quad I_i=\sum_{k=1}^{n} a_{ki}F_i,$$
$$i,\ j=1,\ 2,\ \ldots,\ n. \qquad (11)$$

Given a linear activation function, and (11), the system (10) takes the form

$$\frac{d}{dt}\mathbf{U}=-\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{U}+\mathbf{A}^{\mathrm{T}}\mathbf{F}. \qquad (12)$$

From the theory of solving systems of linear algebraic equations on analog computation-inflammatory-machines it is known that the system

$$\frac{d}{dt}\mathbf{U}=-\mathbf{A}\mathbf{U}+\mathbf{F}$$

has a sustainable solution, if the matrix **A** is a positive definite. Matrix $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ is symmetric definite positive, which ensures stability of the solution.

In the paper [11] optimization functionality (errors functionality) is used traditionally in the theory of neural networks.

$$H(\mathbf{U})=\frac{1}{2}(\mathbf{R},\mathbf{R})=\frac{1}{2}\mathbf{R}^{\mathrm{T}}\mathbf{R},$$

Where $\mathbf{R}=\mathbf{F}-\mathbf{A}\mathbf{U}$ – nullity vector.

4. Concerning research's margin of errors

The proposed mathematical approach for solving airport security's aviation management tasks cannot be attributed to the class of simple tasks. At the mathematical formalization of the researcher is faced with many challenges. The key ones are the following:
- Considerable uncertainty in the linguistic description of the subject area,
- Methodological difficulties in finding adequate mathematical apparatus for formalization,
- Multicriteriality of the solved problem, defined by a high order matrix-criteria optimization of management, whose elements correspond to a plurality of types of object's technical protection,
- The inevitable inadequacy of any of the proposed models, including the format of boundary value problem resulting from the ambiguity of the conceptual apparatus,
- Difficulties in setting the boundary and initial conditions associated with indeterminacy of source information,
- Practically insurmountable complexity of the study of the dynamics of control processes aviation security, since the introduction of the time factor into the model leads to uncontrolled growth of the complexity of the mathematical apparatus, or transforms the task into the class of NP-complex tasks without any solution.

It follows that the proposed approach concerning aviation security management does not provide accurate, even relatively accurate, solution.

However, the terms of the practical usage of the proposed approach to aviation security management do not require an exact solution taking into account the following circumstances:
- Absolute security does not exist; it can be only strived for, what is meant here is an acceptable level of aviation security,
- The quality, the quantitative value of which is determined by their qualitative (expert) methods, initially giving an approximation of (subjective) assessment is a controlled object protection field's parameter,
- An actuator in the security management system is the ergatic, where the decision maker plays the key role.

Thus, the question of the margin of errors of the proposed approach is practically removed. It should be noted, however, that in this case aviation security control procedures should be complemented by risk assessment procedures occurrence negative events, which should be adequately integrated into the overall control scheme.

Conclusion

1. An approach to the modeling of management processes of aviation airport security in civil aviation is proposed, based on a usage of parallel computing processes and neuro-computers and presentation of the field of object protection in the format of a boundary value problem.
2. Practical application of the method is most effectively in supporting decision-making systems, where the leading role is given to the decision-makers.
3. The system of decision support for the management of aviation security should include risk assessment subsystem of occurrence of negative events.

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