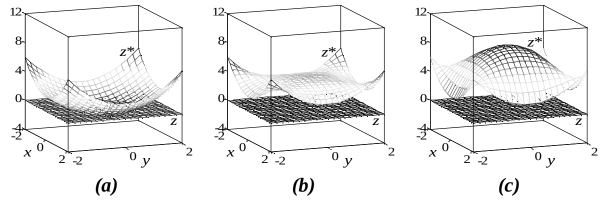

Fig.2. *The original surface z and the similar to it surface z* satisfying constraints (11) and (12) for* **(a)** *ΔV=600;* **(b)** *ΔV=1600; and* **(c)** *ΔV=2400.*

5. Conclusion

This work defined constrained similarity between surfaces via minimizing the L^2 norm of the gradient of the difference between the surfaces. An exact general solution was obtained for discreized surfaces under linear constraints. The results agree with what is expected from similarity of surfaces under constraints.

6. Appendix

In this appendix a MATLAB code for solving Example 1(c) is presented. The variables `A_`, `c_`, and `L_` are used for $\overline{A}$, $\overline{c}$, and $\overline{L}$, while `zs`, `us`, `dz`, and `du` are used for z^*, u^*, Δz, and Δu, respectively. The variable `lambda` is used for λ. To define the needed vectors and matrices first the corresponding vectors and matrices composed of zeros and having the required size are defined. The obtained graph needs to be rotated to be seen from aside.

```
function main

 Nx=21; Ny=21; xa=-2; ya=-2; h=0.2;
 M=2*Nx+2*Ny-4; N=Nx*Ny;

 x=zeros(Nx,1); y=zeros(Ny,1);
 for k=1:Nx
  x(k)=xa+h*(k-1);
 end
 for l=1:Ny
  y(l)=ya+h*(l-1);
 end

 z=zeros(Nx,Ny); u=zeros(N,1);
 dz=zeros(Nx,Ny); du=zeros(N,1); i=1;
 for l=1:Ny
  for k=1:Nx
   z(k,l)=-x(k)*x(k)+y(l)*y(l); u(i)=z(k,l);
   dz(k,l)=x(k)*x(k)+y(l)*y(l); du(i)=dz(k,l);
   i=i+1;
  end
 end

 A=zeros(M,N); c=zeros(M,1); j=1;
 for k=1:Nx
  i=k; A(j,i)=1; c(j)=u(i)+du(i); j=j+1;
 end
 for l=2:Ny-1
  i=Nx*(l-1)+1; A(j,i)=1; c(j)=u(i)+du(i);
  j=j+1;
  i=Nx*(l-1)+Nx; A(j,i)=1; c(j)=u(i)+du(i);
  j=j+1;
 end
 for k=1:Nx
  i=Nx*(Ny-1)+k; A(j,i)=1; c(j)=u(i)+du(i);
  j=j+1;
 end

 A_=zeros(N,N); c_=zeros(N,1);
 for j=1:M
  c_(j)=c(j);
  for i=1:N
   A_(j,i)=A(j,i);
  end
 end

 L_=zeros(N,N);
 L_(1,1)=-2; L_(Nx,Nx)=-2;
 L_(N-Nx+1,N-Nx+1)=-2; L_(N,N)=-2;
 for n=2:(Nx-1)
  L_(n,n)=-3; L_(N-Nx+n,N-Nx+n)=-3;
  L_(n+1,n)=1; L_(N-Nx+n+1,N-Nx+n)=1;
  L_(n-1,n)=1; L_(N-Nx+n-1,N-Nx+n)=1;
  L_(n+Nx,n)=1; L_(N-Nx+n-Nx,N-Nx+n)=1;
 end
 L_(2,1)=1; L_(Nx-1,Nx)=1;
 L_(1+Nx,1)=1; L_(Nx+Nx,Nx)=1;
 L_(N-Nx+2,N-Nx+1)=1; L_(N-Nx+1-Nx,N-Nx+1)=1;
 L_(N-1,N)=1; L_(N-Nx,N)=1;
 for n=Nx+1:N-Nx
  L_(n,n)=-4; L_(n+1,n)=1; L_(n-1,n)=1;
  L_(n+Nx,n)=1; L_(n-Nx,n)=1;
 end
 for n=1:Ny-2
  L_(Nx*n+1,Nx*n+1)=-3;
  L_(Nx*n+Nx,Nx*n+Nx)=-3;
  L_(Nx*n,Nx*n+1)=0; L_(Nx*n+Nx+1,Nx*n+Nx)=0;
 end

 H=inv(L_+A_); d=A_*u-c_;

 lambda=(A*H*A')\(A*u-c-A*H*d)*2;
 us=u-H*(A'*lambda/2+d);

 i=1;

 for l=1:Ny
  for k=1:Nx
   zs(k,l)=us(i);
   i=i+1;
  end
 end

 hold on; surface(x,y,z'); surface(x,y,zs');

end
```

7. References

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