DEFINING THE WORK AREA OF A ROBOT WITH PARALLEL KINEMATICS WITH 3 DEGREES OF FREEDOM

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Abstract: *This article proposes an approach to generate a work area of a three-axis delta robot in programming environment of MATLAB.*

Keywords: *Delta robot, work area, MATLAB*

1. Introduction

Practice has shown that in some cases the use of traditional robot systems is ineffective in solving important practical problems. Such a problem often occurs in aerospace, aviation, healthcare, and technology tasks for processing of parts with complex geometry. Examples of such surfaces can be vanes of gas turbine engines for aircraft turbines and wind farms. The solution to these problems is the use of parallel robots. The most important advantages of this type of robots over the traditional industrial robots are accuracy and payload.

Delta robots are parallel mechanisms which are characterized by high dynamics of the end effector. This comes along with the requirements for minimum weight of the moving components which are of sufficient size, including planetary gears, stiffness of the individual components, especially a minimum clearance in couplings.

The Movement of the robot in the working area can be limited by several factors such as design constraints of passive kinematic couples, the limitations given by the actuators, cohesive restrictions derived from the structural elements of the robot, as well as the points or areas of singularity, which may to divide the working area of the various components. An important parameter during the study of the working area of the robot is the number of degrees of freedom. Usually the number of drive motors correspond to the number of degrees of freedom. In this case, three electric motors and three pairs of parallel passive couples building the robot provides all three degrees of freedom, that meet for positioning the output unit.

The use of three such parallelograms restrain completely the orientation of the movable platform which remains only with three purely translational degrees of freedom. The main advantage of these robots are exactly high positioning speeds [1][2].

Figure 1 shows the CAD model of the robot with parallel structure with three degrees of freedom created by the leading author in this work. The structure of a delta robot with three degrees of freedom is constituted by: a base plate -1, -2 movable platform. The base plate is connected to the movable platform through three kinematic couples. From a structural point of view all three kinematic couples are identical. Each pair is driven by an electric motor -3. The drive system transfers the movement to shoulder -5, through clutch -4. Shoulder -5 brings movement to the kinematic couples -7 via self-aligning spherical bearing -6. Kinematic couples -7 transmit the movement of the movable platform -9 via Self-aligning spherical bearing -8.

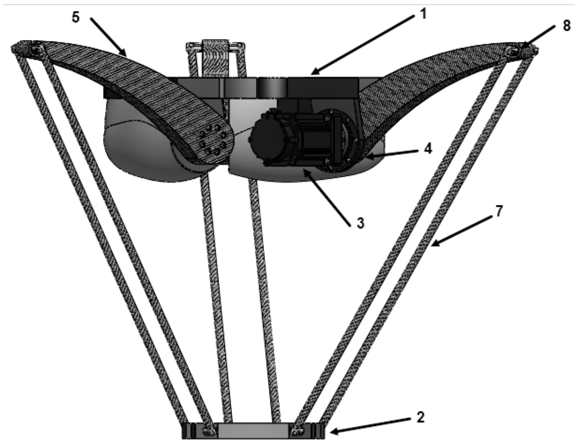


Fig. 1

2. Determination of geometrical characteristics

The reference frame {R} is chosen as shown in Figure 1, axis Z pointing upwards and X perpendicular to the axis of motor 1. Because of delta robot kinematic couples are symmetrical and each arm may be treated separately the model of the shoulder is shown in Figure 2. The index i (i=1,2,3) is used to identify the three arms. Each arm is separated by an angle of 120°. For each arm, a corresponding frame is chosen, located at the same place as the reference frame {R} but rotated α1 = 0°, α2 = 120° and α3 = 240° for the three arms respectively. The different frames {Ri} can be described by a rotation matrix around z axis of the reference frame {R}. The rotation matrix is given by:

$$ {}_{i}^{R}R_{z}=\begin{pmatrix} \cos a_i & -\sin a_i & 0 \\ \sin a_i & \cos a_i & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

The assumption above that the three kinematic couples can be connected in one point, allows us to consider the distance from the reference frame {R} to one of the motors as R=Ra - Rb. Thus, the kinematic couples are connected in one point P and each angle of each of the three upper arms has its initial value, 0°, parallel with the x axis of the frame {Ri}. The angle θi value is then increasing when the upper arm is moving downwards and decreasing when the upper arm is moving upwards. The parameter la represents the length of the upper arm and the parameter lB represents the length of the forearm. With the information, above, a direct and an inverse geometric model of the Delta-3 Robot can be established. These two models are also called the forward and the inverse kinematics [2].

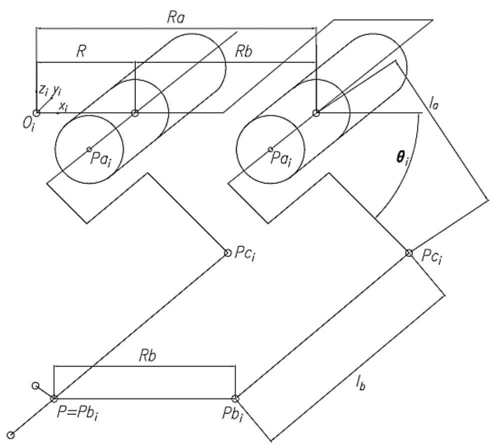


Fig. 2

3. Generating a work area of the delta robot with three degrees freedom

Calculations and visualizations approach is implemented in the software environment of MATLAB. An approach is presented in the block diagram of fig. 3 [3] [4].

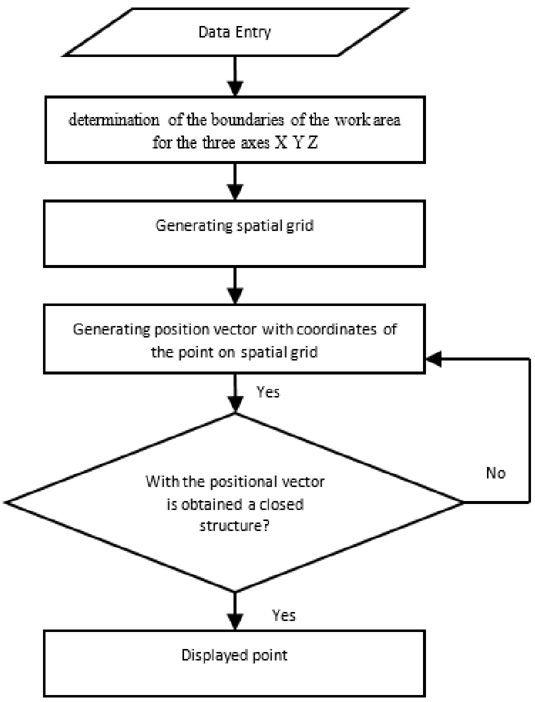


Fig. 3

Code represent the mathematical description of the geometry of the robot positioned consistently in many points with coordinates taken from a spatial grid. For each item is calculated by inverse kinematics angles locked between the units of the robot. The program works in the following sequence:

A. Admission at building the model

Unlike the more common serial mechanisms, all three actuators of the parallel mechanism work together in order to obtain a translation in a Cartesian coordinate system with three degrees of freedom. If necessary, the fourth axis of rotation TCP is added to the fourth motor to the base plate and be able to control the, θr degree of freedom.

Because delta-3 robot is a closed system Figure 1 is more complicated to calculate kinematics to simplify and reduce the number of parameters have been made these simplifications. The movable plate always remains parallel to the base plate and its orientation around the axis is perpendicular to the base plate and is constantly zero. So, this type of parallelogram joints (arms) can be replaced with simple bars without changing the kinematic behavior of the robot.

The joints between the base plate and shoulders as well as movable plate and kinematic couples are identical (fig. 1)[6].

B. Define the geometric dimensions of the robot

In the first step of the approach, introduces the following geometric dimensions of the robot:

ra - distance from the shoulder to the center of the base plate (fig.4);

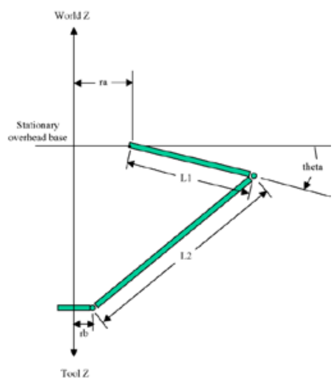
l_1 - length of the arm (fig.4);

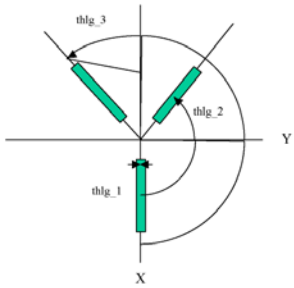
l_2 - length of kinematic couples (fig.4);

rb - the distance from the kinematic couples to the middle of the movable platform (fig.4);

$thlg$ - distance between shoulders (fig.5);

Fig. 4 ***Fig. 5***





C. Inverse Kinematics

The inverse kinematics give multiple solutions of the θ vector with the three revolute joint angles that all satisfies the specific travel plate position. This may cause problems because the system must be able to choose one of them. The criteria upon which to base a decision vary, but a very reasonable choice would be the closest solution, which is the solution where the manipulator moves the links as little as possible.

For the 3-DOF parallel manipulator with the system structure as in Figure 1, each kinematic pair can satisfy the same TCP with two different approaches. Together the three arms of the 3-DOF parallel manipulator result in eight different combinations of the θ vector for a single goal, see fig. 6.

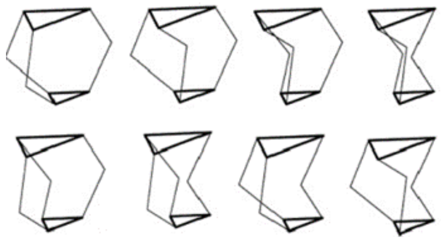


Fig. 6

The inverse kinematic model is obtained by using the three closure equations, constraints, of the kinematic chains:

$$\left\| Pc_i Pb_i \right\|_2^2 - l_B^2 = 0 \rightarrow i = 1,2,3$$

Then for each arm the angle is chosen so it is inside the angle constraints for the robot. This will convey to the first configuration, upper left corner in fig. 6.

D. Velocity kinematics

The Jacobian matrix specifies a mapping from velocities in joint space to velocities in Cartesian space. To calculate the Jacobian matrix for a Delta-3 robot one can use a set of constraint equations linking the Cartesian space variables to the joint space variables. The three constraints equations for the Delta-3 robot can be chosen as:

$$\left\| Pc_i Pb_i \right\|_2^2 - l_B^2 = 0 \rightarrow i = 1,2,3$$

assuming that the length of the forearms is constant. Let s_i denote the vector Pc_iPb_i then can the Euclidean norm be written as $s_i^t s_i$. Consider Figure 2 for the following calculations. The vector s_i can be written as:

$$s_i = O_i S_{b_i} - \left(O_i P_{A_i} + P_{A_i} P_{C_i} \right) = \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} - {}_i^R R_z \left(\begin{bmatrix} R \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} l_A \cos(\theta_i) \\ 0 \\ l_A \sin(\theta_i) \end{bmatrix} \right) \rightarrow i = 1,2,3$$

$$s_i^T s_i - l_B^2 = 0 \rightarrow i = 1,2,3$$
$$s_i^T \dot{s}_i + \dot{s}_i^T s_i = 0 \rightarrow i = 1,2,3$$
$$s_i^T \dot{s}_i = 0 \rightarrow i = 1,2,3$$

$$\dot{s}_i = \begin{bmatrix} \dot{x}_n \\ \dot{y}_n \\ \dot{z}_n \end{bmatrix} - {}_i^R R_z \left(\begin{bmatrix} R \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -l_A \sin(\theta_i) \\ 0 \\ l_A \cos(\theta_i) \end{bmatrix} \right) \dot{\theta}_i = \dot{X}_n - b_i \dot{\theta}_i \rightarrow i = 1,2,3$$

$$b_i = {}_i^R R_z \begin{bmatrix} -l_A \sin(\theta_i) \\ 0 \\ l_A \cos(\theta_i) \end{bmatrix} \rightarrow i = 1,2,3$$

For one pair, kinematic equation takes the form:

$$s_i^T = \begin{bmatrix} \dot{x}_n \\ \dot{y}_n \\ \dot{z}_n \end{bmatrix} - s_i^T b_i \dot{\theta}_i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow i = 1,2,3$$

Jacobian matrix for a Delta-3 robot for all three kinematic couples is:

$$\begin{bmatrix} s_1^T \\ s_2^T \\ s_3^T \end{bmatrix} \dot{X}_n - \begin{pmatrix} s_1^T b_1 & 0 & 0 \\ 0 & s_2^T b_2 & 0 \\ 0 & 0 & s_3^T b_3 \end{pmatrix} \theta = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{X}_n = J \dot{\theta}$$

$$J = \begin{bmatrix} s_1^T \\ s_2^T \\ s_3^T \end{bmatrix}^{-1} \begin{pmatrix} s_1^T b_1 & 0 & 0 \\ 0 & s_2^T b_2 & 0 \\ 0 & 0 & s_3^T b_3 \end{pmatrix}$$

The Jacobian matrix J is not only depending of θ, but also a function of the TCP position X_n, which can be calculated with the forward kinematic model of the Delta-3 robot [2][5][6].

E. Calculation of accelerations

Kinematics of accelerations shows accelerations of joints in Cartesian coordinate system.

$$\begin{bmatrix} s_1^T \\ s_2^T \\ s_3^T \end{bmatrix} \ddot{X}_n + \begin{bmatrix} s_1^T \\ s_2^T \\ s_3^T \end{bmatrix} \dot{X}_n - \left(\begin{pmatrix} s_1^T b_1 & 0 & 0 \\ 0 & s_2^T b_2 & 0 \\ 0 & 0 & s_3^T b_3 \end{pmatrix} \ddot{\theta} + \begin{pmatrix} s_1^T b_1 + s_1^T \dot{b}_1 & 0 & 0 \\ 0 & s_2^T b_2 + s_2^T \dot{b}_2 & 0 \\ 0 & 0 & s_3^T b_3 + s_3^T \dot{b}_3 \end{pmatrix} \dot{\theta} \right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\ddot{X}_n = \begin{bmatrix} s_1^T \\ s_2^T \\ s_3^T \end{bmatrix}^{-1} - \begin{bmatrix} \dot{s}_1^T \\ \dot{s}_2^T \\ \dot{s}_3^T \end{bmatrix} J + \begin{pmatrix} \dot{s}_1^T b_1 + s_1^T \dot{b}_1 & 0 & 0 \\ 0 & \dot{s}_2^T b_2 + s_2^T \dot{b}_2 & 0 \\ 0 & 0 & \dot{s}_3^T b_3 + s_3^T \dot{b}_3 \end{pmatrix} \dot{\theta} + J \ddot{\theta}$$

The derivative of the Jacobian matrix [$\dot{J}$] can be identified as the term multiplying $\dot{\theta}$. And finally, the relationship between the Cartesian acceleration and the acceleration in joint space can be expressed as: $\ddot{X}_n = \dot{J}\dot{\theta} + J\ddot{\theta}$

4. Results

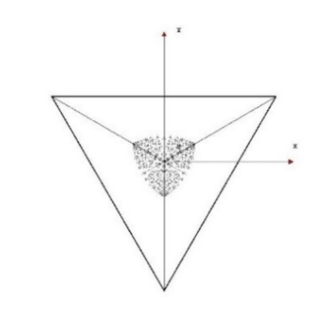
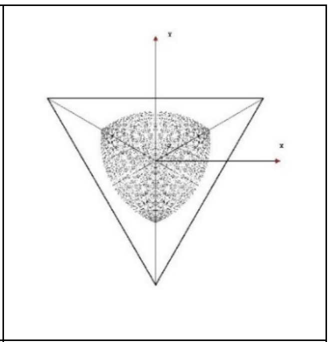
In order to graphically visualize the results, it is convenient to use horizontal sections of volume work area of the robot (Table 1). Points with a lighter color represent the coordinates of the position vectors which is the obtained closed kinematic structure. The dark regions present coordinates in which the robot can be positioned.

5. Conclusion

This is a suggested an approach for generating work zone of three axis delta robot programming in MATLAB environment considering the limiting criteria for positioning.

Through the proposed approach, software can be created that can easily explore the multiple variations of kinematic size of the robot, achieved with their work areas.

Table 1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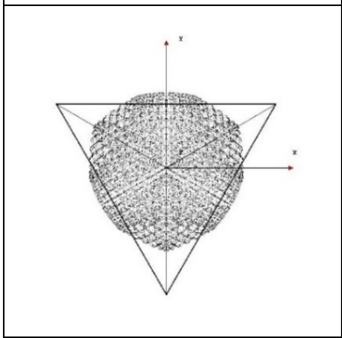	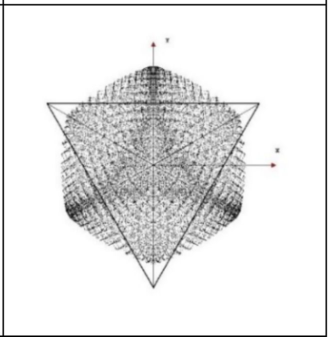
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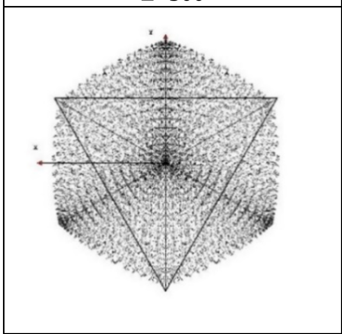	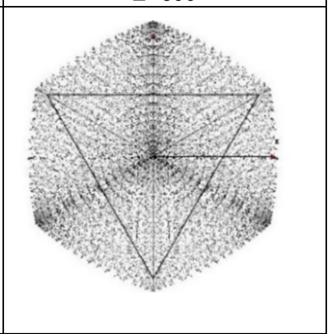
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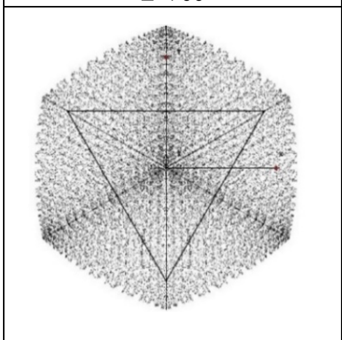	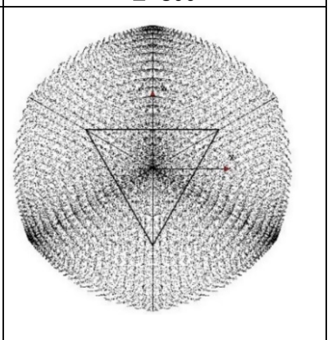
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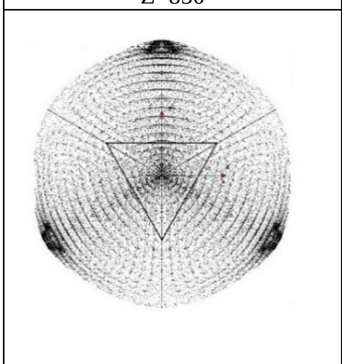	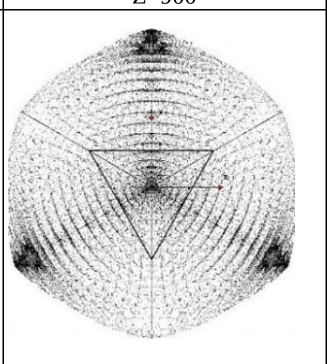
Z=950	Z=1000

On Fig.7 illustrates a view of the working zone in the ZX coordinate, and Fig.8 illustrates a view in the ZY.

As regards the concept of the parallel mechanism is necessary to take into consideration possibility of existence of a singularity, as in reaching it will lead to loss of power or loss of one of the degrees of freedom (DOF). From the standpoint of practical use of the device, these positions are undesirable. Is therefore necessary to consider all positions where possible robot to gets into singularity, so that limiting criteria can be set for positioning. The results are presented in fig. 9, fig, 10 and fig. 11.

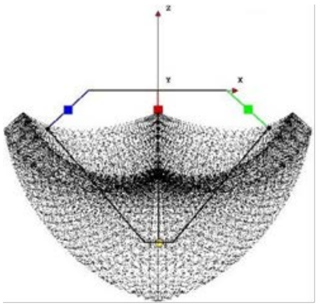

Fig. 7

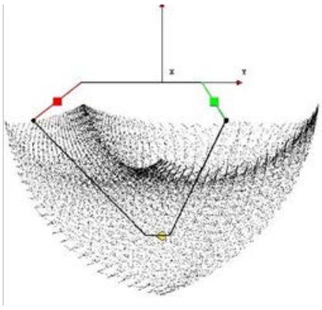

Fig. 8

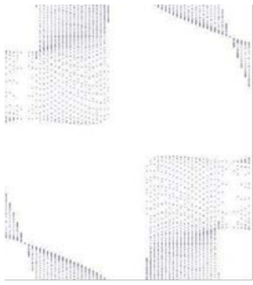

Fig. 9

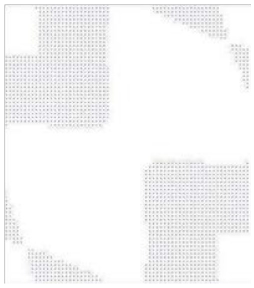

Fig. 10

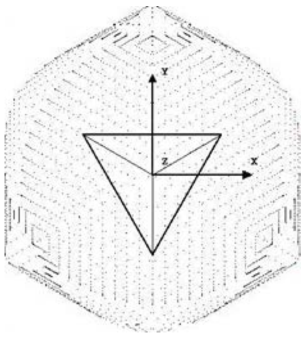

Fig. 11

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