BALANCE CONTROL OF SEGWAY ROBOTS USING ADAPTIVE-ROBUST CONTROLLER

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Abstract: *Due to its compatibility and functionality, segways have been widely used in many countries. It was first introduced in December 2001.Yet, segway robots are faced with problems such as friction and external disturbances. Therefore, some controllers are designed to overcome with these problems. In previous studies, traditional controllers are used to balance a two-wheeled segway robot. The aim of this study is to minimize the trajectory tracking error. Due to external disturbances, such as wind, force and torque, robot parameters cannot be calculated exactly. Hence, the parameters of the robot are assumed to be unknown. In such situations, adaptive and robust controllers give better results. Adaptive and robust control laws were examined and adaptive-robust system was designed for the segway robot. Then Lyapunov function was defined and this adaptive-robust controller was derived from the Lyapunov function. And this control system applied to a two-wheeled segway robot model.*

Keywords: LYAPUNOV THEORY, ADAPTIVE CONTROL, ROBUST CONTROL

1. Introduction

In this study adaptive-robust and fuzzy logic controllers are developed for balancing Segway robot. There are lots of studies about balancing Segway robots.

Grepl [1], deals with the modelling and control of balanced wheeled autonomous mobile robot. In his study SimMechanics is used for modelling mobile robot. LQR and feedback linearization controllers are compared. Kim and Jung [2], used fuzzy logic control system for two-wheeled mobile robot. PID and fuzzy logic controllers are used to control both position and balance of two-wheel mobile robot. Performance of the PID and fuzzy logic controllers are compared through extensive experimental studies. Sangfeel, Eunji, KyungSik and ByungSeop [3], presented the fuzzy logic controller for inverted pendulum type mobile robot. They designed conventional fuzy logic controller. Chiu and Peng [4], designed fuzzy logic control system for two-wheel transporter control system. In their study, experimental results show that the fuzzy logic controller can control the whole system very well. Xu, Guo, and Lee [5], presented a Takagi–Sugeno-type fuzzy logic controller on a two-wheeled mobile robot. Their model consists of two wheels in parallel and an inverse pendulum. Finally, the results shows that fuzzy logic controller shows superior performance.

It is not easy to control inverted pendulum type systems because this type of system is a typical complex nonlinear systems. Kwak and Choi [6], designed two fuzzy logic control systems for the control of a Segway mobile robot. First they introduce the Segway robot and then analyze the system. Then they propose the design of two fuzzy logic control system for the position and balance control of the Segway mobile robot. A software fuzzy logic controller was implemented using a PIC microcontroller in Reid's [7] project. Hadiya, Rai, Sharma, More [8], describes the design and construct a fully functional two wheeled balancing vehicle. In this paper, the vehicle is designed for a single person. And the vehicle is driven by forward and backwards movements. Goher, Tokhi and Siddique [9], designed a two wheeled robotic vehicle with virtual payload. In this paper, two types of control techniques are developed and implemented on the system. They are proportional-derivative control and fuzzy logic control systems. Also an external disturbance force is applied to the road. Finally, the results are analyzed. Grasser, D'arrıgo, Colombı and Rufer [10], had built a prototype of a revolutionary two-wheeled vehicle. Two decoupled state space controllers are used to control the system.

In this paper, adaptive-robust and fuzzy logic controllers are developed for balancing Segway robot. First adaptive-robust control system is designed for the Segway model. The Segway is based on the principle of inverted pendulum that will keep an angle of Zero degrees with vertical at all times. Fuzzy logic control system is developed to keep the system in equilibrium. Then we introduce the two wheeled Segway robot and applied control laws to this model. Finally, results show that using adaptive-robust and fuzzy logic controller together, system gives better results.

2. Derivation of the Control Law

In the absence of friction or other disturbances, Spong writes the dynamic model of an n-link manipulator as [11];

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau \tag{2.1}$$

where q denotes generalized coordinates, τ is the n-dimensional vector of applied torques (or forces), $M(q)$ is the nxn symmetric positive definite inertia matrix, $C(q,\dot{q})\dot{q}$ is the n-dimensional vector of centripetal and Coriolis terms and G(q) is the n-dimensional vector of gravitational terms. Equation (2.1) can also be expressed in the following form.

$$Y(q,\dot{q},\ddot{q})\pi = \tau \tag{2.2}$$

where π is a p-dimensional vector of robot parameters and Y is an nxp matrix which is a function of joint position, velocity and acceleration. For any specific trajectory, the desired position, velocity and acceleration vectors are q_d, $\dot{q}_d$ and $\ddot{q}_d$. The measured actual position and velocity errors are $\tilde{q} = q - q_d$, and $\dot{\tilde{q}} = \dot{q} - \dot{q}_d$. Using the above information, the corrected desired velocity and acceleration vectors for nonlinearities and decoupling effects are proposed as:

$$\dot{q}_r = \dot{q}_d - \Lambda\tilde{q}\ ; \qquad \ddot{q}_r = \ddot{q}_d - \Lambda\dot{\tilde{q}} \tag{2.3}$$

Then, σ is given as [11];

$$\sigma = \dot{q} - \dot{q}_r = \dot{\tilde{q}} + \Lambda\tilde{q} \tag{2.4}$$

where Λ is a positive definite matrix. Then the following nominal control law is considered:

$$\begin{aligned}\tau_0 &= M_0(q)\ddot{q}_r + C_0(q,\dot{q})\dot{q}_r + G_0(q) - K\sigma \\ &= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\,\pi_0 - K\sigma\end{aligned} \tag{2.5}$$

The control input τ can be defined in terms of the nominal control vector τ_0

$$\begin{aligned}\tau &= \tau_0 + Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\,u = M_0(q)\ddot{q}_r + C_0(q,\dot{q})\dot{q}_r + G_0(q) - K\sigma \\ &= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\,(\pi_0 + u) - K\sigma\end{aligned} \tag{2.6}$$

where $\pi_0 \in R^p$ represents the nominal parameters in dynamic model and $K\sigma$ is the vector of PD action.

$$\tilde{\pi} = (\pi_0 - \pi) \leq \rho \tag{2.7}$$

where $\rho \in R^p$, $\delta \in R$ are the upper uncertainty bound on the parametric uncertainty. Let us define the control input u

Theorem 1: [11]

$$u = \begin{cases} -\hat{\rho}\dfrac{Y^T\sigma}{\left\|Y^T\sigma\right\|} & \text{ve} \quad \left\|Y^T\sigma\right\| > \varepsilon \\ -\hat{\rho}^2\dfrac{Y^T\sigma}{\varepsilon} & \text{ve} \quad \left\|Y^T\sigma\right\| \leq \varepsilon \end{cases} \tag{2.8}$$

The Lyapunov function candidate is defined as;[11]

$$V = \frac{1}{2}\sigma^T M(q)\sigma + \frac{1}{2}\tilde{q}^T \Lambda^T K \tilde{q} \tag{2.9}$$

$$V \geq 0$$

Derrivative of the Lyapunov function is:

$$\begin{aligned} \dot{V} &= -\dot{\tilde{q}}^T K \dot{\tilde{q}} - \tilde{q}^T \Lambda^T K \Lambda \tilde{q} + Y^T\theta(\tilde{\pi} + u) \\ &= -x^T Q x + Y^T\theta(\tilde{\pi} + u) \end{aligned} \tag{2.10}$$

Where $x^T = [\tilde{q}^T, \dot{\tilde{q}}^T]$ and $Q = \mathrm{diag}(\Lambda^T K \Lambda, K)$ the rest of the proof is given in [11].

$$\dot{V} \leq 0 \quad \text{for } \|x\| \rangle w \quad \text{where} \tag{2.11}$$

$$w^2 = \varepsilon\rho / 2\lambda_{min}(Q) \tag{2.12}$$

Where $\lambda_{min}(Q)$ denotes the minimum eigenvalue of Q. The argument proceeds as follows. Examining the second term in (2.10), we see that if then;[11]

$$\begin{aligned} Y^T\sigma(\tilde{\pi} + u) &= Y^T\sigma\left(\tilde{\pi} - \rho\frac{Y^T\sigma}{\left\|Y^T\sigma\right\|}\right) \\ &\leq \left\|Y^T\sigma\right\|\left(\left\|\tilde{\pi}\right\| - \rho\right)\langle 0 \end{aligned} \tag{2.13}$$

from the Cauchy-Schwartz inequality $\left\|Y^T\sigma\right\|\rangle\varepsilon$ and our assumption on $\|\pi\|$. If $\left\|Y^T\sigma\right\| \leq \varepsilon$ we have

$$\begin{aligned} Y^T\sigma(\tilde{\pi} + u) &\leq Y^T\sigma\left(\rho\frac{Y^T\sigma}{\left\|Y^T\sigma\right\|} + u\right) \\ &\leq Y^T\sigma\left(\rho\frac{Y^T\sigma}{\left\|Y^T\sigma\right\|} - \frac{\rho}{\varepsilon}Y^T\sigma\right) \end{aligned} \tag{2.14}$$

This last term achieves a maximum value of $\varepsilon\rho/2$ when $\left\|Y^T\sigma\right\| = \varepsilon/2$. Thus we have that

$$\dot{V} \leq -x^T Q x + \varepsilon\rho/2 \tag{2.15}$$

To complete the proof, it suffices to notice the following. With class-K functions γ_1 (.) and γ_2 (.) such that

$$\gamma_1(\|x\|)I \leq M(q) \leq \gamma_2(\|x\|)I \tag{2.16}$$

it can be shown that there exist class-K functions α_1 (.) and α_2 (.) such that

$$\alpha_1(\|x\|) \leq V \leq \alpha_2(\|x\|) \tag{2.17}$$

Equation (2.15) shows that:

$$\dot{V} \leq -\alpha_3\|x\|^2 + \varepsilon\rho/2 \tag{2.18}$$

3. Fuzzy Logic Controller

With fuzzy logic, like small, medium and large vague linguistic expressions can be expressed as by membership functions. These membership functions are triangular, trapezoidal or bell curved shape. (Fig. 1). They take the values between [0,1].

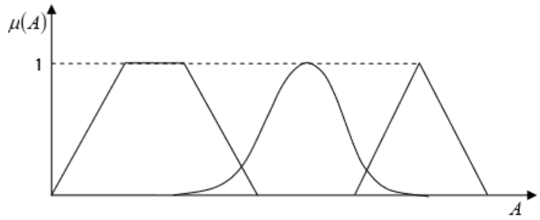

Fig. 1 *Different shapes of membership functions* [12]

Fuzzification, Rule Evaluation and Defuzzification are the steps of the fuzzy logic control. In the first stage, membership functions are defined for the variables. Thus, certain values are converted to fuzzy values. The second stage is the rule evaluation. The rules have been prepared based on the knowledge of the system. And the output of the system is decided by the input of the system.

Fuzzy Logic Controller has two inputs and one output. These are error of theta, it's derivative and output is the control force respectively. Linguistic variables which implies inputs and outputs have been classified as: NB, NS, Z, PS, PB. Inputs and outputs are all normalized in the interval of [0, 1] as shown in Fig.2

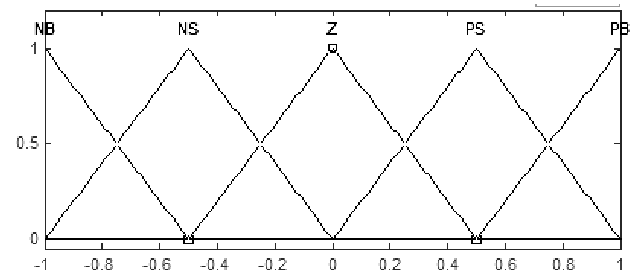

Fig. 2 *Membership functions of inputs (et, det) and output (u)*

The linguistic labels used to describe the Fuzzy sets were "Negative Big" (NB), "Negative Small" (NS), "Zero" (Z), "Positive Small" (PS), "Positive Big" (PB). Rules are written in a rule base look-up table which is shown in Table 1

Table 1: *Decision Table (et, det, u)*

etheta\detheta	NB	NS	Z	PS	PB
NB	NB	NB	NS	NS	Z
NS	NB	NS	NS	Z	PS
Z	NS	NS	Z	PS	PS
PS	NS	Z	PS	PS	PB
PB	Z	PS	PS	PB	PB

4. Equations of Motion

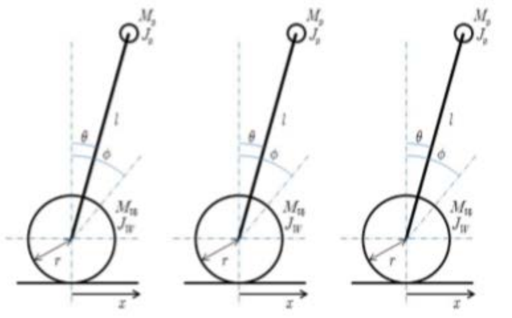

Fig. 3 *Schematics of Segway type mobile robot. [6]*

$$M_w \ddot{x} = H + H_w$$
$$J_w \ddot{\phi} = -rH_w + \tau \qquad (4.1)$$

The rotational angle of the wheel and the displacement of the robot have the following relationship:[6]

$$r\ddot{\phi} = \ddot{x} \qquad (4.2)$$

From Eqs. (4.1) and (4.2), the dynamic equations of the Segway robot are; [6]

$$(M_w + M_p + \frac{J_w}{r^2})\ddot{x} + M_p l \cos\theta\ddot{\theta} - M_p l \sin\theta\dot{\theta}^2 = \frac{\tau}{r} \qquad (4.3)$$

$$(J_p + M_p l^2)\ddot{\theta} + M_p l\ddot{x}\cos\theta - M_p g l \sin\theta = -\tau \qquad (4.4)$$

where M_w is mass of wheel, J_w is the inertia of wheel, θ: Angle of pole, x is displacement of the robot, r: radius of wheel, M_p :Mass of center of gravity of pole, J_p Moment of inertia of center of gravity of pole and ϕ: Rotational angle of wheel.

Also the torques are represented in a matrix form like in (4.5)

$$\tau = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} G_{11} \\ G_{21} \end{bmatrix}\begin{bmatrix} x \\ \theta \end{bmatrix} \qquad (4.5)$$

This new control law applied to two wheeled segway robot. Simulations have been done by using control law (2.8). We simulate the position and balance control of the Segway mobile robot using adaptive-robust and fuzzy logic controller together. The results of the angle of the pole, the angle error of the pole, the displacement, the change in the displacement are shown in Figures 4, 5, 6 and 7 respectively.

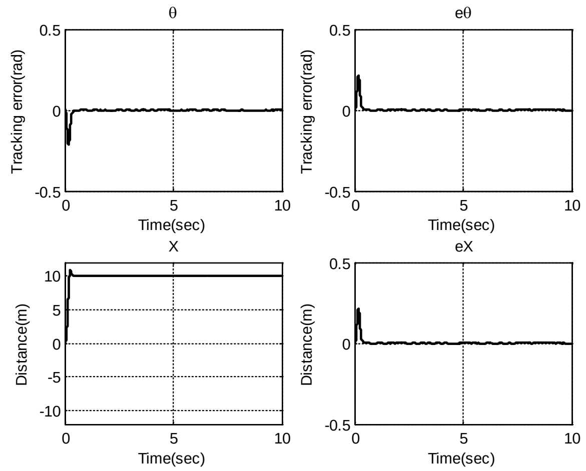

Fig. 4 *Response using the adaptive-robust and fuzzy logic control law when Λ=diag(20 20), K=diag(20 20). The target position is 10 [m].*

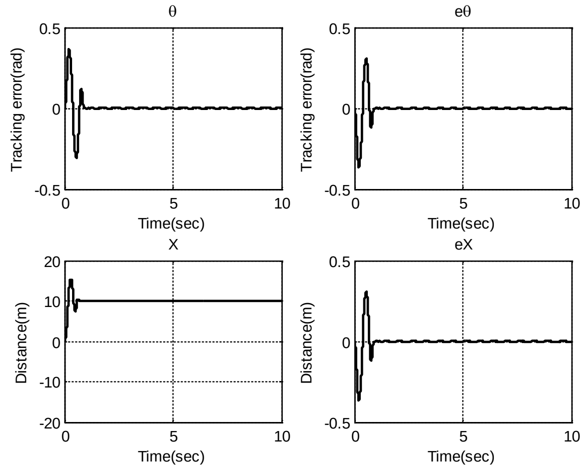

Fig. 5 *Response using the adaptive-robust and fuzzy logic control law when Λ=diag(50 50), K=diag(50 50), The target position is 10 [m].*

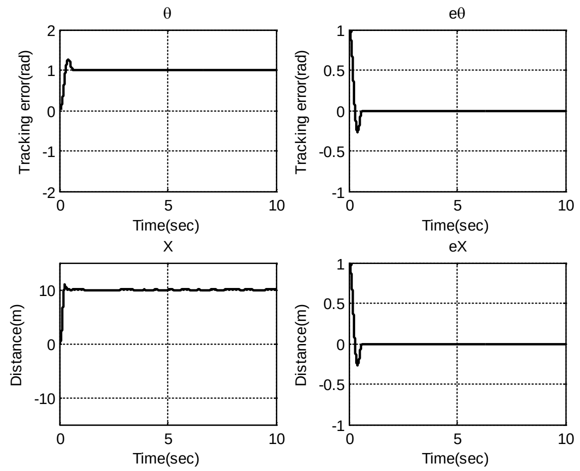

Fig. 6 *Response using the adaptive-robust and fuzzy logic control law when Λ=diag(20 20), K=diag(20 20), The target position is 10 [m]. The reference rod is 1 rad.*

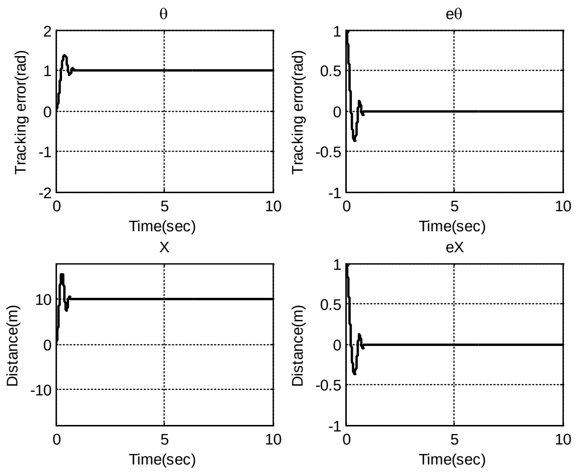

Fig. 7 *Response using the adaptive-robust and fuzzy logic control law when Λ=diag(50 50), K=diag(50 50), The target position is 10 [m]. The reference rod is 1 rad.*

5. Conclusion

The aim of this study is to develop a novel fuzzy logic control and adaptive-robust control law to minimize the trajectory tracking error and balance the rod. We develop adaptive-robust controller for position control then we design fuzzy logic controller for balancing the rod. As seen in the figures 4 - 7, the optimal values for K and Λ system gives better results. The main important part of this study is the parameters of robot are assumed to be unknown.

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APPENDIX

$$M_{11} = M_w + M_p + \frac{J_w}{r^2}$$
$$M_{12} = M_{21} = M_p l \cos\theta$$
$$M_{22} = J_p + M_p l^2$$

$$C_{11} = C_{21} = C_{22} = 0$$
$$C_{12} = -M_p l \sin\theta\dot{\theta}$$

$$G_{11} = C_{12} = C_{21} = 0$$
$$G_{22} = -M_p g l$$

And some parameters of the Segway mobile robot are as follows:[13]

$$M_w = 0.076\text{kg}$$
$$J_w = 3.42\text{x}10^{-5}\text{kgm}^2$$
$$M_p = 0.6\text{kg}$$
$$J_p = 1.34\text{x}10^{-2}\text{kgm}^2$$
$$g = 9.81\text{m}/\text{s}^2$$
$$r = 0.03\text{m}$$
$$l = 0.15\text{m}$$

$$M = \begin{bmatrix} \pi_1 & \pi_2 \\ -\pi_2 & -\pi_3 c_2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & \pi_3 s\theta\dot{\theta} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix}$$
$$G = \begin{bmatrix} 0 \\ \pi_4 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix}$$

$$\pi_1 = (M_w + M_p + \frac{J_w}{r^2}) \qquad \pi_2 = M_p l$$
$$\pi_3 = J_p + M_p l^2 \qquad \pi_4 = M_p g l$$