SIMPLIFIED ANALYTIC SOLUTION OF THE PROBLEM OF AUTOMATED MOTION CONTROL OF AN AUTONOMOUS RIGID BODY WITHOUT ITS OWN PROPULSION SYSTEM IN INCOMPRESSIBLE STRATIFIED VISCOUS FLUID

УПРОЩЕННОЕ АНАЛИТИЧЕСКОЕ РЕШЕНИЕ ЗАДАЧИ АВТОМАТИЗИРОВАННОГО УПРАВЛЕНИЯ ДВИЖЕНИЕМ АВТОНОМНОГО ТВЕРДОГО ТЕЛА БЕЗ СИЛОВОЙ УСТАНОВКИ В СТРАТИФИЦИРОВАННОЙ ВЯЗКОЙ НЕСЖИМАЕМОЙ ЖИДКОСТИ

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Резюме: *This paper results are based on the mathematical model of the motion control of an autonomous solid body moving in incompressible stratified viscous fluid which was presented by the authors at and XIII MTM Congress held in March 2016 and XIII MTM Congress held in September 2016. It is assumed that the body does not have its own propulsion system, but is equipped with controlled rudders - wings of finite span. It is moved by the influence of the buoyancy force and wings lift effect. The control is produced by the angle of attack of the wing change for reaching to a neighborhood of the given point by this solid body. This body motion is considered to be plane-parallel motion. At this paper authors present a simplification of this mathematical model in order to find an analytical solution of the differential equation describing the object motion and a necessity and acceptability analysis of the simplification.*

КЛЮЧЕВЫЕ СЛОВА: MATHEMATICAL MODEL, MOTION OF SOLIDS IN A FLUID, MOTION COTROL, BUOYANCY FORCE, ENSURING ACCESS TO THE GIVEN POINT, WINGS OF FINITE SPAN, WINGS LIFT

1. *Introduction*

The effectiveness of observations and measurements obtained in the study of the underwater world via underwater vehicles, in particular, unmanned, depends on minimizing the impact of these submersible crafts to surrounding underwater environment. First of all, it refers to a moving apparatus, which movement is carried out by various power plants (screw propeller or other propulsion). Therefore, the reduction or elimination of such effects is an important application. The ideal situation would obviously be the complete lack of engine. This means that movement control of the body can be carried out only by natural hydrodynamic forces, for instance, the Archimedes buoyancy or an wing lift effect (the body can be equipped with wing). Basic terminology and classical results for the body's motion in continuum can be found in the books [1, 2].

2. *Mathematical model*

As an autonomous rigid body, the authors propose to consider a research submersible – a uniform sphere-shaped rigid body with two similar symmetrically located around the ball centre wings (fig. 1). Actually other modifications of mutual bracing of the sphere-shaped body and wings are possible. However, the proposed mathematical model can be taken as a basis for whole these alternatives.

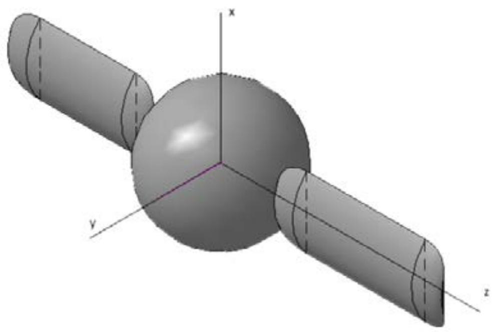

Fig. 1. Schematic submersible craft image.

The motion of submersible craft is assumed to happen in a limitless borehole bottom reservoir with an ideal incompressible non-conducting stratified liquid with viscosity effect. The viscosity is taken into account as a Stokes' drag force. It is also assumed that each layer has own density, which is known. Furthermore, liquid in each layer can move rectilinearly and uniformly with known velocity along the horizontal axis, which is perpendicular to a wingspread.

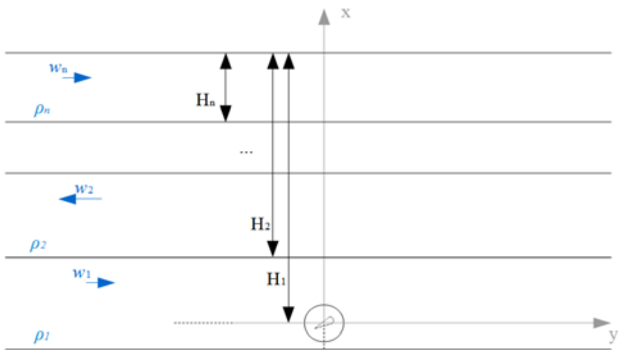

Fig. 2. Stratified continuous medium figure.

In this paper the authors consider plane-parallel motion of submersible craft case. At the initial time this body is located in stationary state at a predetermined depth (fig. 2). It is necessary to define the obtaining solution algorithm in a one-layer liquid for building a similar solution in stratified liquid..

At the previous authors paper [3,4] mathematical model of the submersible craft plane-parallel motion was constructed. It allows controlling the body through wings angle of attack α modifications:

$$\begin{cases}\left(m+\frac{2}{3}\rho\pi R^3\right)\frac{d^2x}{dt^2}=F_{arch}-2F_i\cos\delta-\left(F_{drag}^{(1)}+2F_{drag}^{(2)}\right)\cos\delta-2F_{lift}\sin\delta-F_g;\\ \left(m+\frac{2}{3}\rho\pi R^3\right)\frac{d^2y}{dt^2}=-2F_i\sin\delta-\left(F_{drag}^{(1)}+2F_{drag}^{(2)}\right)\sin\delta-2F_{lift}\cos\delta.\end{cases}$$

Here F_{arch} – is the buoyancy force, $F_{drag}^{(j)}=C_X^{(j)}S^{(j)}\frac{\rho v^2}{2}$ – the head resistance force for a sphere (j=1) and wings (j=2), $F_{lift}=$

$\rho v^2 S \frac{k\alpha}{1+\mu_0}$ – the wing lift, $F_i = \frac{\rho}{2} v^2 S \frac{\mu_0}{2k}\left(\frac{2k\alpha}{1+\mu_0}\right)^2$ – the induced drag force (details can be found in [3]).

Using the standard change of variables:

$$x = z_1 \qquad y = z_3$$
$$\dot{x} = \dot{z_1} = z_2 \quad \dot{y} = \dot{z_3} = z_4,$$

initial mathematical model is reduced to the nonlinear differential equation system:

$$(1) \quad \begin{cases} \dot{z_1} = z_2, \\ \dot{z_2} \cdot b_0 = b_1 - (b_2 \cdot \alpha^2 + b_3 + 2b_4) \cdot z_2 \cdot \sqrt{{z_2}^2 + {z_4}^2} - \\ -2b_5 \cdot \alpha \cdot z_4 \cdot \sqrt{{z_2}^2 + {z_4}^2} \\ \dot{z_3} = z_4, \\ \dot{z_4} \cdot b_0 = -(b_2 \cdot \alpha^2 + b_3 + 2b_4) \cdot z_4 \cdot \sqrt{{z_2}^2 + {z_4}^2} + \\ +2b_5 \cdot \alpha \cdot z_2 \cdot \sqrt{{z_2}^2 + {z_4}^2} \end{cases}$$

Here coefficients are defined as:

$b_0 = m + \frac{2}{3}\rho\pi R^3$, $\quad b_1 = \rho g V - mg$, $\quad b_2 = \rho S_{kp} \frac{2k\mu_0}{(1+\mu_0)^2}$, $\quad b_3 = c_{0\_sph} \frac{\rho\pi R^2}{2}$, $b_4 = c_{0\_w} \frac{\rho S_w}{2}$, $b_5 = \rho S_{kp} \frac{k}{1+\mu_0}$.

From a practical standpoint, it is presented that the system (1) is too lengthy for operational applications. The authors show below, that this system can be simplified without the loss of required accuracy in significant cases from the applied point of view.

3. *Simplified mathematical model*

By analyzing numerical solution of differential equation system (1) in homogeneous liquid with zero initial conditions [4], it can be noted that $z_2 \gg z_4$. Therefore $\sqrt{{z_2}^2 + {z_4}^2}$ can be expand this function in a Taylor series near the point $z_4/z_2 = 0$:

$$\sqrt{{z_2}^2 + {z_4}^2} = z_2\sqrt{1 + \frac{{z_4}^2}{{z_2}^2}} \approx z_2\left(1 + \frac{z_4}{2z_2} - \frac{1}{8}\left(\frac{z_4}{z_2}\right)^3 + \cdots\right).$$

It is assume that $\sqrt{{z_2}^2 + {z_4}^2} \approx z_2$. By excluding a term $-2b_5 \cdot \alpha \cdot z_2 \cdot z_4$ from the second equation of (1) under the assumption of $z_2 \gg z_4$, differential equation system is transformed into

$$(2) \quad \begin{cases} \dot{z_1} = z_2, \\ \dot{z_2} \cdot b_0 = b_1 - (b_2 \cdot \alpha^2 + b_3 + 2b_4) \cdot {z_2}^2 \\ \dot{z_3} = z_4, \\ \dot{z_4} \cdot b_0 = -(b_2 \cdot \alpha^2 + b_3 + 2b_4) \cdot z_2 \cdot z_4 + 2b_5 \cdot \alpha \cdot {z_2}^2 \end{cases}$$

If the function of attack angle is time-invariant (constant value), the system (2) can be solved [5] with zero initial conditions:

$$(3) \begin{cases} z_1 = -\sqrt{\frac{k_1}{k_2}} + \frac{1}{k_2}\ln\left(\frac{exp\{2\sqrt{k_1 k_2}t\}+1}{2}\right), \\ z_2 = \sqrt{\frac{k_1}{k_2}} \cdot \frac{exp\{2\sqrt{k_1 k_2}t\}-1}{exp\{2\sqrt{k_1 k_2}t\}+1} \\ \dot{z_3} = z_4, \\ z_4 = \frac{2exp\{\sqrt{k_1 k_2}t\}}{1+exp\{2\sqrt{k_1 k_2}t\}} \cdot \frac{k_1 k_3}{k_2} \int_0^t \left(\frac{exp\{2\sqrt{k_1 k_2}t\}-1}{exp\{2\sqrt{k_1 k_2}t\}+1}\right)^2 \cosh\sqrt{k_1 k_2}t \, dt \end{cases}$$

Here coefficients are defined as:

$k_1 = \frac{b_1}{b_0}$, $k_2 = \frac{(b_2 \cdot \alpha^2 + b_3 + 2b_4)}{b_0}$, $k_3 = \frac{2b_5 \cdot \alpha}{b_0}$.

Below it is shown that original and simplified system numerical solutions coincide virtually.

4. *Numerical examples*

Software called MATLAB 7.10.0 (R2010A) is used for numerical solution.

Motion of the submersible craft in homogeneous (single-layer) ideal incompressible viscous fluid with shear flow in the line of horizontal axis is considered. Authors examine three cases of the variation law of attack angle:

1) α = 0.3;
2) α = 0;
3) α = – 0.3.

4.1. *Original system numerical solution*

Fourth-order of accuracy Runge-Kutta method was applied for numerical solution of the differential equation system (1).

Motion trajectories of the submarine craft corresponding to the angle of attack can be calculated by solving the system (1) with zero-initial condition (fig. 3).

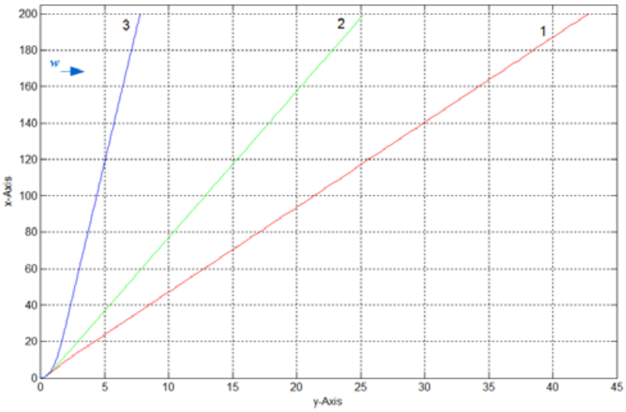

Fig. 3. Motion trajectories of the submarine craft in accordance with system (1).

4.2. *Simplified system solution*

In the system (3) the first two equations represent in an explicit form dependence of the first (z_1) and the second (z_2) components on time. In order to solve the third and the fourth equations, the authors use the method of numerical integration of single variable function – trapezium method.

Motion trajectories of the submarine craft corresponding to the angle of attack can be calculated by solving the system (3) with zero-initial condition (fig. 4).

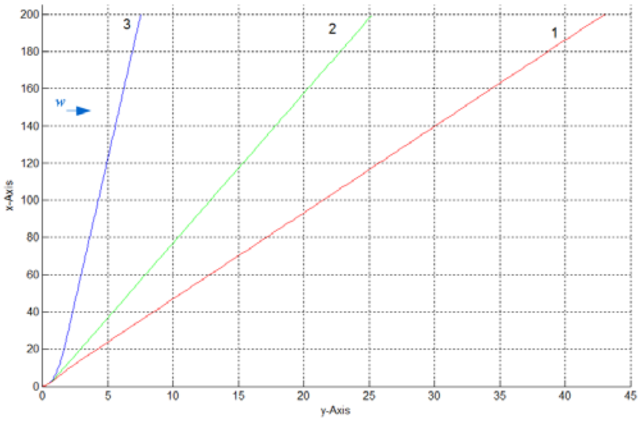

Fig. 4. Motion trajectories of the submarine craft in accordance with system (3).

From figure 3 and 4 it can be noted that the nature of the trajectories of the submersible craft motion does not change during the transformation from system (1) to the system (3). Component-wise divergences of the system (1) and (3) solutions are presented in a figure 5.

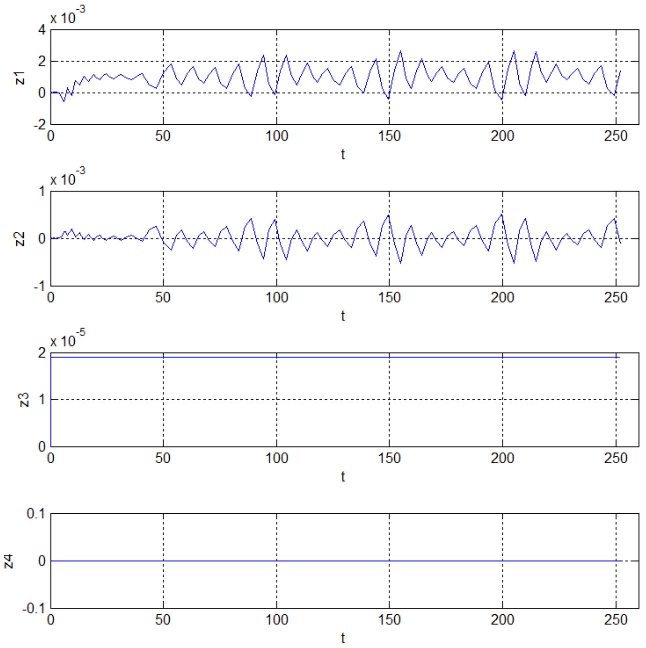

Fig. 5. Component-wise divergencse of the system (1) and (3) solutions.

In the figure 5 the largest variances are obtained for the first (z_1) and the second (z_2) components. This is explained by the fact that for the system (1) these components are calculated numerically while for the system (3) they are defined analytically. The third (z_3) and the fourth (z_4) components have the least differences due to the use of numerical methods in both cases.

At these examples, the following values of quantities are offered. The diameter of the surfaced body (ball) is 1 meter; its mass is calculated like $m = 0.98\rho V$, where ρ is averaged body density. It is assumption to consider rectangular wings with wingspan 1 meter, aspect ratio of the wing 5 and relative maximum thickness 16 %. An initial immersion depth H_1 equals 200 meters. In case of homogeneous liquid its density is supposed to be 1038 kg/m^3, shear flow velocity – $|\vec{w}| = 0.1$ m/s.

5. Conclusion

At this paper analysis of the possibility of simplifying the original system which describes the motion of the submersible craft without its own propulsion system in homogeneous (single-layer) incompressible viscous fluid with shear flow in the line of horizontal axis is presented. In the transformation to stratified (layered) liquid it is necessary to solve the corresponding differential equation system sequentially for each layer starting with the bottom layer. Its initial conditions are supposed zero conditions. For other layers initial conditions are recalculated depending on coordinates of inertia center of the submarine craft at the transitional point from layer to layer.

Thus, the authors solved 2 problems:

1. The transformation to the simplified equation system allowing analyzing the behavior of the system by analytical methods is justified.
2. Development algorithm of numerical solution of the system is simplified. It has substantial practical value.

6. Literature

1. Vallander S.V. Lekcii po gidroaeromehanike [Lectures on hydrodynamics]. – St. Petersburg, St. Petersburg Univ. Press, 2005. 304 p. (In Russian)
2. Kochin N.E., Kibel' I.A., Roze N.V. Teoreticheskaja gidromehanika [Theoretical Hydromechanics]. Moskow, Fizmatlit Publ., 1963. Vol. 1-2. (In Russian)
3. Firsov A.N., Kuznetcova L.V. The solution of the synthesis problem of partial motion control of a rigid body in incompressible viscous fluid // Machines, Technologies, Materials 2016. (Borovetz, Bulgaria, 16-19.03.2016) – 2016. Vol. 5/2016. – P. 21-22
4. Firsov A.N., Kuznetcova L.V. The numerical-analytic substantiation of the possibility of automated motion control of an autonomous rigid body without its own propulsion system in incompressible stratified viscous fluid // Machines, Technologies, Materials 2016. (Varna, Bulgaria, 14-17.09.2016) – 2016. Vol. 9/2016. – P. 30-32.
5. Krasnov M.L. Obiknovennie differenstialnie uravnenia [Ordinary differential equations]. – Moskow.: Vysshaya Shkola Publishers, 1983