COMPUTER AIDED ANALYSIS OF KINEMATICS AND KINETOSTATICS OF SIX-BAR LINKAGE MECHANISM THROUGH THE CONTOUR METHOD

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Abstract: *In this paper is presented a six-bar linkage mechanism of the pump for oil extrusion. In this mechanism are introduced higher kinematic pairs. Dimensions and other incoming links are adopted as necessary. For the six-bar linkage mechanisms is carried out the kinematic analysis and for all linkages are shown the displacement, velocity and acceleration. The analysis is performed by Math Cad software, while kinetostatic analysis is carried out using Contour Method, comparing results of two different software‘s Math CAD and Working Model. The simulation parameters are computed for all points of the contours of mechanism*

Keywords: *MECHANISM, CONTOUR, DISPLACEMENT, VELOSITY, ACCELARATION*

1. Introduction

This paper has been completed using Math Cad and Working Model software’s. In this paper is presented a six-bar linkage ABCD and a simple crank mechanism DEF as shown in the figure below. Firstly, in this mechanism is determined DF_y and θ_5. Derivative DF_y represents the speed of the slider F. From the given picture $0.1 \le \theta_2 \le 360^o$ should be determined. The masses are adopted, since the moments of inertia need to calculate. The kinematic part of the paper will be completed by finding the velocities and accelerations of each point ***A, B, C, D, E, F*** for the centres from C_1 to C_6. In this way are determined the angular accelerations and velocities of the linkages 2, 3, 4 and 5. Whereas for the kinetostatic part will be determined the reaction forces of the points: X_A, Y_A, X_B, Y_B, X_C, Y_C, X_D, Y_D, X_E, Y_E, X_F, Y_F, $N=X_{F6}$ and M_{tr} which are acting on the leading link 2. In the picture are shown 5 bodies: ***AB, BC*** right triangle, rod ***EF***, slider bar ***F***.

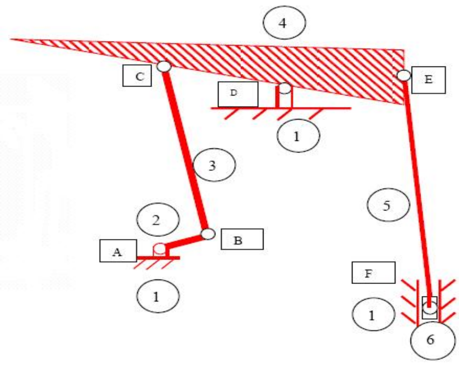

Fig. 1 *Six bar-linkage mechanism ABCDEF*

Given data:

$$\left| \begin{array}{l} \theta_2 = 0.5\,\text{deg} ... 360[\text{deg}], \omega_2 = 2, r_2 = 1.7 \\ r_3 = 6.9, r_{4a} := 4.2, r_D = \sqrt{4.2^2 + 6.4^2}, \alpha = 56.725 \end{array} \right| \text{.........(1)}$$

Determination of angle α of r_D with x_1: Linkage I

$$\left| \vec{r_2} + \vec{r_3} + \vec{r_{4a}} = \vec{r_D} \right| \text{...(2)}$$

From then designed outline at x and y we have the following initial conditions:

$$\left| \begin{array}{l} r_2 \cdot \cos(\theta_2) + r_3 \cdot \cos(\theta_3) + r_{4a} \cdot \cos(\theta_4) = r_D \cdot \cos(\alpha) \\ r_2 \cdot \sin(\theta_2) + r_3 \cdot \sin(\theta_3) + r_{4a} \cdot \sin(\theta_4) = r_D \cdot \sin(\alpha) \end{array} \right| \text{...... (3)}$$

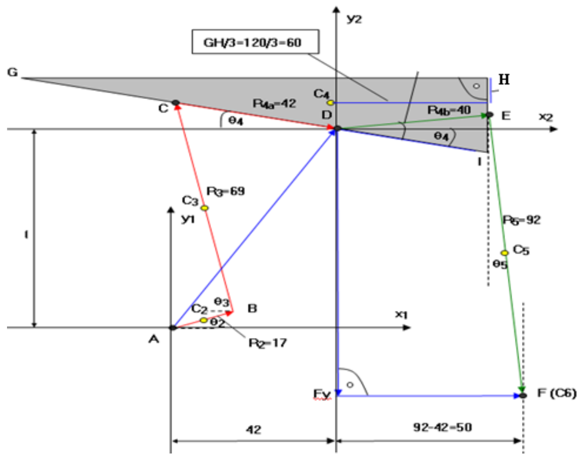

Fig. 2 *Four-bar linkage ABCD and crank mechanism DEF*

As shown in the Figure 2, there are 14 unknown sizes, 12 equations for the first four bodies, $dy(x, y)$ for the slider 14 equations.

2. The analysis of the positions, velocities and accelerations of six-bar linkage mechanism

In the figure 3 is presented the velocity plan for the points A, B, C and D. Also are presented angular velocities and angular accelerations for the points A, B and C. After finding the velocities and accelerations of these points, the displacements, velocities and accelerations of these points with angles θ_3 and θ_4 in function of θ_2 are shown graphically. Outline is shown in this form:

$$\left| \theta_3 = 180[\text{deg}], \theta_4 = 90[\text{deg}] \right| \text{... (3)}$$

The vector equation without the **contour I** is:

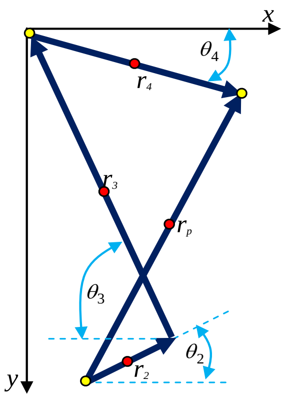

***Fig. 3** Velocity plan for the points A, B and C*

$$\left|\begin{pmatrix} r_2\cos(\theta_2) \\ r_2\sin(\theta_2) \\ 0 \end{pmatrix} + \begin{pmatrix} r_3\cos(\theta_3) \\ r_3\sin(\theta_3) \\ 0 \end{pmatrix} + \begin{pmatrix} r_4\cos(\theta_4) \\ r_4\sin(\theta_4) \\ 0 \end{pmatrix} = \begin{pmatrix} r_p\cos(\alpha) \\ r_p\sin(\alpha) \\ 0 \end{pmatrix}\right| \quad (4)$$

$$\left| \begin{array}{l} Zgj(\theta_2) = Find(\theta_3, \theta_4), \\ \theta_3 = 104.18[\text{deg}], \theta_4 = -3.96[\text{deg}], \beta = 59.48[\text{deg}] \\ \omega_3 = \dfrac{d}{d\theta_2}\theta_3(\theta_2)\cdot\omega_2, \omega_4 = \dfrac{d}{d\theta_2}\theta_4(\theta_2)\cdot\omega_2 \\ \varepsilon_3 = \dfrac{d^2}{d\theta_2{}^2}\theta_3(\theta_2)\cdot\omega_2{}^2, \varepsilon_4 = \dfrac{d^2}{d\theta_2{}^2}\theta_4(\theta_2)\cdot\omega_2{}^2 \end{array} \right| \quad ....(5)$$

After calculation of the values by Math Cad software, in the following are shown graphically the values of positions, velocities and accelerations respectively for $\theta_3, \omega_3, \varepsilon_3$ and $\theta_4, \omega_4, \varepsilon_4$:

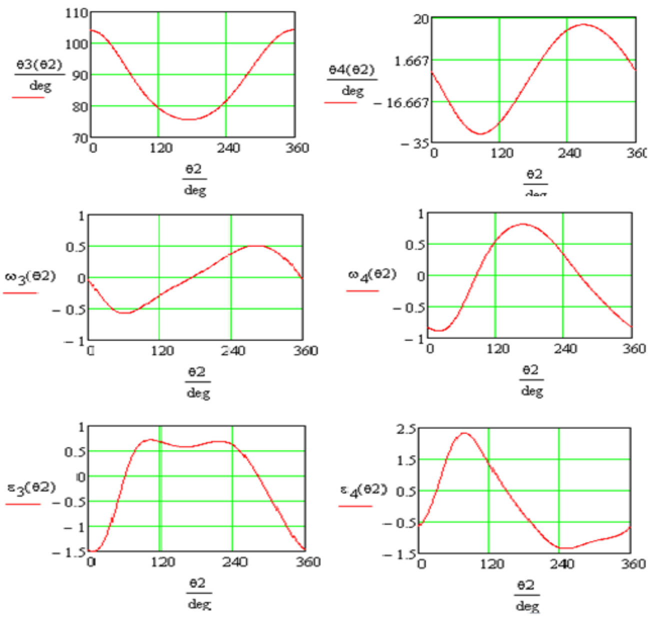

***Fig. 4** Diagrams for positions, velocities and accelerations*

2.1 Position of the point B

The position of the point B is calculated in direction of x and y :

$$\left| \begin{array}{l} x_B = r_2\cdot\cos(\theta_2), y_B = r_2\cdot\sin(\theta_2) \\ v_{Bx} = -r_2\cdot\sin(\theta_2)\cdot\omega_2, v_{By} = r_2\cdot\cos(\theta_2)\cdot\omega_2 \\ v_B = \sqrt{v_{Bx}(\theta_2)^2 + v_{By}(\theta_2)^2} = 10.03[cm/s] \end{array} \right| \quad ....(6)$$

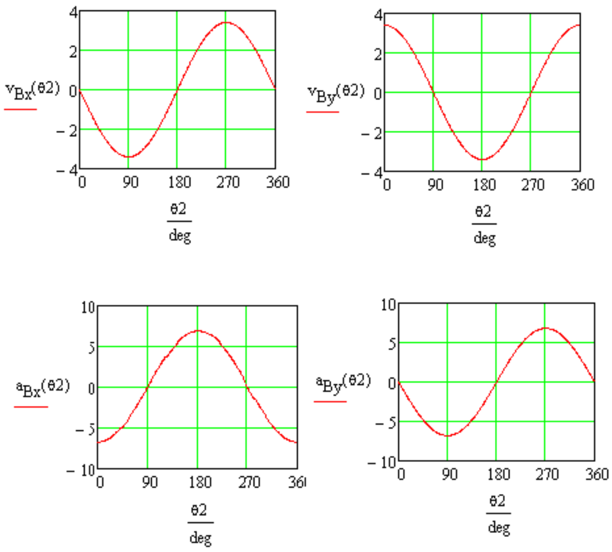

***Fig. 5** The position diagrams for the position equations of the point B in direction of x and y*

$$\left| \begin{array}{l} a_{Bx} = -r_2\cdot\cos(\theta_2)\cdot\omega_2^2, v_{By} = -r_2\cdot\sin(\theta_2)\cdot\omega_2^2 \\ a_B = \sqrt{a_{Bx}(\theta_2)^2 + a_{By}(\theta_2)^2}\,[cm/s^2] \end{array} \right| \quad ....(7)$$

Angular velocity and acceleration for the point B:

$$\left| \begin{array}{l} \omega_4 = \dfrac{d}{d\theta_2}\theta_4\cdot\omega_2, \varepsilon_4 = \dfrac{d^2}{d\theta_2{}^2}\theta_4\cdot\omega_2{}^2 \\ \omega_4 = -0.87[\text{deg}], \varepsilon_4 = 0.45[\text{deg}^2] \end{array} \right| \quad ....(8)$$

3. The equations for position (displacement), velocity and acceleration of the point C

In the following is shown the displacement of point C, along with velocities and accelerations which are presented graphically by the diagrams via Math Cad program:

$$\left| \begin{array}{l} x_c = 0 y_c = r_2\cdot\cos(\theta_2) + r_3\cdot\sin(\theta_3(\theta_2)) \\ v_{cx} = 0, v_{cy} = r_2\cdot\sin(\theta_2) + r_3\cdot\cos(\theta_3(\theta_2)) \\ v_c = v_{cy}(\theta_2);_c a_{cx}(\theta_2) = 0 \\ a_{cy} = (-r_2\cdot\sin(\theta_2) - r_3\cdot\cos(\theta_3(\theta_2)))\cdot\omega^2 \\ a_c = a_{cy}(\theta_2) \end{array} \right| \quad ....(9)$$

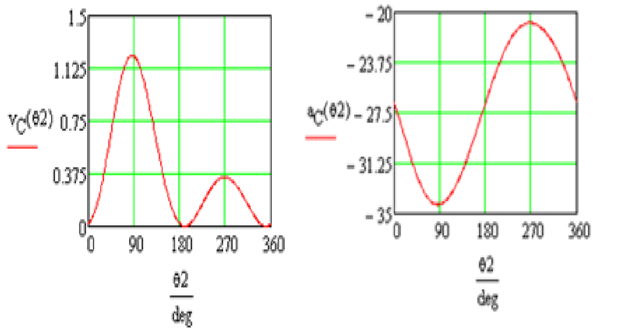

Fig. 6 *Velocities and acceleration of point C*

3.1 The expression for the middle point of the bars AB, BC and CD

In the following are extracted the displacement for the middle position of the bars AB, BC and CD and other displacement which belong to these bars. Are also presented the velocities and accelerations of these bars and their diagrams separately:

$$\left| \begin{array}{l} AB: \\ x_{c2} = \frac{r_2}{2}\cos(\theta_2); y_{c2} = \frac{r_2}{2}\sin(\theta_2) \\ a_{xc2} = \frac{-r_2}{2}\sin(\theta_2)\cdot\omega^2; a_{yc2} = \frac{r_2}{2}\cos(\theta_2)\cdot\omega^2 \end{array} \right| \quad ....(9)$$

$$\left| \begin{array}{l} BC: \\ x_{c3} = \frac{r_3}{2}\cos(\theta_3(\theta_2)); y_{c3} = \frac{r_3}{2}\sin(\theta_3(\theta_2)) \\ a_{xc2} = \frac{-r_3}{2}\sin(\theta_3(\theta_2))\cdot\omega_2^2; a_{yc3} = \frac{r_3}{2}\cos(\theta_3(\theta_2))\cdot\omega_2^2 \end{array} \right| \quad ...(10)$$

$$\left| \begin{array}{l} CD: \\ x_{c4} = \frac{r_4}{2}\cos(\theta_4(\theta_2)); y_{c4} = \frac{r_3}{2}\sin(\theta_4(\theta_2)) \\ a_{xc4} = \frac{-r_4}{2}\sin(\theta_4(\theta_2))\cdot\omega_2^2; a_{yc4} = \frac{r_4}{2}\cos(\theta_4(\theta_2))\cdot\omega_2^2 \end{array} \right| \quad ..(11)$$

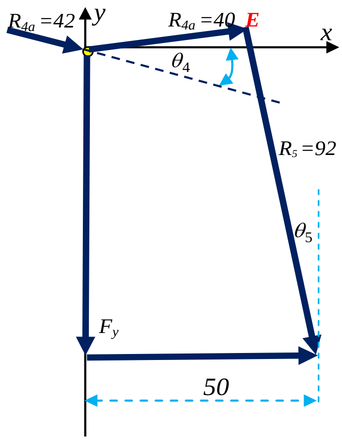

Fig. 7 *Crank mechanism*

3.2 The equations for position (displacement), velocity and acceleration of the point E

For the point ***E*** of Crank Mechanism determined the displacement, and in the following pictures are shown velocities and accelerations in the direction ***x*** and ***y***:

$$\left| \begin{array}{l} x_E = r_{4b}\cos(\theta_4(\theta_2)); y_E = r_{4b}\sin(\theta_4(\theta_2)) \\ x_E = -r_{4b}\sin(\theta_4(\theta_2))\cdot\omega_2; y_E = r_{4b}\cos(\theta_4(\theta_2))\cdot\omega 2 \\ a_{Ex} = -r_{4b}\cos(\theta_4(\theta_2))\cdot\omega_2^2; a_{Ey} = -r_{4b}\sin(\theta_4(\theta_2))\cdot\omega_2^2 \end{array} \right| \quad ..(12)$$

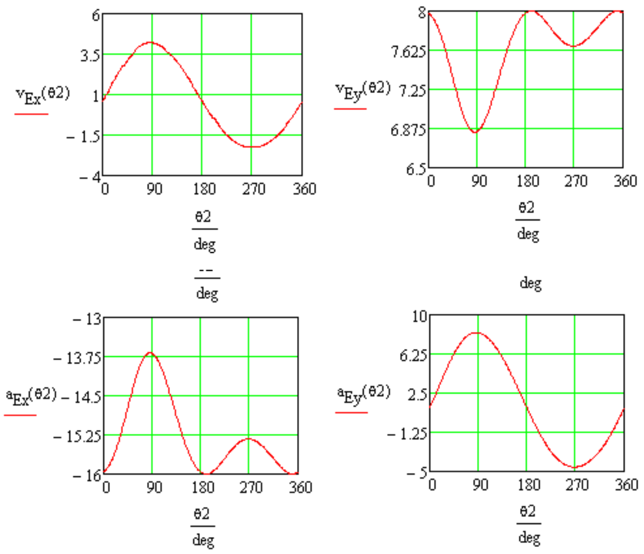

Fig. 8. *Diagrams of the velocities and acceleration for the E*

4. Kinetostatic analysis of the six-bar linkage mechanism

For the kinetostatic part will be presented the kinetostatic analysis of six-bar linkage mechanism by Math Cad software. Moments of inertia, masses of bodies are used in the following:

$$\left| \begin{array}{l} m_2 = 10; m_3 = 15; m_4 = 20; m_5 = 15; m_6 = 5; g = 9.807 \\ J_{C2} = \frac{1}{2}\cdot m_2 \cdot r_2{}^2; J_{C3} = \frac{1}{2}\cdot m_3 \cdot r_3{}^2 \\ J_{C4} = \frac{1}{12}\cdot m_4 (2.4^2 - 1.2^2); J_{C5} = \frac{1}{12}\cdot m_5 \cdot r_5{}^2 \end{array} \right| \quad .(13)$$

Linkage I:

The equilibrium conditions for the point A are equal to zero. Six linkages are used for the kinetostatic analysis. For the first linkage are given the following equilibrium conditions $\boldsymbol{X_A, Y_A}$ and $\boldsymbol{X_B, Y_B}$ for the middle points of the linkage AB, point C2 and for the body mass $\boldsymbol{m_2}$.

$$\left| \begin{array}{l} X_A - X_B = m_2 \cdot a_{xc2}(\theta_2); Y_A - Y_B = m_2 \cdot a_{yc2}(\theta_2) \\ M_{tr} = Y_A \frac{r_2}{2}\cos(\theta_2) - X_A \frac{r_2}{2}\cos(\theta_2) + \\ \qquad Y_B \frac{r_2}{2}\cos(\theta_2) - X_B \frac{r_2}{2}\sin(\theta_2) \end{array} \right| \quad ......(14)$$

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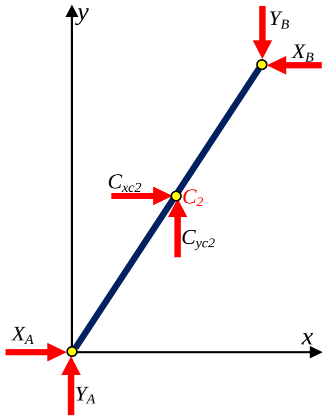


Fig. 9. *Linkage I–Equilibrium conditions for the point AB*

Linkage II

Also, for the linkage II are written the equilibrium conditions BC, which are $\boldsymbol{X_B, Y_B}$, $\boldsymbol{X_C, Y_C}$ center $\boldsymbol{C_3}$ body mass ***BC*** and moment of inertia for the point $\boldsymbol{C_3}$.

$$\left| \begin{array}{l} X_C - X_B = m_3 \cdot a_{xc3}(\theta_2); Y_C - Y_B = m_3 \cdot a_{yc3}(\theta_2) \\ J_{C3} \cdot \varepsilon(\theta_2) = X_C \cdot \frac{r_3}{2} \cdot \sin(\theta_3(\theta_2)) - Y_C \cdot \frac{r_3}{2} \cdot \cos(\theta_3(\theta_2)) \\ - Y_B \cdot \frac{r_3}{2} \cdot \theta_3(\theta_2)) - X_B \cdot \frac{r_3}{2} \cdot \sin(\theta_3(\theta_2)) \end{array} \right| \quad .(15)$$

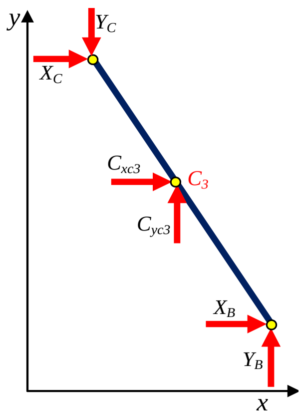


Fig.10. *Linkage II – Equilibrium conditions for the part* BC

Linkage III

$$\left| \begin{array}{l} X_C + X_D - X_E = m_4 \cdot a_{xC4}(\theta_2) \\ Y_C + Y_D - Y_E = m_4 \cdot a_{yC4}(\theta_2) \\ J_{C4} \cdot \varepsilon_4(\theta_2) = -Y \cdot r_{4a} + Y - r_{4b} - EH - \\ \qquad Y \cdot r_z(\theta_2) + X_D \cdot r_m(\theta_2) \end{array} \right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

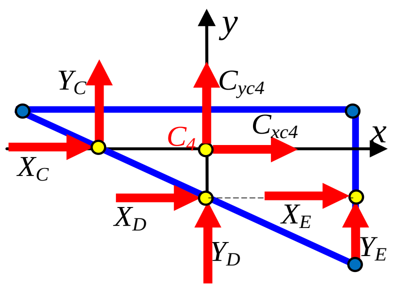


Fig.11. *Linkage III of the points C, D, E and center* C_4

$$\left| \begin{array}{l} X_E - X_F = m_5 \cdot a_{xC5}(\theta_2) \\ Y_C + Y_D - Y_E = m_4 \cdot a_{yC4}(\theta_2) \\ J_{C5} \cdot \varepsilon_5(\theta_2) = (X_E + X_F) \cdot \frac{r_5}{2} \cdot \sin(\theta_5(\theta_2)) - \\ \qquad (Y_E + Y_F) \cdot \frac{r_5}{2} \cdot \sin(\theta_5(\theta_2)) \end{array} \right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

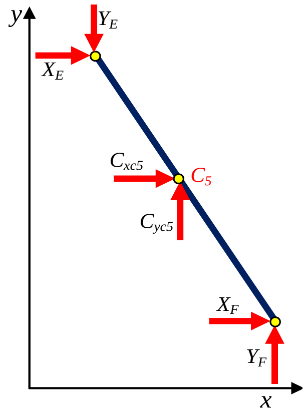
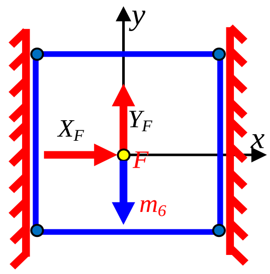


Fig. 12. *Linkage IV of the part EF in direction of x and y*

Linkage V

$$\left| Y = -(m_6 \cdot a_{yF}(\theta_2) + m_6 \cdot g) \right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

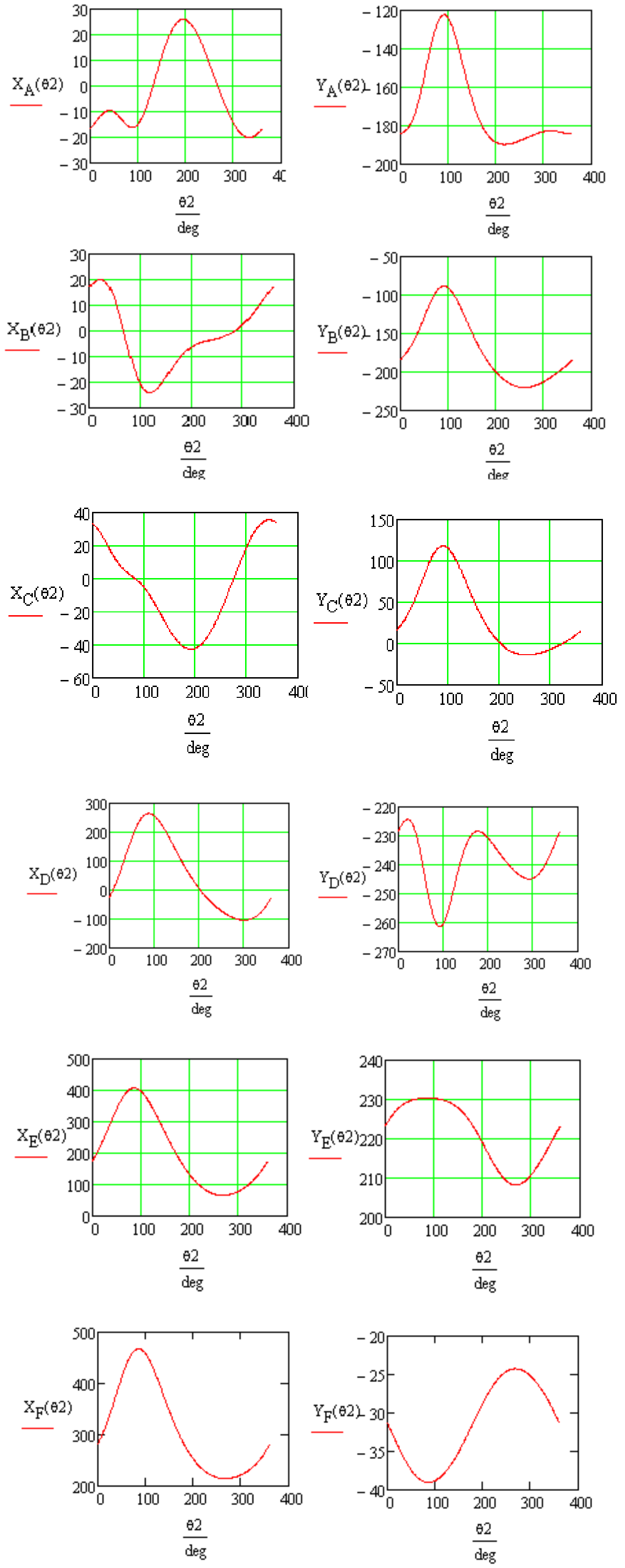


Fi.g.13. *Linkage V of the point F in direction of* x *and* y

All unknown parameters for the linkages in both directions ***x*** and ***y*** are calculated by Math Cad. From point A to the point Find the Transmission moment M_{tr}:

$$\left| \begin{array}{l} Solve(\theta 2) := Find(X_A \cdot Y_A \cdot X_B \cdot Y_B \cdot X_C \cdot \\ \qquad Y_C \cdot X_D \cdot Y_D \cdot X_E \cdot Y_E \cdot X_F \cdot Y_F \cdot M_{tr} \end{array} \right| \quad \ldots\ldots (18)$$

In the following diagrams are given the values of all unknown parameters for the positions (0; 0.5; 1) by Math Cad software. This it was really a tough work since the working process by Math Cad was very slow especially to follow the procedure which is given by kinetostatic analysis.



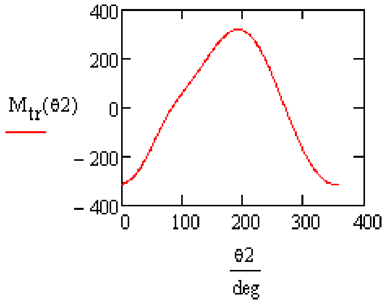


Fig.14**. Diagrams of kinetostatic analysis of the given mechanism for the points **A, B, C, D, E, F** and transmission moment **Mtr

5. Simulation for six-bar mechanism by Working Model software

In the second part of this paper is performed the simulation for all points ***A,B,C,D,E,F*** and ***Mtr*** (time dependent) of the six bar linkage mechanism by Working Model, which is shown in the following.

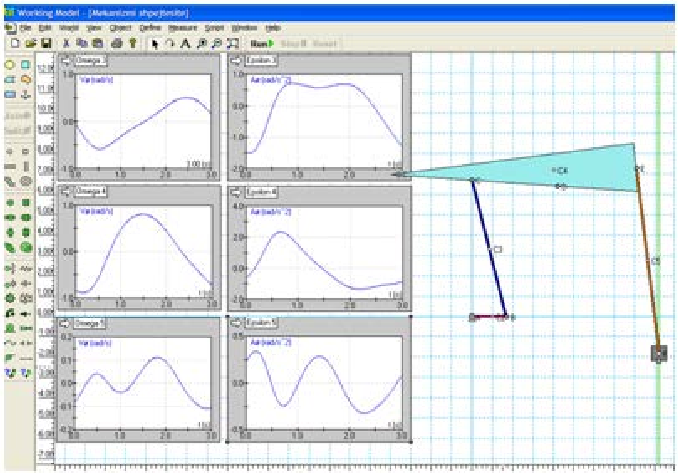


***Fig.15**.Diagrams in Working Model for angular velocity and acceleration 3, 4 and 5*

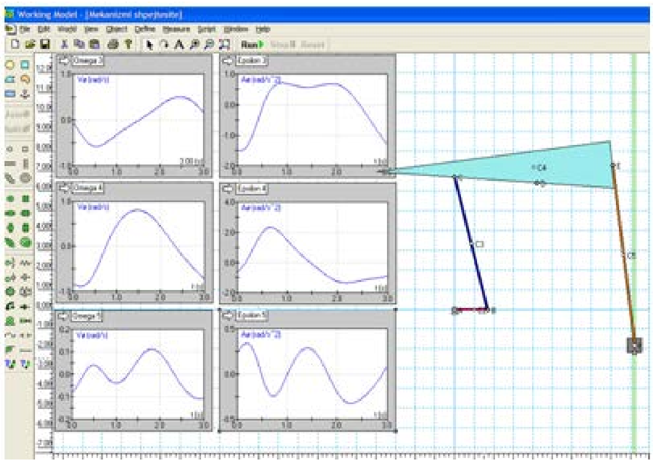


Fig.16**:. Diagrams in Working Model for equilibrium points **A, B, C, D, E, F** and transmission moment M_{tr} in direction **x

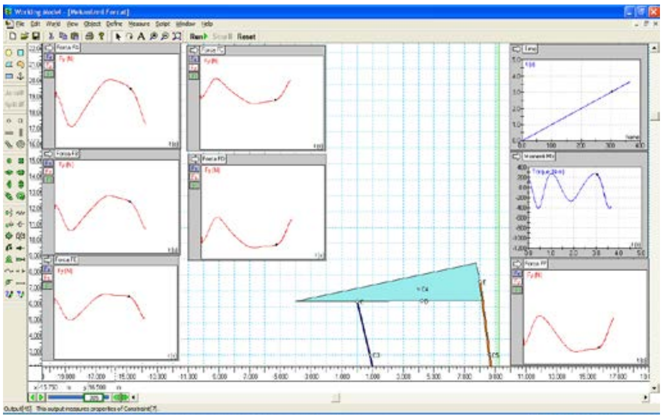


Fig.17.Diagrams in Working Model for equilibrium points A,B,C,D,E,F and transmission moment (Mtr) in direction y

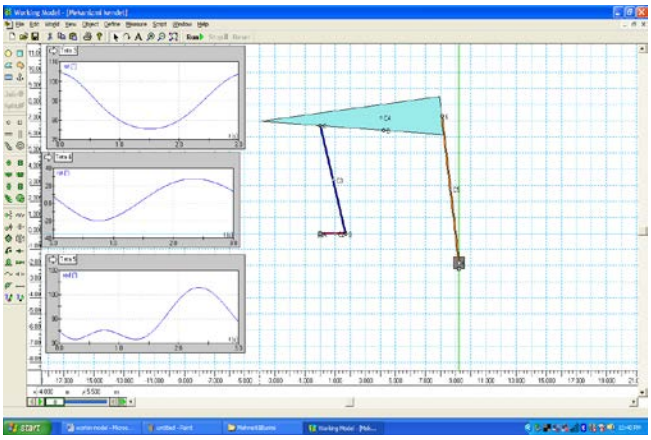


Fig.18. *Diagrams for θ_3, θ_4 and θ_5 (time dependent), under command RUN*

6. Conclusions and recommendations

By application of the software's Math Cad or Matlab we have reached to carry out the full analysis of this paper. In this paper are made the calculations of all positions (displacement) for the whole mechanism, and also are determined the plans for velocities and accelerations for each point.

However, in this paper are shown the outline planes of the mechanism same as the diagrams for each linkage through Math Cad and Working Model software, Six-bar linkage mechanism diagrams which are derived by Working Model are almost similar to the diagrams derived by Math Cad, same as the derived results, Through Working Model software, are derived the results of reactions from the equilibrium conditions of six bar linkage mechanism, angular velocities and accelerations for the points 3, 4 and 5, for the angles θ_3, θ_4 and θ_5 in time domain through the command Run,

As the general conclusion; the results derived by both software, same as for their diagrams, for all points of the six bar linkage mechanism are within the reasonable boundaries. Our expectations are that similar results will be derived by application of Matlab software.

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