VEHICULAR TRAFFIC FLOW MODELLING BY MEANS OF CAR-FOLLOWING APPROACH

Ph.D. student Kurtc V.[1], Ph. D. Anufriev I.[2]

Institute of Applied Mathematics and Mechanics – Peter the Great St. Petersburg State Polytechnic University, St. Petersburg, Russia

E-mail: [1]kurtsvv@gmail.com, [2]Igor.Anufriev@kronshtadt.ru

Abstract: This paper concerns the problem of vehicular traffic flows simulation. We present a continuous car-following model with explicit reaction-time delay. This model is based on the Intelligent Driver Models and eliminates some drawbacks like unrealistic distance between vehicles in a homogeneous flow and instant reaction of a driver. The model is defined by a couple of delayed differential equations. Introducing reaction time explicitly one can simulate traffic flow instabilities such as kinematic waves with a greater degree of realism. We calculate conditions for that the linear stability occurs on a ring and in the finite platoon of cars. The parameters of the model are calibrated according to NGSIM trajectory data. Another method described in this paper allows to estimate parameters according to technical characteristics of the specific vehicle type. Simulating vehicular traffic for large cities with microscopic approach one needs to solve tremendous systems of differential equations. The speed of changes in the size of the components of such systems usually lies in a wide range, as the dynamics and behaviour of the vehicles can strongly differ. We introduce a multirate numerical scheme with a self-adjusting time stepping strategy. The step size for each component is determined by its own local temporal variation. The stability analysis for the developed scheme is performed and the stability conditions are obtained. The use of multiple time steps allows parallel computing.

Keywords: VEHICULAR TRAFFIC, MICROSCOPIC MODELS, TRAFFIC FLOW INSTABILITIES, CALIBRATION, MULTIRATE SOLVERS

1. Introduction

Nowadays, the overall traffic demand has been increasing from day to day. The existing road network, as well as traffic management schemes, do not cope with the growing traffic flows. This fact leads to formation of traffic jams and, as a result, environmental pollution and increase of travel times. To tackle this problem, traffic flow modelling is used nowadays. Vehicular traffic simulation is one of the possible tool for studying phenomena of traffic flows oscillations [1-3] and testing schemes for optimal transportation planning.

Traffic flow models can be categorized with the respect to the way how reality is presented [4]. Macroscopic (hydrodynamical) models [5-8] describe traffic flow as liquids or gases in motion in terms of traffic density $\rho(x,t)$, flow $Q(x,t)$ and mean speed $V(x,t)$. Microscopic models include cellular automata [9,10] and car-following models [11-16] and describe "driver-vehicle particles" individually with such dynamical variables as vehicle acceleration, speed and position. Traffic flow models mostly microscopic ones are an integral part of commercial software products (Aimsun, MITSIM, VISSIM) used for investigating of vehicular dynamics, testing control algorithms of traffic management and for upgrading the existing or designing a new road network. Another application of vehicular flows simulation is driving simulators which admit only car-following approach. In spite of diversity of traffic flow models, development of realistic and elaborated models is required.

Driving styles, types of vehicles and traffic regulation rules vary from country to country. Consequently, parameters of the model should be adapted to the specific conditions, that is, calibrated [17,18]. Moreover, one needs to check if the calibration procedure was performed successfully by means of validation [17].

Nowadays**,** microscopic traffic data have become more available and provides information about thousands of vehicle trajectories [19,20]. As a result, the problem of analysing and comparing microscopic traffic flow models with real microscopic data has become more actual. In this paper, we consider the NGSIM I80 data set [19] for calibration. In addition, for realistic simulation of system containing diverse of "driver-vehicle particles" (inter-driver variation) model parameters should be properly calibrated according to technical characteristics of the specific vehicle model [21].

When simulating trajectories by integrating time-continuous car-following models, standard integration schemes such as the simple Euler method, ballistic update, Heun's method and the the fourth-order Runge–Kutta are used [31]. In case of simulation of vehicular traffic on large scales in large cities, the number of vehicles could reach tens of thousands, which, for the microscopic approach, corresponds to the solution of tremendous systems of differential equations. Such systems have such a feature as slow and fast components. This fact suggests the use of multirate approach for numerical integration [32, 33]. The multirate solver for numerical integration is described in this paper. The solver is based on automatic identification of an integration step (microstep) for each component separately. The microstep is determined on the basis of the required accuracy and the estimate of the numerical integration error. Due to the multiplicity conditions several consecutive microsteps for each equation of the system are carried out within one macrostep. This scheme is used to solve large systems which describes the dynamics of the traffic flows.

The paper is organized as follows. In section 2 we specify the car-following model with explicit reaction time delay. Section 3 contains some results on stability analysis of the underlying model. Calibration methodology with respect to trajectory data is described in Section 4. In Section 5 we introduce the approach for quantitative assessment of car-following models by means of comparison of fundamental diagrams. Finally, Sections 6 finishes with discussions and conclusion.

2. Car-following Model

Oscillations of traffic flow are well-known phenomenon in congested car traffic [1-3, 22], which is usually described in terms of linear and nonlinear string instabilities [4, 23]. One of the reason of emergence of traffic flow oscillations is finite driver perception [4, 24]. This fact speaks in favor of considering the driver reaction time explicitly.

The specified car-following model is the modification of the well-know "Intelligent Driver Model" (IDM) [13]. The acceleration function is defined as continuous function of the vehicle's speed v and distance to its leader h [25]

$$\dot{v}(t) = w(v,h) \cdot a\left(1 - \left(\frac{v(t)}{V^0}\right)^\delta\right) + (1 - w(v,h)) \cdot a\left(1 - \left(\frac{s^*(v(t))}{h(t-\tau)}\right)^2\right),$$

(1)

where $s^*(v) = s_0 + Tv + cv^2$ is the "desired" distance, τ is driver's reaction time and a, b, V^0, s_0, T are the model parameters. The continuously differentiable function $w(v,h)$ is defined as follows

$$w(v,h) = \begin{cases} 0, h \in (-\infty, s^*) \\ -2t^3 - 3t^2 + 1, , h \in [s^*, s^* + D], \\ 1, h \in (s^* + D, +\infty) \end{cases} \quad (2)$$

where $t = \frac{h - s^*}{D} - 1$.

The distance between vehicles in a steady-state flow according to the IDM demonstrates unrealistic increase as the velocity approaches the desired speed V^0 [21] (Fig. 1). One of the feature of the model (1) is that the distance in a steady flow moving with speed v is equal to "desired" distance $s^*(v)$ which is defined as a quadratic velocity function. Taking into account reaction time explicitly allows sufficiently increase the realism of simulation results.
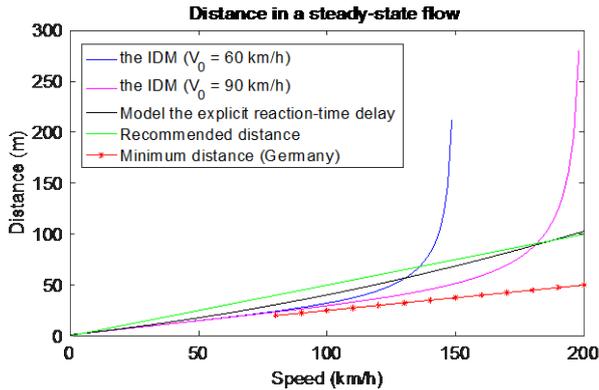

Fig. 1. *Distance in a steady state flow: the IDM, the model (1), recommended and minimum distance (Germany)*

3. Stability Analysis

In case of microscopic approach, the dynamics of many-body system is defined with system of differential equations (ordinary or delayed). Such systems admit uniform states, that is, all speeds and gaps are constant and equal. One of the method for benchmarking of the models is the stability analysis of the uniform solutions [4, 26]. Real observations of traffic flows proof the existence of non homogeneous configurations which are the formation and propagation of stop-and-go waves in case of high density values [22]. To sum up, elaborated models should admit unstable uniform solutions as well. It is worth to add, that oscillating solutions of corresponding systems of differential equations can have amplitudes which lead to backward motion and even crashes. Obviously, it is in contradiction with reality and should be controlled by means of stability conditions.

This section contains the results of stability analysis performed for car-following model (1) [25]. The local and string stability analysis of uniform solution is performed.

To study local stability we consider one vehicle following its leader. The acceleration of the follower is defined with (1) whereas the leader moves with constant speed v_L. The stability of the following stationary solution is investigated

$$(v = v_L, h = s^*(v)). \quad (3)$$

The following statement determines conditions on the parameters of the model for that the uniform solution is linearly stable.

Statement 1. The uniform solution (3) is linearly stable if and only if $\tau < \tau_{cr}, \tau_{cr} = \min_n\{\tau_{1,2}(n): \tau_{1,2}(n) > 0\}$. Here $\tau_{1,2}(n) = \frac{\pm arccos\left(-B^2/2C + \sqrt{B^4/4C^2 + 1}\right) + 2\pi n}{\beta}, n = 0, \pm1, \pm2, ..., \quad \beta^2 = \frac{-B^2 + \sqrt{B^4 + 4C^2}}{2}, B = -\frac{2a(T + 2cv)}{s_0 + Tv + cv^2}, C = \frac{2a}{s_0 + Tv + cv^2}$.

The local stability analysis (for a finite vehicle platoon with a leader travelling at a fixed speed) is different to the string stability, for vehicles on an infinite line or on a ring. The conditions for string

stability are more restrictive because they include convective perturbations as well, which can locally vanish.

String stability analysis performed for N vehicles on a ring of length L. Dynamics of this system is described as follows

$$\ddot{x}_i(t) = w(\Delta x_i(t), \dot{x}_i(t)) \cdot a\left(1 - \left(\frac{\dot{x}_i(t)}{V^0}\right)^\delta\right) +$$

$$+ \left(1 - w(\Delta x_i(t), \dot{x}_i(t))\right) \cdot a\left(1 - \left(\frac{s^*(\dot{x}_i(t))}{\Delta x_i(t - \tau)}\right)^2\right), \quad (4)$$

$$i = 1, ..., N,$$

$$\Delta x_i(t) = x_{i+1}(t) - x_i(t), \quad i = 1, ..., N - 1,$$

$$\Delta x_N(t) = x_1(t) - x_N(t) + L.$$

The uniform configuration

$$x_{i+1}^H(t) - x_i^H(t) = x_1^H(t) - x_N^H(t) + L = {}^1\!/_\rho,$$

$$x_i^H(t) = x_i^H(0) + v_\rho t \quad (5)$$

is a stationary solution of the system (4).

Statement 2. The uniform solution (5) of the system (4) is string stable if and only if $\frac{a(T + 2cv)}{s_0 + Tv + cv^2} > 1$ and $\tau < \tau_{cr}, \tau_{cr} = \min_n\{\tau_{1,2}(n): \tau_{1,2}(n) > 0\}$, where

$$\tau_{1,2}(n) = \frac{\frac{\theta}{2} \pm arccos\left(-\frac{\beta C}{2B\sin(\theta/_2)}\right) + 2\pi n}{\beta}, n = 0, \pm1, ..., \quad \beta =$$

$$\pm\sqrt{\frac{-C^2 + \sqrt{C^4 + 16B^2\sin^2(\theta/_2)}}{2}}, B = -\frac{2a(t + 2cv)}{s_0 + Tv + cv^2}, C = \frac{2a}{s_0 + Tv + cv^2}.$$

4. Calibration to Microscopic Observations

First calibration method allows to calibrate model parameters according to technical characteristics of the specific type of vehicle [21]. It could be applied to any car-following model with or without delay. Let the vehicle dynamics is defined by the following system with initial conditions for speed v and gap h

$$\begin{cases} \dot{v} = A(v, v_L, h, \theta) \\ \dot{h} = v - v_L \\ h(0) = h_0, v(0) = v_0 \end{cases}, \quad (6)$$

where θ is the vector of the model parameters, $h = s_L - s$. By means of numerical integration we can formulate the optimization problem with constraints

$$\begin{cases} \min_{\theta, \bar{s}, \bar{v}}\{w_1\|\tilde{s} - \bar{s}\|^2 + w_2\|\tilde{v} - \bar{v}\|^2\} \\ s_{i+1} - s_i = \Delta t \cdot (v_i - v_{L,i}) \\ v_{i+1} - v_i = \Delta t \cdot A(v_i, v_{L,i}, h_i, \theta), \\ i = 1, ..., N - 1 \\ s_1 = \tilde{s}_1, v_1 = \tilde{v}_1 \\ \theta \in [\theta_l, \theta_u] \\ stability \ conditions \end{cases}, \quad (7)$$

where $\|\cdot\|$ is the Euclidean norm, w_1 and w_2 are weight factors, $\{\tilde{s}_i\}_{i=1}^N$ и $\{\tilde{v}_i\}_{i=1}^N$ are real trajectory data, $\bar{s} = \{s_i\}_{i=1}^N, \bar{v} = \{v_i\}_{i=1}^N$ mean simulated positions and speeds according to the model, θ_l, θ_u designate low and upper bound for parameter values. Stability conditions are incorporated in optimization problem in order to get parameters which guarantee stable solution.

We consider two subsequent problems which are free acceleration and the emergency deceleration to avoid crash. Solution of the free acceleration problem provides values of two parameters a and δ while the desired speed V^0 is fixed to the maximum vehicle speed. Emergency deceleration problem provides values of two other parameters T and c, whereas reaction time τ and minimum distance to the leader are not related to the dynamic properties of the underlying car.

Seconds approach exploits NGSIM I-80 data set which provides information about 5648 vehicle trajectories on a road section of approximately 500 m [19]. To find the optimal parameter values of a car-following model with a non-linear acceleration function (1), one needs to solve a non-linear optimization problem numerically. Initializing the microscopic model with the empirically given speed

and gap, one computes the trajectory of the following car with a real trajectory of its leader [17, 18, 27]. Afterwards it is directly compared to the speeds $v^{data}(t)$ or gaps $s^{data}(t)$ provided by the trajectory data. Here we use global and platoon methods in order to estimate the ratio between inter-driver and intra-driver variations [27].

The calibration procedure aims at minimizing the difference between the measured and simulated dynamic variables. Any quantity which represents aspects of the driving behaviour can serve as an objective function, such as the gap s, speed v, speed difference Δv, or acceleration a [28,29]. To assess quantitatively the error between measured and simulated data, different objective functions can be used. The absolute error measure reads

$$S_s^{abs} = \frac{\sum_{i=1}^{n}\left(s_i^{sim} - s_i^{data}\right)^2}{\sum_{i=1}^{n}\left(s_i^{data}\right)^2}. \tag{8}$$

In case of platoon calibration the error measure must have a double sum, one over the trajectory pairs j, and one over time i.

5. Multirate numerical integration scheme for time-continuous car-following models

Let the dynamics of N vehicles moving one by one is described with the following system of ordinary differential equations

$$\begin{cases} \dot{v}_1 = a^{(1)}(v_1, h_1) \\ \dot{h}_1 = v_L - v_1 \\ \langle ... \rangle \\ \dot{v}_N = a^{(N)}(v_N, h_N) \\ \dot{h}_N = v_{N-1} - v_N, \end{cases} \tag{9}$$

with initial conditions $v_I(0) = v_{I0}, h_I(0) = h_{I0}, I = 1, ..., N$. For brevity, we rewrite the system (9) in the following way

$$\dot{x}(t) = F(x(t)), \qquad x(0) = x^{(0)} \tag{10}$$

where $x = (v_1, h_1, ..., v_N, h_N)^T$, $F_{2I-1} = a^{(I)}(v_I, h_I)$, $F_{2I} = v_{I-1} - v_I$.

Let us consider one macrostep of the underlying multirate scheme, which connects the solutions at the neighboring macrolevels t_{n-1} and $t_n = t_{n-1} + \Delta T$. Within one macrostep ΔT for each component x_i of the solution vector x one carries out k_i consecutive microsteps $\Delta t_i = \Delta T / k_i$ according to the basic numerical scheme, which is the explicit Euler method [30]. The multiplicity factors $\{k_i\}_{i=1}^{2N}$ are identified so as to satisfy the condition

$$\|x_n - \tilde{x}(t_n)\|_\infty < \varepsilon. \tag{11}$$

Here $\tilde{x}$ is the exact solution of the problem

$$\dot{x}(t) = F(x(t)), \qquad x(t_{n-1}) = x^{(n-1)} \tag{12}$$

One macrostep for the ith component is

$$\begin{cases} x_i^{(m+1)} = x_i^{(m)} + \Delta t_i \cdot F_i\left(x_1^{(m)}, ..., x_i^{(m)}, ..., x_{2N}^{(m)}\right) \\ x_i^{(m+2)} = x_i^{(m+1)} + \Delta t_i \cdot F_i\left(x_1^{(m)}, ..., x_i^{(m+1)}, ..., x_{2N}^{(m)}\right) \\ ... \\ x_i^{(m+k_i)} = x_i^{(m+k_i-1)} + \Delta t_i \cdot F_i\left(x_1^{(m)}, ..., x_i^{(m+k_i-1)}, ..., x_{2N}^{(m)}\right) \end{cases}$$

Performing some analytical transformations one obtains the estimation of local integration error for the speed of Ith vehicle after k_{2I-1} consecutive microsteps

$$est_{2I-1, k_{2I-1}} = \frac{\Delta T^2}{2}\left|\frac{1}{k_{2I-1}} a_v^{(I)} a^{(I)} + a_h^{(I)}(v_{I-1} - v_I)\right| \tag{13}$$

and for the gap of Ith vehicle after k_{2I} consecutive microsteps

$$est_{2I, k_{2I}} = \left|\frac{\Delta T^2}{2}\left(a^{(I-1)} - a^{(I)}\right) + \frac{\Delta T^3}{6}\left(jrk^{(I-1)} - jrkI\right)\right| \tag{14}$$

where $jrk^{(I)} = a_v^{(I)} a^{(I)} + a_h^{(I)}(v_{I-1} - v_I)$, $a_v^{(I)}$ and $a_h^{(I)}$ are partial velocity and gap derivatives of acceleration function $a^{(I)}$. Let ε_h and ε_v are relative accuracy for gap and speed respectively. The time stepping strategy for ΔT and Δt_{2I-1} is

$$\begin{cases} \left|\frac{\Delta T^2}{2}\left(a^{(I-1)} - a^{(I)}\right) + \frac{\Delta T^3}{6}\left(jrk^{(I-1)} - jrk^{(I)}\right)\right| < h_I \varepsilon \\ \frac{\Delta T^2}{2}\left|\frac{1}{k_{2I-1}} a_v^{(I)} a^{(I)} + a_h^{(I)}(v_{I-1} - v_I)\right| < v_I \varepsilon_v, \\ \qquad I = 1, ..., N \end{cases} \tag{15}$$

The value of Δt_{2I} is directly identified from (14). Values of $\Delta t_{2I-1}, \Delta t_{2I}$ and ΔT are calculated analytically.

Stability investigation is an integral part of the analysis of any numerical scheme [33]. The numerical scheme is stable if and only if all eigenvalues of the transition matrix are less than unity in modulus. Due to specific structure of the system (9) it is possible to identify all eigenvalues analytically. Consequently, stability conditions for the scheme with multiple steps are

$$\left|1 + r_{2I-1}^{k_{2I-1}} \pm \sqrt{(1 + r_{2I-1}^{k_{2I-1}})^2 - 4\frac{r_{2I-1}^{k_{2I-1}} - 1}{r_{2I-1} - 1} a_h^{(I)} \frac{\Delta T^2}{k_{2I-1}}}\right| < 2$$

where $r_{2I-1} = \left(1 + a_v^{(I)} \frac{\Delta T}{k_{2I-1}}\right), I = 1, ..., N$.

6. Discussion and Conclusions

The model with explicit reaction-time delay represents plausible free-acceleration and following dynamics of car traffic. Regarding traffic flow instabilities, observed in reality, one can simulate propagation of stop-and-go waves using this model [25]. The speed of the backward propagation of the wave in a computer experiment is consistent with real observations.
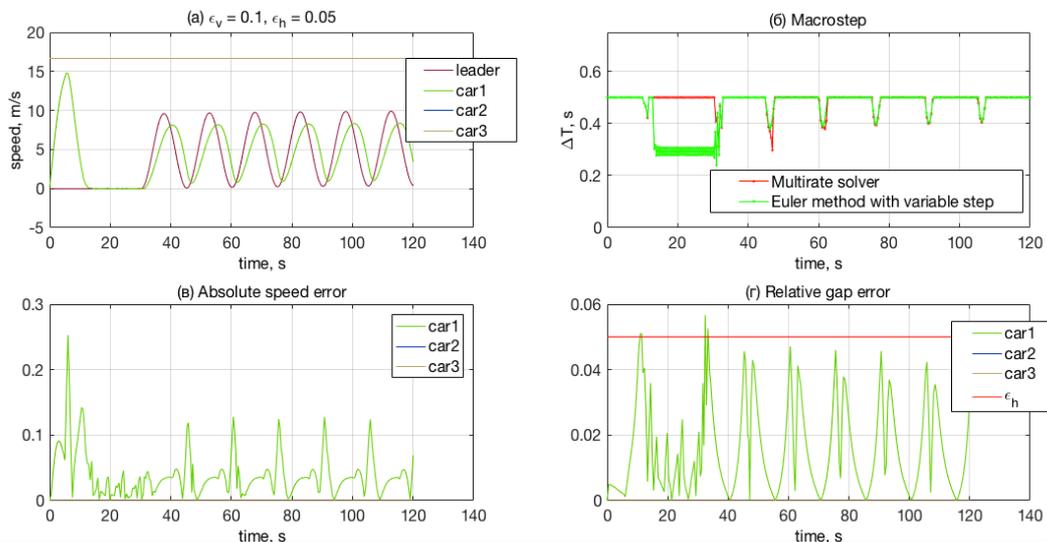

Fig. 2. *Speed time-series of 4 vehicles (upper-left). Macrosteps (upper-right). Absolute speed error (lower-left) and relative gap error (lower-right)*

We calculated stability conditions for the finite vehicles platoon (local stability) and for vehicles on a ring (string stability). Conditions on model parameters are obtained analytically. Numerical experiments confirm the correctness of these conditions [25]. Moreover, it is possible to find feasible values of reaction time τ, which simultaneously guarantee stability of the solutions.

Calibration of the underlying car-following model with explicit reaction-time delay according to NGSIM data set indicates its good fitting power. In other words, absolute minimums of the objective function are in the typical error range obtained in previous studies [17, 18]. Calibration approach exploiting technical characteristics of a car allows to obtain parameter values for specific vehicle models. In application to driving simulators, the quality of vehicular traffic simulation plays a major role, that is, each car should move according to its real dynamics, drivers behave in predictable manner and in accordance with traffic rules.

The numerical experiments show that the local error of the multirate integration method at the end of each macrostep does not exceed the required accuracy (Fig. 2). The results confirm that the speed of the method can be increased while preserving the requirements for the accuracy of the numerical solution if, at the same time, the speed of each component value is considered. Moreover, this multirate solver admits parallel implementation which will further accelerate calculations.

References

[1]   M. Mauch, M.J. Cassidy, Freeway traffic oscillations: observations and predictions. Transportation and traffic theory in the 21st century: Proceedings of the Symposium of Traffic and Transportation Theory, p. 653-672, 2002.

[2]   M. Schonhof, D. Helbing, Empirical features of congested traffic states and their implications for traffic modeling. Transportation Science, Vol. 41, Issue 2 (2007), p. 135-166.

[3]   B. Zielke, R. Bertini, M. Treiber, Empirical Measurement of Freeway Oscillation Characteristics: An International Comparison. Transportation Research Record, No 2088 (2008), p. 57–67.

[4]   M. Treiber, A. Kesting, Traffic Flow Dynamics: Data, Models and Simulation, Springer-Verlag Berlin Heidelberg, 2013.

[5]   M.J. Lighthill, G.B. Whitham, On kinematic waves: II. Theory of traffic flow on long crowded roads. Proc. R. Soc. London, Ser. A. Vol. 229 (1955), p. 281-345.

[6]   H.J. Payne, Models of freeway traffic and control. Simulation Council Proc. 28, Mathematical Models of Public Systems ed. by G.A. Bekey, Vol. 1 (1971), p. 51–61.

[7]   B.S. Kerner, P. Konhäuser, Cluster effect in initially homogeneous traffic flow. Phys. Rev. E., Vol. 48 (1993), p. 2335-2338.

[8]   A.A. Chechina, N.G. Churbanova, M.A. Trapeznikova, Two-dimensional hydrodynamic model for traffic flow simulation using parallel computer systems. Proceedings of the international conference of the numerical analysis and applied mathematics 2014. AIP Conference Proceedings, 1648 (2015), p. 530007.

[9]   K. Nagel, M. Schreckenberg, A cellular automaton model for freeway traffic. J. Physique I France, Vol. 2 (1992), p. 2221–2229.

[10]   J. Tian, G. Li, M. Treiber, R. Jiang, N. Jia, Sh. Ma, Cellular automaton model simulating spatiotemporal patterns, phase transitions and concave growth pattern of oscillations in traffic flow. Transportation Research Part B: Methodological, Vol. 93 (2016), p. 560-575.

[11]   M. Bando, K. Hasebe, Dynamical model of traffic congestion and numerical simulation. Physical Review E, Vol. 51(2) (1995), p. 1035-1042.

[12]   M. Bando, K. Nakanishi, A. Nakayama, Analysis of Optimal Velocity Model with explicit delay. Physical Review E Vol. 58(5) (1998), p. 5429-5435.

[13]   M. Treiber, A. Hennecke, D. Helbing, Congested traffic states in empirical observations and microscopic simulations. Physical Review E, Vol. 62(2) (2000), p. 1805-1824.

[14]   I. Lubashevsky, P. Wagner and R. Manhke, A bounded rational driver model. European Physical Journal B. Vol. 32 (2003), p. 243-247.

[15]   A. Tordeux, S. Lassarre, M. Roussignol. An adaptive time gap car-following model. Transportation Research Part B, Vol. 44 (2010), p. 1115-1131.

[16]   R. Jiang, Q. Wu, Z. Zhu, Full velocity difference model for a car-following theory. Physical Review E, Vol. 64(1) (2001), p. 017101.1-017101.4.

[17]   E. Brockfeld, D. Kuhne, P. Wagner, Calibration and Validation of Microscopic Traffic Flow Models. In Transportation Research Record, 1876 (2004), p. 62-70.

[18]   A. Kesting, M. Treiber, Calibrating Car-Following Models by Using Trajectory Data. In Transportation Research Record, Vol. 2088 (2008), p.148-156

[19]   NGSIM: Next Generation Simulation. FHWA, U.S. Department of Transportation. www.ngsim.fhwa.dot.gov. Accessed May 5, 2007.

[20]   Deutsches Zentrum für Luft- und Raumfahrt. www.dlr.de/cs/. Accessed May 5, 2007.

[21]   V. Kurtc, I. Anufriev, Local stability conditions and calibrating procedure for new car-following models used in driving simulators. Traffic and Granular Flow'13, p. 453-461, 2015.

[22]   Y. Sugiyama, M. Fukui, M. Kikushi, K. Hasebe, A. Nakayama, K. Nishinari, and S. Tadaki. Traffic jams without bottlenecks. Experimental evidence for the physical mechanism of the formation of a jam. New J. Phys., Vol. 10(3) (2008), p. 033001.

[23]   G. Orosz, R. Wilson, R. Szalai, G. Stepan, Exciting traffic jams: Nonlinear phenomena behind traffic jam formation on highways.   Physical Review E, Vol. 80 (2009), p. 046205.

[24]   P. Wagner, I. Lubashevsky, Empirical basis for car-following theory development. preprint cond-mat/0311192 (2003).

[25]   V. Kurtc, I. Anufriev, Car-following model with explicit reaction-time delay – linear stability analysis of the uniform solution on a ring. Mathematical Models and Computer Simulations, Vol. 29, No 4 (2017).

[26]   G. Orosz, R.E. Wilson, and G. Stepan, Traffic jams: dynamics and control. Proc. R. Soc. A, Vol. 368 (2010), p. 4455-4479.

[27]   V. Kurtc, M. Treiber, Calibrating the Local and Platoon Dynamics of Car-following Models on the Reconstructed NGSIM Data. Traffic and Granular Flow'15, p. 515-522, 2016.

[28]   V. Punzo, B. Ciuffo, M. Montanino, May we trust results of car-following models calibration based on trajectory data? TRB 2012 Annual Meeting (2012).

[29]   V. Punzo, M. Montanino, B. Ciuffo, Do we really need to calibrate all the parameters? Variance-based sensitivity analysis to simplify microscopic traffic flow models. IEEE Transaction on Intelligent Transportation Systems, 2014.

[30]   V. Kurtc, I. Anufriev, Multirate numerical scheme for large-scale vehicle traffic simulation. Mathematical Models and Computer Simulations, Vol. 8, No. 6 (2016), p. 744–751.

[31]   M. Treiber, V. Kanagaraj, Comparing numerical integration schemes for time-continuous car-following models. Physica A: Statistical Mechanics and its Applications, Vol. 419 (2015), p. 183-195.

[32]   M. Gunther, A. Kvœnø, P. Rentrop, Multirate partitioned Runge-Kutta methods. *BIT*, Vol. 41 (2001), p. 504-514.

[33]   V. Savcenco, W. Hundsdorfer, J.G.Verwer, A multirate time stepping strategy for stiff ordinary differential equations, BIT, Vol. 47 (2007), p. 137-155.