ROTATING OF A BALL IN CHAMBER FILLED WITH A FLUID

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Abstract: *Influece of form errors of a chamber filled with a liquid on the movement and stability of a ball, rotating in the chamber ([1-3]), is studied. Two cases of the influence of a chamber form errors on the forces, acting on the ball, are defined. The first case describes the situation when limitations on the rotor shift are not imposed and disturbances of the chamber form are set by spherical harmonics not above the first order. Then the chamber of a disturbed form, form the point of view of the reaction forces of the liquid and their moments, does not differ from a similar spherical chamber. In the second case disturbance of a chamber form are arbitrary and the rotor shift is supposed small. Then the force, acting on the rotor, depends on its displacement only, and the momentum does not depend on shift. A chamber of any form is equivalent to an ellipsoid. A rising here diflective moment tends to direct the angular speed vector along the small semiaxis of the ellipsoid, i.e., a stable position of the rotor appears.*

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1. Let us consider a Cartesian system of coordinates, the origin of which is located in the center of a rotating ball - a rotor. Otherwise, the system is arbitrary. Direction of angular speed vector $\vec{\Omega}$ is defined with the angle α (between the axis OZ and $\vec{\Omega}$ and β (between the axis OX and the projection of $\vec{\Omega}$ on the plane x, y). Also we will concider spherical coordinates $(r,\ \theta,\ \varphi)$, related to Cartesian formulas:

$$x = r\sin\ \theta\cos\ \varphi\ ,\ y = r\sin\ \theta\sin\ \varphi\ ,\ z = r\cos\ \theta.$$

Let us introduce characteristic thickness of the gap $\delta = (R_2 - R_1)/R_1$, where R_2 – is the chamber radius. Then we will move on to dimensionless variables, choosing the rotor R_1 radius as a unit of distance and $1/\Omega$ is a unit of time. The equation for the rotor surface is $r = 1$ and the equation for the chamber inner surface is $r = r(\theta,\ \varphi)$. The problem of viscous incompressible fluid flow in the gap between the rotor and the chamber within the Stokes approximation outside the field of mass forces is the following [4]:

$$(1)\qquad \Delta\vec{\omega} = 0,\ \ \vec{\omega} = \mathrm{rot}\vec{v},$$

$$\mathrm{div}\vec{v} = 0.$$

Boundary conditions are:

$$\vec{v}|_{r=1} = -\sin\ \alpha\sin\,(\varphi-\beta)\vec{e}_\theta - [\sin\ \alpha\cos\ \theta\cos\,(\varphi-\beta) - \cos\ \alpha\sin\ \theta]\vec{e}_\varphi,$$

$$\vec{v}|_{r=r(\theta,\,\varphi)} = 0.$$

Now let us use the assumption of small thickness of the gap between the layer. To do that, we will introduce a new radial variable ξ, that is, when the inner surface of the chamber is a sphere of radius R_2, we suppose that

$$\xi = (r-1)/\delta.$$

Equation for the rotor surface now is $\xi = 0$, and equation for the inner surface of the chamber is:

$$\xi = h(\theta,\ \varphi),\ \ h(\theta,\ \varphi) = (r(\theta,\ \varphi)-1)/\delta.$$

As for the equations of motion (1) and the boundary conditions we will retain principal terms of their asymptotics only at $\delta \to 0$. Then equations (1) can be rewritten with a precision of terms of order δ.

$$(2)\qquad \frac{\partial^2\omega_r}{\partial\xi^2} = 0,\ \ \frac{\partial^2\omega_\theta}{\partial\xi^2} = 0,\ \ \frac{\partial^2\omega_\varphi}{\partial\xi^2} = 0,$$

$$(3)\qquad \frac{1}{\delta}\frac{\partial v_r}{\partial\xi} + \frac{1}{sin\ \theta}\frac{\partial}{\partial\theta}(\sin\ \theta\, v_\theta) + \frac{1}{sin\ \theta}\frac{\partial v_\varphi}{\partial\varphi} = 0,$$

$$(\Delta = \frac{1}{\delta^2}[\partial^2/\partial\xi^2 + O(\delta)]),$$

where

$$(4)\qquad \omega_r = \frac{1}{sin\ \theta}[\frac{\partial}{\partial\theta}(\sin\ \theta\, v_\varphi) - \frac{\partial v_\theta}{\partial\varphi}],$$

$$(5)\qquad \omega_\theta = -\frac{1}{\delta}\frac{\partial v_\varphi}{\partial\xi},\ \ \omega_\varphi = \frac{1}{\delta}\frac{\partial v_\theta}{\partial\xi}.$$

Now the boundary conditions are as follows:

$$(6)\qquad v_r|_{\xi=0} = 0,\ \ v_\theta|_{\xi=0} = -\sin\ \alpha\sin\,(\varphi-\beta),$$

$$v_\varphi|_{\xi=0} = -\sin\ \alpha\cos\ \theta\cos\,(\varphi-\beta) + \cos\ \alpha\sin\ \theta,$$

$$v_r|_{\xi=h} = v_\theta|_{\xi=h} = v_\varphi|_{\xi=h} = 0.$$

The solution of equations (2) looks like

$$\omega_\theta = A(\theta,\ \varphi)\xi + B(\theta,\ \varphi),$$

$$\omega_\varphi = C(\theta,\ \varphi)\xi + D(\theta,\ \varphi).$$

Differentiating equation (4) twice in respect to ξ and using the first equation (2) and the resultant solutions for ω_θ and ω_φ, we will obtain the following equation

$$(7)\qquad \frac{\partial}{\partial\theta}(A\sin\ \theta) + \frac{\partial C}{\partial\varphi} = 0.$$

Now it is possible to express the components of the field of velocities v_θ and v_φ in terms of the introduced coefficients A, B, C, D, using equations (5) and the boundary equations

$$(8)\qquad v_\theta = -\sin\ \alpha\sin\,(\varphi-\beta) + \delta(C\xi^2/2 + D\xi),$$

$$(9)\qquad v_\varphi = -\sin\ \alpha\cos\ \theta\cos\,(\varphi-\beta) + \cos\ \alpha\sin\ \theta - \delta(A\xi^2/2 + B\xi).$$

Coefficients B and D can be obtained with the found above components of velocity and conditions on the chamber surface

$$(10)\qquad D = \frac{1}{\delta h}\sin\ \alpha\sin\,(\varphi-\beta) - \frac{Ch}{2},$$

$$B = -\frac{1}{\delta h}[\sin\ \alpha\cos\ \theta\cos\,(\varphi-\beta) - \cos\ \alpha\sin\ \theta] - \frac{Ah}{2}.$$

So, angular velocity components of the fluid can be expressed in terms of coefficients A and C

$$(11)\qquad v_\theta = (\frac{\xi}{h}-1)[\sin\ \alpha\sin\,(\varphi-\beta) + \frac{\delta}{2}Ch\xi],$$

$$(12)\qquad v_\varphi = (\frac{\xi}{h}-1)[\sin\ \alpha\cos\ \theta\cos\,(\varphi-\beta) - \cos\ \alpha\sin\ \theta - \frac{\delta}{2}Ah\xi]$$

and basing on the equation of continuity and condition $v_r|_{\xi=0} = 0$

$$v_r = \frac{\delta}{sin\ \theta}[\frac{\xi^2}{2}(\frac{sin\ \alpha}{h^2}(\frac{\partial h}{\partial\theta}\sin\ \theta\sin\,(\varphi-\beta) + \frac{\partial h}{\partial\varphi}\cos\ \theta\cos\,(\varphi-\beta)) - \frac{cos\ \alpha}{h^2}\frac{\partial h}{\partial\varphi}\sin\ \theta +$$

(13)
$$+\frac{\delta}{2}(\frac{\partial}{\partial\theta}(\sin\,\theta\, hC)-\frac{\partial}{\partial\varphi}(hA)))--\frac{\xi^3\delta}{6}(\frac{\partial}{\partial\theta}(\sin\,\theta\, C)-\frac{\partial A}{\partial\varphi})].$$

Using condition $v_r|_{\xi=h}=0$, we obtain one more equation for coefficients A and C

$$\frac{\partial}{\partial\theta}(\sin\,\theta\, h^3C)-\frac{\partial}{\partial\varphi}(h^3A)=-\frac{6\,sin\,\alpha}{\delta}[\frac{\partial h}{\partial\theta}\sin\,\theta\sin\,(\varphi-\beta)+$$

$$+\frac{\partial h}{\partial\varphi}\cos\,\theta\cos\,(\varphi-\beta)]+\frac{6\,cos\,\alpha}{\delta}\frac{\partial h}{\partial\varphi}\sin\,\theta. \tag{14}$$

We can define coefficients A and C, solving equation (14) together with equation (7), then it is possible to find the whole field of velocities.

Let us consider, first, a special case when function h does not depend on φ, that is the chamber has axial symmetry as related to axis oz. It is possible when the chamber is spherical and axis oz is directed along the line of the rotor and the chamber centres. In this case it is sufficient to differentiate the latter equation with respect to φ and substitute expression $\frac{\partial C}{\partial\varphi}$ from (7) into it. Then we will get the equation for function A:

$$\frac{\partial}{\partial\theta}[\sin\,\theta\, h^3\frac{\partial}{\partial\theta}(\sin\,\theta\, A)]+h^3\frac{\partial^2A}{\partial\varphi^2}=\frac{6\,sin\,\alpha}{\delta}\frac{dh}{d\theta}\sin\,\theta\cos\,(\varphi-\beta),$$

the solution of which should be in the following form:

$$A(\theta,\,\varphi)=\frac{6\,sin\,\alpha}{\delta}f(\theta)\cos\,(\varphi-\beta). \tag{15}$$

As a result we obtain an ordinary differential equation for $f(\theta)$:

$$[\sin\,\theta\, h^3(\sin\,\theta\, f)']'-h^3f=h'\sin\,\theta \tag{16}$$

(here prime means a derivative with respect to θ). It is required to find solution for this equation which is continuous at $0\le\theta\le\pi$. Actually, possibility to find such a solution depends on function $h=h(\theta)$, that is, on the chamber form. For a spherical chamber $h=1+\lambda\cos\,\theta$, where λ is relative displacement of centers (the distance between the centers is $|\lambda|\delta$) and the solution for equation (16) is as follows (compare [9]):

$$f(\theta)=\frac{\lambda}{\lambda^2+4}(\frac{1}{h}+\frac{1}{h^2}),\;\; h=1+\lambda\cos\,\theta. \tag{17}$$

In the general case when h depends on φ it is possible to "integrate" equation (7) first, representing it as condition of equality of mixed second derivatives of a new function $E=E(\theta,\,\varphi)$:

$$\sin\,\theta\, A=\frac{\partial E}{\partial\varphi},\quad C=-\frac{\partial E}{\partial\theta}. \tag{18}$$

The following expression can be chosen as E

$$E(\theta,\,\varphi)=-\int_0^\theta C\,(\bar\theta,\,\varphi)d\bar\theta,$$

for which the conditions written above can be verified directly if A and C are connected with equation (7). Let us substitute this function $E(\theta,\,\varphi)$ into equation (14):

$$\frac{\partial}{\partial\theta}(\sin\,\theta\, h^3\frac{\partial E}{\partial\theta})+\frac{\partial}{\partial\varphi}(\frac{h^3}{sin\,\theta}\frac{\partial E}{\partial\varphi})=$$

$$=\frac{6\,sin\,\alpha}{\delta}[\frac{\partial h}{\partial\theta}\sin\,\theta\sin\,(\varphi-\beta)+\frac{\partial h}{\partial\varphi}\cos\,\theta\cos\,(\varphi-\beta)]-\frac{6\,cos\,\alpha}{\delta}\frac{\partial h}{\partial\varphi}\sin\,\theta. \tag{19}$$

2. Let us consider the case when the chamber shape differs little from spherical. Let us set spherical form of the chamber in the form of $h_0=1+\lambda\cos\,\theta$ (axis oz along the line of centers). Then we will set the form, which differs little from spherical, by function

$$h=h_0+\mu h_1,\;\; |\mu|\ll 1$$

and we will look for a solution of equation (19) in the form of

$$E(\theta,\,\varphi)=E_0(\theta,\,\varphi)+\mu E_1(\theta,\,\varphi)+O(\mu^2).$$

Substituting h and E into equation (19) and equating the coefficients at the same degrees μ. We obtain equations for definition E_0 and E_1, where E_0, which satisfies the condition of norming, is already known from (15), (17) and (18):

$$E_0(\theta,\,\varphi)=\frac{6\,sin\,\alpha}{\delta}\frac{\lambda}{\lambda^2+4}(\frac{1}{h_0}+\frac{1}{h_0^2})\sin\,\theta\sin\,(\varphi-\beta). \tag{20}$$

Now the right part of the equation E_1

$$\frac{\partial}{\partial\theta}(\sin\,\theta\, h_0^3\frac{\partial E_1}{\partial\theta})+\frac{\partial}{\partial\varphi}(\frac{h_0^3}{sin\,\theta}\frac{\partial E_1}{\partial\varphi})=$$

$$=-3\frac{\partial}{\partial\theta}(\sin\,\theta\, h_0^2h_1\frac{\partial E_0}{\partial\theta})-3\frac{\partial}{\partial\varphi}(\frac{h_0^2h_1}{sin\,\theta}\frac{\partial E_0}{\partial\varphi})+$$

$$+\frac{6\,sin\,\alpha}{\delta}[\frac{\partial h_1}{\partial\theta}\sin\,\theta\sin\,(\varphi-\beta)+\frac{\partial h_1}{\partial\varphi}\cos\,\theta\cos\,(\varphi-\beta)]-\frac{6\,cos\,\alpha}{\delta}\frac{\partial h_1}{\partial\varphi}\sin\,\theta \tag{21}$$

is completely known and because it is lineary dependent on function h_1, which can be expanded into series with respect to spherical function, it is sufficient to consider spherical functions themselves as h_1 ([5])

$$h_0=P_n^m(\cos\,\theta)\cos\,m\,\varphi,\;\; h_1=P_n^m(\cos\,\theta)\sin\,m\,\varphi, \tag{22}$$
$$0\le m\le n,\;\; n=0,\,1,\,2,\ldots$$

Here P_n^m are adjoint Legendre functions of first kind. Let us substitute function h_1 and its derivatives into the right part of (21) and expand this part of the equation into Fourier series with respect to variable φ. If

$$E_1(\theta,\,\varphi)=\frac{a_0(\theta)}{2}+\sum_{m=1}^{\infty}[\,a_m(\theta)\cos\,m\,\varphi+$$
$$+b_m(\theta)\sin\,m\,\varphi],$$

then equation (21) decomposes into a finite system of ordinary differential equation. The system is finite because for functions (20), (22) in the right part of (21) a finite number of nonzero Fourier components is retained. From the condition of norming ($E(0,\,\varphi)=0$) $a_m(0)=0$. Let us look for continuous $0\le\theta\le\pi$ solution for equations in the form of Fourier series with respect to the orthogonal system of adjoint Legendre functions of appropriate weight with a fixed superscript. Let us clarify what we get at small values of n, that is in the first harmonics with respect to angle φ. If $m=0$, then h_1 = const and the equation for the chamber surface takes the following form:

$$h=1+\lambda\cos\,\theta+\mu h_1=(1+\mu h_1)[1+\frac{\lambda}{1+\mu h_1}\cos\,\theta],$$

If $n=1$, then for m two values are possible: $m=0,\,1$. At $m=0$ we get

$$h_1=\cos\,\theta\,,\;\; h=1+\lambda\cos\,\theta+\mu\cos\,\theta=1+(\lambda+\mu)\cos\,\theta$$

and function h is reduced to h_0 at $\lambda\to\lambda+\mu$. Let us take an arbitrary spherical function of order $n=1$ as h_1:

$$h=1+\lambda\cos\,\theta+\mu_1\sin\,\theta\cos\,\varphi+\mu_2\sin\,\theta\sin\,\varphi+\mu_3\cos\,\theta=$$

$$=1+(\lambda+\mu_3)\cos\,\theta+\mu_1\sin\,\theta\cos\,\varphi+\mu_2\sin\,\theta\sin\,\varphi.$$

Performing a turn, converting axis oz into axis oz' with the directing vector

$$\vec e=\frac{1}{\sqrt{(\lambda+\mu_3)^2+\mu_1^2+\mu_2^2}}\{\mu_1,\,\mu_2,\,\lambda+\mu_3\},$$

we will convert function h into function $h'=1+\lambda'\cos\,\theta'$, where

$$\lambda'=\sqrt{(\lambda+\mu_3)^2+\mu_1^2+\mu_2^2},\;\;\cos\,\theta'=\frac{z'}{r'}=$$

$$=\frac{1}{\sqrt{(\lambda+\mu_3)^2+\mu_1^2+\mu_2^2}}[\mu_1\sin\,\theta\cos\,\varphi+\mu_2\sin\,\theta\sin\,\varphi+(\lambda+\mu_3)\cos\,\theta].$$

So, for disturbance of the shape of chamber inner surface, which is set by spherical function h_1 of zero or first order, we have exact solution for equation (19), deriving from (20) with simple substitution of parametres. Geometrically zero order h_1 means small extension or compression of the chamber, and the first order means a small shift with a small turn, so that the spherical shape of the chamber does not change, though the chamber undergoes some deformation and shift. Therefore for any disturbance of the chamber surface with spherical function h_1 of the order not more than one there exits an effetive spherical chamber with close meanings of relative gap δ and relative shift $\lambda\delta$, giving the same values of the main forces of the fluid response to the rotor and the main vector of moment of these forces and hence leading to the same equations of the rotor motion and the same perturbating moment. Specifically, central position of the rotor equilibrium at small disturbances of the chamber shape of kind ϵh_1 remains unstable, as in the problem for the spherical chamber.

3. Let us consider a chamber of arbitrary shape which differs little from spherical. Taking into consideration that the equation for the arbitrary chamber inner surface $r = r(\theta, \varphi) > 1$ (absense of contact of rotating ball and the chamber is supposed). As a measure of the relative gap δ value, we choose average thickness of the layer with respect to the sphere

$$\delta = \frac{1}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\pi} [r(\theta, \varphi) - 1]\sin\theta\, d\theta.$$

Radial variable ξ is determined as before $\xi = (r-1)/\delta$. Then the chamber inner surface is set with equation $\xi = (r(\theta, \varphi) - 1)/\delta = h(\theta, \varphi)$. Let us suppose that function $r(\theta, \varphi)$, together with $h(\theta, \varphi)$ are twice continuously differentiated and decompose ξ into uniformely convergent series with respect to spherical functions:

(23) $$h(\theta, \varphi) = \sum_{n=0}^{\infty}\sum_{m=0}^{n} P_n^m(\cos\theta)(a_n^m \cos m\varphi + b_n^m \sin m\varphi),$$

where $P_n^m(\cos\theta)$ are adjoint Legender functions [5,6]

$$P_n^m(x) = \frac{(-1)^m}{2^n n!}(1-x^2)^{\frac{m}{2}}\frac{d^{m+n}(x^2-1)^n}{dx^{m+n}}, \quad 0 \le m \le n.$$

It is obvious from orthogonality correlation of Legendre functions and trigonometric functions the average value of function $h(\theta, \varphi)$ with respect to sphere

$$\bar{h} = \frac{1}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\pi} h(\theta, \varphi) P_0^0(\cos\theta)\sin\theta\, d\theta =$$

$$= \frac{1}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\pi} a_0^0 [P_0^0(\cos\theta)]^2 \sin\theta\, d\theta = a_0^0,$$

but it is obvious from definition ξ and δ that $\bar{h} = 1$, that is $a_0^0 = 1$ and

(24)

$$h(\theta, \varphi) = 1 + \sum_{n=1}^{\infty}\sum_{m=0}^{n} P_n^m(\cos\theta)(a_n^m \cos m\varphi + b_n^m \sin m\varphi).$$

Now, this function should be substituted into equation (19). Let us solve it, supposing that the chamber differs little from a sphere, concentric regarding the rotor. As for geometry it means not only that the chamber shape is close to spherical, but that the rotor center is close to the chamber center, that is coefficients a_n^m and b_n^m are small. As a measure of the chamber deviation from the sphere, which is concentric regarding the rotor, we will choose function $h-1$, equal to zero if and only if the chamber is spheric and its centre coincides with the rotor centre. We will consider as the value of this function its norm in Hilbert space L_2 of functions, which are square-integrable on the sphere

$$\| h - 1 \| = \sqrt{\int_0^{2\pi} d\varphi \int_0^{\pi} [h(\theta, \varphi) - 1]^2 \sin\theta\, d\theta}.$$

Because

$$a_n^m = \frac{(2n+1)}{2\pi}\frac{(n-m)!}{(n+m)!}\int_0^{2\pi} d\varphi \int_0^{\pi} [h(\theta, \varphi) - 1] P_n^m(\cos\theta)\cos m\varphi \sin\theta\, d\theta,$$

evaluating the latter integral with the help of Cauchy-Bunyakovskii inequality, we will obtain

$$|a_n^m| \le \sqrt{\frac{(2n+1)}{2\pi}\frac{(n-m)!}{(n+m)!}}\, \| h - 1 \|,$$

that is, every coefficient a_n^m is a small value, not less than the first order regarding $h-1$. At $a_n^m = 1$

(25)
$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial E_n^m}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 E_n^m}{\partial\varphi^2} = \frac{3\sin\alpha}{\delta}[(m+n)(n-m+1)P_n^{m-1}(\cos\theta)\sin((m-1)\varphi + \beta) + P_n^{m+1}(\cos\theta)\sin((m+1)\varphi - \beta)] + \frac{6\cos\alpha}{\delta} m P_n^m(\cos\theta)\sin m\varphi.$$

Solution of equation (25), which is continuous on the sphere, is the following:

(26)
$$E_n^m = -\frac{3\sin\alpha}{\delta n(n+1)}[(n+m)(n-m+1)P_n^{m-1}(\cos\theta)\sin((m-1)\varphi + \beta) + P_n^{m+1}(\cos\theta)\sin((m+1)\varphi - \beta] - \frac{6\cos\alpha}{\delta n(n+1)} m P_n^m(\cos\theta)\sin m\varphi.$$

Solution for the equation similar to (25), but at $b_n^m = 1$, is obtained from (26) with the turn for $\pi/2$.

In terms of main force $\vec{F}$ vector definition, acting on the rotor, the chamber of arbitrary shape, which differs little from the sphere with the centre in the centre of the rotor in the first assumption regarding $h-1$ does not differ from a spherical chamber with the same value of the gap δ, the centre of which is displaced regarding the rotor centre for the appropriate value in the appropriate direction. In particular, it is impossible to make the position of the rotor equilibrium stable, using selection of the chamber shape, if this position was unstable for the spherical chamber.

4. Let us place the beginning of Cartesian system of coordinates in the centre of spherical rotor. Let the chamber have the shape of an ellipsoid close to a sphere with semiaxis $1+\delta_1,\ 1+\delta_2,\ 1+\delta_3$. Let us direct the axis of coordinates along the main axis of the chamber. We will define coordinates of the chamber centre as (x_0, y_0, z_0). Then the equation for the chamber inner surface is the following:

(27) $$\frac{(x-x_0)^2}{(1+\delta_1)^2} + \frac{(y-y_0)^2}{(1+\delta_2)^2} + \frac{(z-z_0)^2}{(1+\delta_3)^2} = 1.$$

Let us consider $\delta_1, \delta_2, \delta_3, x_0, y_0, z_0$ to be so small that their squares and pair products can be neglected in comparison with themselves. Then equation (27) can be transformed into this equation

$$x^2 + y^2 + z^2 - 2(\delta_1 x^2 + \delta_2 y^2 + \delta_3 z^2) - 2(xx_0 + yy_0 + zz_0) = 1 + O(\delta_1^2 + \ldots + z_0^2).$$

In spherical coordinates this equation has the following form:

(28)
$$r^2(1 - 2\delta_1 \sin^2\theta \cos^2\varphi - 2\delta_2 \sin^2\theta \sin^2\varphi - 2\delta_3 \cos^2\theta) - 2r(x_0 \sin\theta\cos\varphi + y_0 \sin\theta \sin\varphi + z_0\cos\theta) = 1 + O(\delta_1^2 + \ldots + z_0^2).$$

Then we will define its approximate solution, linear with respect to $\delta_1, \delta_2, \delta_3, x_0, y_0, z_0$:

$$r = 1 + \delta_1 r_1 + \delta_2 r_2 + \delta_3 r_3 + x_0 s_1 + y_0 s_2 + z_0 s_3 + O(\delta_1^2 + \ldots + z_0^2).$$

Substituting r into equation (28) and equating coefficients at $\delta_1, \ldots, z_0$, to zero, we will find $r_1, \ldots, s_3$. So, with the accuracy up to small values of the second order:

$$r(\theta, \varphi) = 1 + \delta_1 \sin^2\theta \cos^2\varphi + \delta_2 \sin^2\theta \sin^2\varphi + \delta_3 \cos^2\theta + x_0 \sin\theta\cos\varphi + y_0 \sin\theta\sin\varphi + z_0\cos\theta.$$

Let us introduce average thickness of the gap δ according the formula: $\delta = (\delta_1 + \delta_2 + \delta_3)/3$, therefore:

$$h = \frac{r-1}{\delta} = \frac{1}{2}[\frac{\delta_1 + \delta_2}{2} \sin^2\theta + \delta_3 \cos^2\theta + x_0 \sin\theta\cos\varphi +$$

$$+ y_0 \sin\theta \sin\varphi + z_0 \cos\theta + \frac{\delta_1 - \delta_2}{2} \sin^2\theta \cos 2\varphi].$$

Decomposition of function h with respect to spherical harmonics is the following:

(29)

$$h = 1 + \frac{1}{\delta}[z_0 P_1^0(\cos\theta) - x_0 P_1^1(\cos\theta)\cos\varphi - y_0 P_1^1(\cos\theta)\sin\varphi + (\delta_3 - \delta)P_2^0(\cos\theta) + \frac{\delta_1-\delta_2}{6} P_2^2(\cos\theta)\cos 2\varphi],$$

where $P_1^0(\cos\theta) = \cos\theta$, $P_1^1(\cos\theta) = -\sin\theta$, $P_2^0(\cos\theta) = (3\cos^2\theta - 1)/2$, $P_2^2(\cos\theta) = 3\sin^2\theta$; that is the chamber centre shift regarding the rotor centre, assigned with parametres x_0, y_0, z_0, contributes to h in the form of harmonics with $n = 1$ only, though does not influence the momentum. Let us define the deviating moment $\vec{M}^p$, that is the component of $\vec{M}$, orthogonal to angle vector velocity $\vec{\Omega}$:

$$\vec{M}^p = \vec{M} - \frac{1}{\Omega^2}(\vec{M}, \vec{\Omega})\vec{\Omega},$$

$$M_x^p = -\frac{8\pi\mu' \Omega R_1^3}{15\delta^2}[(\delta_1 - \delta_2)\sin^2\alpha \sin^2\beta + (\delta_1 - \delta_3)\cos^2\alpha]\sin\alpha\cos\beta,$$

$$M_y^p = -\frac{8\pi\mu' \Omega R_1^3}{15\delta^2}[(\delta_2 - \delta_3)\cos^2\alpha + (\delta_2 - \delta_1)\sin^2\alpha\cos^2\beta]\sin\alpha\sin\beta,$$

$$M_z^p = -\frac{8\pi\mu' \Omega R_1^3}{15\delta^2}[(\delta_3 - \delta_1)\sin^2\alpha\cos^2\beta + (\delta_3 - \delta_2)\sin^2\alpha\sin^2\beta]\cos\alpha.$$

It is obvious that in the ellipsoidal chamber deviation moments occur in the first approximation with regard to small parameters

$$\frac{\delta_1 - \delta}{\delta}, \frac{\delta_2 - \delta}{\delta}, \frac{\delta_3 - \delta}{\delta},$$

but for spherical chamber $(\delta_1 = \delta_2 = \delta_3 = \delta)$ they are absent in the first approximation with regard to λ $(h = 1 + \lambda\cos\theta)$ and occur in the second approximation with regard to λ only. It is known from the exact solution [4]. Moreover, in the case of sphere, deviating moment depends on the rotor shift. If the rotor follows the circular path, the averaged with regard to period deviating moment is equal to zero. In the case of ellipsoidal chamber deviating moment in the first approximation does not depend on the rotor position.

Let us clarify the evolution of the vector angular velocity $\vec{\Omega}$ in the simplest case of isotropic spherical rotor (density from the centre only). In this case inertia tensor is spherical and is set with a scalar I, equal to inertia moment regarding any axis , crossing the centre. Vector $\vec{\Omega}$ evolution is defined with the equation of moments $I\frac{d\vec{\Omega}}{dt} = \vec{M}$. Let us substitute components of the moment into it and obtain the system of equations

(30)

$$\dot{\Omega}_x = -\frac{8\pi\mu' R_1^3}{3\delta I}(1 + \frac{\delta_1 - \delta}{5\delta})\Omega_x,$$

$$\dot{\Omega}_y = -\frac{8\pi\mu' R_1^3}{3\delta I}(1 + \frac{\delta_2 - \delta}{5\delta})\Omega_y,$$

$$\dot{\Omega}_z = -\frac{8\pi\mu' R_1^3}{3\delta I}(1 + \frac{\delta_3 - \delta}{5\delta})\Omega_z.$$

Its solution is the following:

(31)

$$\Omega_x = \Omega_x^0 \exp[-\frac{8\pi\mu' R_1^3}{3I\delta}(1 + \frac{\delta_1 - \delta}{5\delta})t],$$

$$\Omega_y = \Omega_y^0 \exp[-\frac{8\pi\mu' R_1^3}{3I\delta}(1 + \frac{\delta_2 - \delta}{5\delta})t],$$

$$\Omega_z = \Omega_z^0 \exp[-\frac{8\pi\mu' R_1^3}{3I\delta}(1 + \frac{\delta_3 - \delta}{5\delta})t].$$

We can always suppose (probably, renaiming the axes) that

(32) $0 < \delta_1 \le \delta_2 \le \delta_3$.

Then $\delta_3 = 3\delta - \delta_1 - \delta_2 < 3\delta$, whence it follows that:

$$0 < \frac{\delta_1}{\delta} \le \frac{\delta_2}{\delta} \le \frac{\delta_3}{\delta} < 3,$$

or

$$-1 < \frac{\delta_1 - \delta}{\delta} \le \frac{\delta_2 - \delta}{\delta} \le \frac{\delta_3 - \delta}{\delta} < 2,$$

that is addition to vector damping decrement $\vec{\Omega}$ from (31), dependent on non-spherical type of the chamber is not more than 20% in the direction of decrease and 40% in the direction of increase in comparison with a spherical chamber with the same relative thickness of the gap.

To study evolution of the vector direction $\vec{\Omega}$ we will rewrite the system of equations (30) in spherical coordinates. Therefore it is sufficient to find solution for this equation, satisfying initial condition $\beta(0) = \beta_0, 0 < \beta_0 < \frac{\pi}{2}$.

Such a solution is set by the formula

(33) $\beta(t) = \mathrm{arctg}[\mathrm{tg}\beta_0 \exp(-(A_2 - A_1)t)]$,

$$\alpha(t) = \mathrm{arcctg}[\mathrm{ctg}\alpha_0 \sqrt{\frac{1 + tg^2\beta_0}{exp(2(A_2 - A_1)t) + tg^2\beta_0}} \cdot \exp(-(A_3 - A_2)t)],$$

$\alpha(t) \to \frac{\pi}{2}$ for $t \to \infty$.

So, if at initial time the end of vector $\vec{\Omega}$ is in semisphere $-\frac{\pi}{2} < \beta_0 < \frac{\pi}{2}$ with the centre at the ellipsoidal chamber, then as time passes it is attracted to the end of this semiaxis, except the case of elongated ellipsoid of rotation when it is attracted to the equator along its meridian. In the opposite semisphere the same situation takes place: the end of vector $\vec{\Omega}$ evolves to the end of the small semiaxis (or, in case of elongated ellipsoid of rotation - to the equator. In this case every point of which is the end of the small semiaxis).

As a result we can conclude that in case of the ellipsoidal chamber direction of the small semiaxis is stable for vector $\vec{\Omega}$, which, under influence of deviating moment, is attracted to this direction from any initial position.

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