SOFTWARE ASSURANCE OF THE SYNTHESIS AND DESIGN OF HYPERBOLOID GEAR DRIVES

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Abstract: *The study presents a brief description of the type's software, applicable to the synthesis and design of gear transmissions. The main accent is put on the approach to the computer synthesis, for which the optimization process is carried out by the method of directed search of the optimal variant of a synthesized mechanism. The specific features of the programs, applied by the authors, oriented to the synthesis of spatial gear mechanisms with linear contacting teeth with face meshing (Spiroid and Helicon) are studied in detail.*

Keywords: HYPERBOLOID GEAR DRIVES, MATHEMATICAL MODELLING, SYNTHESIS, SOFTWARE ASSURANCE

1. Introduction

The contemporary requirements to the accuracy and reliability characteristics of machine products for the techniques to a great extend dictate the applied scientific methods for the technological synthesis, design and manufacture of gear drives [1 - 3]. In the processes of synthesis and design of the different types of gears, it is necessary to be solved complex set of problems, which considered all together define the desired optimal construction. In this case, an optimal construction means a gear transmission, which is capable to ensure the preliminary given kinematic and strength characteristics at the minimum cost for realization and exploitation. In essence, this is a system of requirements, related to different quality characteristics of the gear, namely [1, 4]:

- **geometric ones**, which control the kinematic exactness, smoothness of the working process, the character of the contact (placement of the contact spot; the orientation of the contact lines and the radius of the curvatures at contact points), related to the loading capacity of the gear sets and etc.;
- **dynamic ones,** which have impact on the noise and vibrations of the gear drives, the conditions for appearance of resonant phenomena and etc.;
- **strength ones** determining the durability and reliability of gear sets, including the transfer of nominal power in the process of rotations transformation with avoidance of "scoring", "pitting" and etc. on the active tooth surfaces of the synthesized mechanism;
- **economic ones**, that define the production costs (e.g., per unit of power), energy loss for the motions transformation (coefficient of efficiency), etc.

The realization of an adequate approach for the creation of the real gear drives requires this approach to be a complex one. This is consisted in considering of the required quality characteristics of the created gear mechanisms with the existing specific technological and manufacturing capabilities.

The choice of the approach should be realized in the process of synthesis and design of gear transmissions.

2. Aspects of the Computer Design of Gear Drives

The wide variety of gear drives used in industry and transport as reduction drives and multipliers, as well as the continuous pursuit of researchers to create new and improved gear mechanisms on one hand and on the other - the different and rapidly vitiating approaches to the mathematical modeling, synthesis and design make it practically impossible to create universal CAD systems. In connection with the mentioned above, a special attention should be paid to the extremely dynamic development of the modern technical computational tools and software applications. This often requires a revaluation not only of the way in which computer programs are organized, but also leads to informal changes in applied mathematical models [5]. The computer design is evolved, forming three types of software [6], in order to realize scientific studies in the field "Theory of gearing" and to provide an adequate scientific support for this type of manufacture.

First type. The programs, included here, are designed to study the influence of the different kinematic, constructive, technological and exploitation parameters on various quality characteristics of the studied gear drives. Essentially, this type of software is not subjected to a particular strategy, associated with the design of CAD systems. The elaborated mathematical models, algorithms and computer programs are designed to determine the influence of one or other real-life existing parameters on the qualitative characteristics of the concrete gearings. However, the programs created in this case can be used as software modules, which are elements of system of criteria for quality control of the synthesized gear mechanisms.

Second type. This group is consisted of computer programs organized on the basis of algorithms, which are contained in standardization documents [7], company methodologies [8] or handbooks [9, 10]. The program products, included here, are developed on the basis of algorithms for geometric and strength calculation of the traditional types of gear drives: cylindrical involute with external and internal mating gears, cylindrical worm gears, bevel gears with straight teeth and so on. It should be noted, that the algorithms used in these cases do not ensure the optimization in the synthesis and design of gear mechanisms. Secondly, this category of software can also include those products, through which the strength characteristics of the already geometrically and technologically synthesized gear drives [7] are examined. In that capacity, these computer programs can be treated as analysis instruments of the gear mechanisms.

Third type. The computer programs included in this category are those, which are based on the mathematical models, developed on the basis of the specially oriented scientific studies. For example, for Bulgaria, these are the computer programs that deal with the synthesis and design of Spiroid and Helicon gear sets [6, 11, 12], and with conical and hypoid gear mechanism – type Gleason [13-15]; and others. For the contemporary gear transmissions, including even classical gear mechanisms, which are treated in terms of current engineering requirements, the construction of new mathematical approaches to their geometrical, technological and strength synthesis is required. The optimization synthesis process in this case is realized by application of the ***method of direct search***. This method gives opportunity to reduce the number of calculated gear pairs, which compose the synthesized gear mechanism. It will be reminded, that the essence of this method is as follows:

- input parameters are defined, as well as those that will not be changed throughout the whole synthesis process;
- the variables parameters are determined as well the way of their variation, respectively;
- the process of changing of the defined variable input parameters compared to their initially given value continues, until the introduced optimization criteria are fulfilled;

- from the calculated pairs of conjugated gear sets, a final variant is chosen for which, there is the best satisfaction of the additional conditions (restrictions) introduced in the mathematical model.

In other words, the process of optimization synthesis and design of the third type of software is based on adequate iterative procedures, by which the desired solution is found by changing certain parameters.

3. Constructing of Computer Programs for Calculation of Hyperboloid Gears with a Linear Contact

The computer programs designed for the synthesis and design of linear contacting hyperboloid gear mechanisms belong to the third type software. Taking into account, the known methodological limitations when constructing this type software, the following sequence is followed, for creation of the system for the computer design of hyperboloid gears with linear contacting tooth surfaces.

3.1. Mathematical Modeling for the Synthesis and Design

When profiling of the kinematically conjugated surfaces, upon which the rotations transformation between crossed axes is carried out, the basic observed principles are the principles of T. Olivier. Thoroughly discussed in [6], it will be summarized only that part of them, which is directly related to the construction of the concrete computer programs. Two applied approaches to the construction of mathematical models for synthesis are formulated here: mathematical modeling, upon which the geometric, technologic and exploitation characteristics of the designed gear sets in a small vicinity of the pitch contact point is defined and optimized and mathematical modeling related to the ensuring of the qualitative characteristics in the entire mesh region.

It is obvious that the methodological difference between the two approaches for the synthesis of spatial gear drives requires to define in advance the adaptability of the future designed hyperboloid gear sets to one of the two approaches. The determination of the adaptability of the planned procedure for building an adequate mechano-mathematical model is a complex creative process, requiring the knowledge of both the theoretical content of the approaches to the synthesis and the specific technological and exploitation requirements characterizing the created products.

3.2. Principles of Organization of the Design Process

Here, the focused will be paid only on those principles, which are determining for the construction of computer programs for the synthesis of Spiroid and Helicon gears.

Determination of the groups of independent and variable input parameters influencing the design conditions. To the group of independent input parameters should be included a set of standardization modules, that determines the technological capabilities of the hobbing machines; coefficients that define the tooth geometry as a function of the modules; coefficients of frictions between the different pairs of materials applicable for producing of the toothing of the conjugated gear pairs; coefficients, linear and angular values associated with the design of the instrumental equipment and etc.

To the input data parameters, among which the variable ones are chosen, as a rule are included those which define the overall geometry of the calculated gear system. Here belongs the parameters determining the dimensions of the gear structure: the offset, distances from the offset to the planes in which the pitch circles lie; the angles defining the orientation of the above said planes relative to the pitch normal and etc. The variable input data include also those, from which the geometry of the conjugated active tooth surfaces depends: the independent coordinates of the tooth surfaces; their helical parameters; parameters which determine face width of the teeth, etc.

Introducing basic analytical relations, which are based on the chosen approach to the mechano-mathematical modeling. Here are in included the solutions of the fundamental tasks of the synthesis upon a pitch contact point and upon a mesh region with the application of the adequate geometric interpretations of the basic equation of meshing, namely [6]: the task for the synthesis of pitch circles; the task for the definition of the geometry of the active tooth surfaces by their linear and angular characteristics in the pitch contact point; the task for defining the singularity in the pitch contact point (without describing the analytical type of the tooth surfaces); the task for analytically defining the entire mesh region; formulations of relations, which are used to determine the optimal dimensions and placement of the region of mesh on the active surface and etc. This principle of organizing the computer design includes also the introduction of geometric and kinematic relations, intended for the reduction of the input parameter sets.

Constructing the complex process for the synthesis and design of hyperboloid gear drives. This is accomplished by defining the separate stages of the synthesis and design in their sequence and interconnection. This principle, applied in the design of each computer program, is in direct dependency by the type of functioning set of criteria. Those criteria determine the defined characteristics of the quality of the gear mechanism in dependence of the accepted approach to the mathematical modelling. A distinctive characteristic of the accepted principle for construction of the complex process for synthesis and design is the chosen approach for the estimation of the calculated option of gear mechanism.

4. Software Programs for Geometric and Technological Synthesis of Spiroid Gears

The shown considerations for construction of software programs, applicable to the synthesis of spatial gear mechanisms [6, 16] are also realized when constructing three types' software products for the design of Spiroid gears, which functional relations are shown in Fig. 1. Each one of the three main directions, illustrated there, has its own importance. It means that the user can restrict himself to use the results of only one program; to analyze and interpret these results and then after an adequate assessment, to go through the entire process shown in the figure.

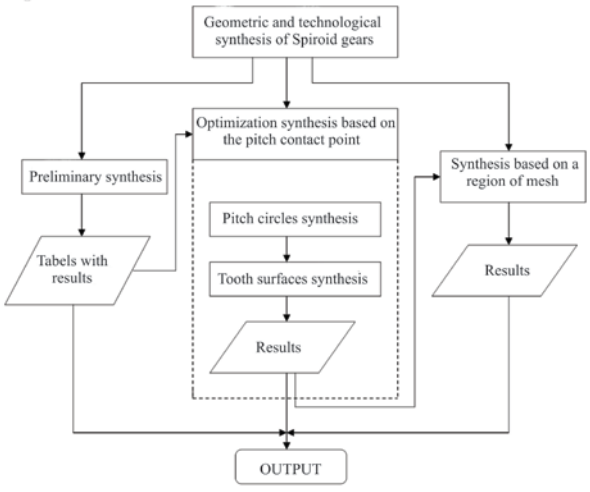

Fig. 1 *General block-scheme of the approaches for the synthesis of Spiroid gears*

Further bellow it will be treated only one of the directions - the central one, which includes the process of optimization geometric and technological synthesis of Spiroid gears.

4.1. Program for Optimization Geometric and Technological Synthesis of Spiroid Gears upon a Pitch Contact Point

This program consists of solving the following tasks:

- synthesis of the geometric pitch circles;

- synthesis of the active tooth surfaces of the Spiroid pinion and of the cutting tool (Spiroid hob);
- verifying the quality criteria to be fulfilled.

From the formulation of the defined tasks, it can be seen that the algorithm of this program corresponds to the approach to mathematical modeling for synthesis upon a pitch contact point. In this regard, when designing Spiroid gears, it is of particular importance to select the location of the pitch contact point in the fixed space. The placement of the pitch contact point (as a common point of the pitch circles and conjugated active tooth surfaces) affects on one hand on the common geometry of the designed gear system (overall dimensions of the gear pair) and on the other- on the geometry and proportions of the gear's teeth, as well as on the gears' quality (through the geometric, kinematic and strength characteristics of the conjugated gear pair).

For the purposes of this study, here will be briefly repeated some of the information contained in [6], which treats the geometric nature of the externally contacting geometric pitch circles. The pair of pitch circles $(H_1^c : H_2^c)$ of the mentioned type are circles H_i^c $(i = 1,\ 2)$, having only one contact point P. Their centers are places on the axes of rotations $i-i$ $(i = 1,\ 2)$ of the movable links, and the corresponding circles are perpendicular to those axes. Mutual position of the crossed axes is uniquely determined by the angle $\delta = constant$ between the skewed axes and by the offset (center distance) $a_w = constant$. The position of the geometric circles H_i^c $(i = 1,\ 2)$ in respect with the rotations axes $i-i$ $(i = 1,\ 2)$ and with their offset line is defined by four parameters for each circle. Five independent scalar equations determine the condition - that two circles have one common point. Hence, the mutual position of two circles is not defined in a unique way. It is a function of 5 independent parameters. The synthesis of geometric pitch circles is preceded by the decision, which five of the eight parameter (for both circles) to be chosen for independent ones and how to choose the intervals for their variation.

These five independent parameters, for this case, are chosen as: an angle δ between the axes of rotations $i-i$ $(i = 1,\ 2)$; the offset a_w; the angle δ_1, which is concluded between H_1^c and the pitch normal $m-m$ (half of the angle at the top of the pitch cone H_1^s of the Spiroid pinion); radius r_1 of H_1^c and the distance a_1 between the offset line of the gear and the plane in which H_1^c lies.

The ambiguity of the solution enables the possibility, that these free parameters to be changed discreetly within a certain limitations and among many pairs of geometric pitch circles to look for those ones, which parameters ensure that the preliminary defined requirements for the quality of the synthesized gear drives in the vicinity of the pitch contact point are satisfied. The criteria used in the program will be discussed below.

The program allows to choose the type of conic linear helicoid, applied as active tooth surfaces of the Spiroid pinion: convolute, Archimedean or involute ones. The calculation of the necessary and sufficient geometrical and technological parameters for the design of the Spiroid gear pair and cutting instruments is realized for the required type of conic helicoid.

The indicators that serve to control the quality of the gearing are significant for the design process. As it has already be mentioned, the dependence on the solution of the task for the synthesis of the pitch circles from concrete free parameters, should be searched among the optimal geometric, kinematic and technological quality characteristics in the vicinity of the pitch contact point. They will be briefly explained.

- **The basic technological criterion**. This is the main criterion related to the technology of elaboration of Spiroid gears. It is related to the decreasement in cutting tools nomenclature by ensuring the conditions for the design of Spiroid hobs with standard modules (hob parameters are functions of this module). This causes the requirement that the calculated in the pitch contact point module to coincide (with a given exactness) with any of the modules contained in the input array of standard modules.
- **Criterion that controls the singularity in the pitch contact point.** The constructed criterion is analytically described in [6, 17]. The insurance of the performance of this criterion helps to reduce the ordinary nodes from the mesh region of the synthesized gear drive. Hence, it leads to the improvement of its loading capacity, of the efficiency and of the durability. It will be reminded, that the elimination of the singularity of first order, by this criterion, is guaranteed in vicinity of the pitch contact point. The optimization, when using this criterion, is realized by the verification of the analytical dependencies introduced for each of the selected combination of the five independent variables.
- **Criterion for controlling the transmission angle of the normal force (pressure angle).** This criterion provides optimization of gear sets in terms of the transmission of normal forces from the pinion to the big gear (crown), when the gear mechanism is operated under the conditions of the rotations transformation at low-side driving.
- **Criterion for controlling the value of the Spiroid pinion spiral angle.** This criterion controls the value of this angle in the pitch contact point. Its values have to belong to definite intervals in accordance to the purpose of the design of the gear mechanism.

Here, it should specially be noted, as it is shown in [6], that the choice of the appropriate values of the pressure angle and the Spiroid pinion spiral angle of the longitudinal line of the active tooth surface $\Sigma_1^{(1)}$ in the pitch contact point substantially affect the efficiency of the gear drives. Therefore, if these geometric characteristics of the tooth surfaces of the Spiroid pinion are appropriately chosen, then an indirect control of the gear mechanism' efficiency is achieved. It also should be mentioned, that from the calculated equivalent variants of the synthesized gear mechanisms, from a geometrical and technological view point, the program allows to select that one which has the highest efficiency value for the computational (pitch) contact point.

- **Criterion related to the durability of the gear drive.** It controls the magnitude of the sliding speed at the computational contact point, depending on the chosen material for the toothing of the Spiroid gear - different types of bronze.
- **Criterion controlling the hydrodynamic conditions of meshing.** This optimization aims that the synthesized gear set has to obtain a maximum as a value –summed circumferential velocity $|\overline{V}_\Sigma|$ in the pitch contact point and minimum value of the angle Ω, which $\overline{V}_\Sigma$ concludes with the normal to the contact line in the pitch contact point.
- **Technological criteria for hobbing.** These criteria are related to the choice of the minimum value of the axial (normal) profile angle of the Spiroid hob, in order to provide optimal conditions for hobbing, both in terms of cutting the metal and in relation to the strength characteristics of the elements of the gear rack of the hob and others.

In number of cases of the design process, some of the initially independent parameters could be fixed due to the specific requirements (or example requirements for maximum sizes of the gear mechanism and the mutual position of the shafts of the gears), which results in reduction of the number of independent variables without limitation to search and find an optimal geometry of the tooth surfaces.

Input parameter of the programs are: number of Spiroid pinion threads; number of Spiroid gear teeth; offset; standard pressure angle; type of the Spiroid pinion (type of the tooth surfaces

of the Spiroid pinion); type of the bearing of the gear shafts (on two bearing supports or console); frequency of revolution and etc. Keys parameters will take values of 1 or 0 depending on whether a given criterion will be taken in consideration for the synthesis or not. For each of the free parameters should be chosen minimum and maximum values as well as the steps of variation. The independent cycles in the computer program are equal to the number of the free parameters.

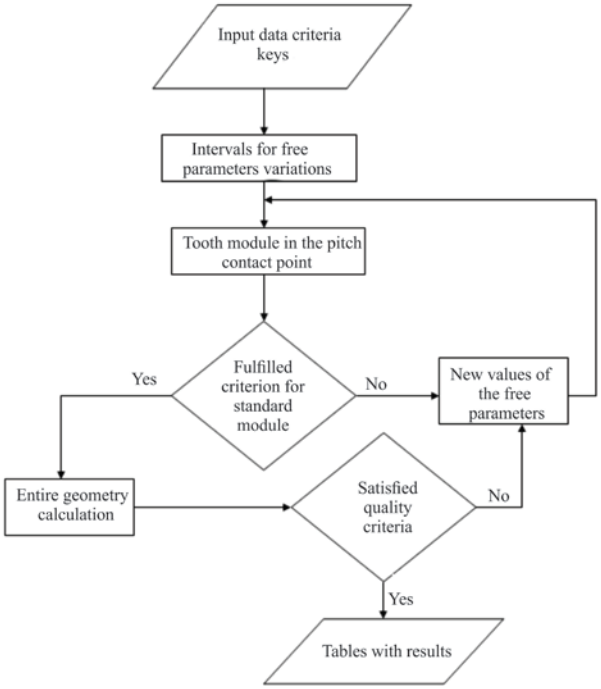

Fig. 2 *Program scheme for an optimization synthesis of a Spiroid gear pair*

In Fig. 2 the main block-scheme of the commented above program can be seen. The table of the results is consisted of: the basic geometric parameters of the Spiroid pinion and of the Spiroid gear, the constructive parameters of the Spiroid hob, geometric and constructive parameters of the gear pair, parameters related to the quality of meshing at the pitch contact point, such as forces in the pitch contact point, distances between bearings and pitch circles planes, efficiency, etc. It should be mentioned that, when the bearing supports are calculated, the distances between them and the pitch circles in real dimensions *mm* or dimensionless - as the ratio of the distances to the diameter of the pitch circle of the Spiroid gear should be given. In the program, all forces, that act in the pitch contact point and at the bearing supports are dimensionless in relation to the peripheral force, acting on the Spiroid gear. If in the beginning of the program, the torque on gear shaft or a torque on a pinion shaft is given, then this peripheral force has a concrete value and all forces and loads are calculated in [N]. Analogically, the sliding velocity is calculated referred to the Spiroid pinion angular velocity. If the Spiroid pinion numbers of driving motor revolutions per minute are given, then the sliding velocity is obtained in $[m/s]$.

5. Conclusion

On the basis of the shown in the study basic principles of constructing computer programs for the gear drive synthesis, the accent is put on the description of the mathematical model and based on it software for an optimization synthesis upon a pitch contact point of gear pairs of type Spiroid and Helicon. The main tasks, which are solved upon this approach for synthesis and design, ate shown briefly. The procedures described there are illustrated with an adequate block-scheme.

References

1. Abadjiev, V., D. Petrova. On the Synthesis and Computer Design of Spiroid Gears: A Review of the Bulgarian Approach. Journal of Theoretical and Applied Mechanic, Vol.33, No 3, Sofia, 2003, pp. 3-30.

2. Minkof –Petrof, K. Automated Technological Synthesis of Spatial Gearings. Proceedings of the Sixth World Congress on "The Theory of Machines and Mechanisms", Volume II, December 15 – 20, 1983, Delhi, India, pp. 807-810.

3. Minkov, K., V. Abadjiev, D. Petrova. Some Aspects of the Geometric and Technological Synthesis of Hypoid and Spiroid Gears, The Theory of Machines and Mechanisms, Proceedings of 7th World Congress of IFToMM, 17-22 September 1987, Seville, Spain, Pergamon Press, Volume 3, pp. 1275-1278.

4. Abadjiev, V. Bulgarian Approach to the Synthesis and, Design and Manufacture of Spiroid and Helicon Gear Drives, Proc. from the First Conference with International Participation. Machine and Machine Elements, 4-6 November 2004, Volume I, Sofia, 2004, pp. 70-78 (in Bulgarian).

5. Goldfarb,V. Aspects of Gears and Reduction Drives Automated Design. Gearing and Transmissions, Izhevsk, Assoc. Engineers, 1991, No 1, 20-24 (in Russian).

6. Abadjiev, V. Gearing Theory and Technical Applications of Hyperboloid Mechanisms, Sc. D. Thesis, Institute of Mechanics, Bulgarian Academy of Sciences, Sofia, 2007, 309p (In Bulgarian).

7. Bulgarian Standard 17108 (CT na CIV 5744 - 86). Cylindrical Involute Gears with Externally Meshing. Standardization Strength Calculation of the Gear's Teeth, G-15, 1999, Sofia, p. 124,(in Bulgarian).

8. Gear Engineering Standard. Bending, Stresses in Bevel Gear Teeth. Gleason Works, Rochester, N.Y., Copyright 1955, 1960, 1965, 28 p

9. Giznbrug, E., N. Golovanov, N. Firun, N. Hlebskiy. Gears Handbook of Mechanical Engineering. Leningrad, 1980, 416 p., (in Russian).

10. Dudley, D., J. Sprengers, D. Schroder, H. Yamashina. Gear Motor Handbook, Springer – Verlag, Berlin Heidelberg, 1995, 607 p

11. Abadjiev, V. On the Synthesis and Analysis of Spiroid Gears. Ph. D. Thesis, 1984, Sofia, 158 p. (in Bulgarian).

12. Abadjiev, V., D. Petrova. Aided Design of Hyperboloid Gears with High Gear Ratios. Theoretical and Applied Mechanics, Vol. 4, Publishing House of the Bulgarian Academy of Sciences, Sofia, 1984, pp. 24-30.

13. Anchev, A., K. Minkov, D. Petrova, Og. Kumchev. Synthesis and Analysis of Bevel Gears with Spiral Teeth, Publishing House of the Bulgarian Academy of Sciences, Sofia, 1980, p. 160

14. Minkov, K. Synthesis and Analysis of Semi-Conjugated Conic and Hypoid Gears. Sc. D. Thesis, LITMO, Leningrad, 1971, p. 170

15. Minkov, K. Mechanical and Mathematical Modeling of Hyperbolic Gears. Sc. D. Thesis, Sofia, 1986, 330 p. (in Bulgarian).

16. Abadjieva, E., Mathematical Models of the Kinematic Processes in Spatial Rack Mechanisms and Their Application, Ph. D Thesis, Institute of Mechanics- BAS, Sofia, Bulgaria, 2010, 165 p.

17. Abadjieva E., V. Abadjiev. Synthesis of Hyperboloid Gear Drives: Controlling of the Singularity on the Active Tooth Surfaces. Proc. of International Conference of Science, Engineering and Technology Innovations. March 15-17, 2017 Taipei, Taiwan, (Published on CD)