THE SUBJECTIVE MODEL OF RATIONAL CHOICE IN MULTI-AGENT SYSTEMS

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Abstract: *The paper considers the problem of modeling agent's choice, which allows explaining made decisions, as well as predicting possible options. The approach is based on the ideas of a subjectively rational choice. The subjectively rational choice supposes that a choice motivation is determined by external and internal factors. Internal factors represent the interests of a subject induced by his needs and ethical system he follows. External factors are induced by accepted obligations. The agent can estimate his satisfaction with the current goal-seeking state situation. The estimation might lead to changing a structure of interests, so the agent can choose it. The paper shows that when making decisions the agent uses three sets of alternatives as follows: controlling, structural and identification. This supposes the existence of three virtual sides, which choose relevant alternatives. The selection rules for such alternatives depending on subject's awareness of the situation and the structure of his interests are formed* by finding a compromise.
KEYWORDS: REFLEXIVE CONTROL, DECISION MAKING, MODEL, COMPROMISE.

1 Introduction

Agent's behavior control, when the agent has mentality and will, has become possible after appearing of decision-making models, which take into account his subjective understanding of a choice situation. These models helped estimating control efficiency before controlling [3]. It should be noted, that a decision-making process was an uncontrolled factor in a normative decision-making theory.

The development of the idea of subjective rational choice [3] helped to:

1) explain decision-making by a subject in specific situations; 2) predict possible reactions of the other subject in various situations (for a decision-maker);

3) solve a problem of active prediction when a control side creates an appropriate image of the future for a controlee.

A subjective rational choice supposes that the choice motivation is determined by external and internal factors. Internal factors reflect subject's interests, which are induced by his needs and an ethical system. External factors are connected with obligations. Affective evaluation of the level of subject's satisfaction of current choice situation, as shown in [4], might lead to changing the structure of subject's interests, so he is able to choose it. Subject's preferences in a selection process reflect his interests, thus we can determine a number of G alternatives of a preference structure (structural alternatives according to [1]).

2 Background assumptions

1. Subject's choice is based on his view of a choice situation.

2. The components of his view reflect various aspects of understanding choice situation by the subject and create an information structure of representations. Many optional representations will be X.

3. For many surrounding conditions S a set of observed surrounding conditions meets the condition $S \cap X \neq \varnothing$, i.e. subject's view might include an objective part, as well as a phantom one.

4. The subject chooses structural alternatives depending one valuation of the level of satisfaction of choice situation property values.

5. Formation of the view is based on the procedures of perception, awareness and analysis according to subject's cognitive capabilities.

According to the abovementioned assumptions, the subject uses three sets of alternatives in decision-making: control C (modes of action), structural G and identification X. Therefore, it is possible to assume the existence of three virtual sides, which choose corresponding alternatives. A choice rule for such alternatives depending on understanding of the surrounding and the structure of interests by a subject will be called *strategies* here.

Let us assume that decision-making has several cyclical steps, and modes of action are chosen at each step $n = 1, 2, ...$ from the set C depending on the view of surrounding conditions $x \in X$. This is due to the fact that a joint over conscious (intuitional) and conscious (formal) analysis of surrounding conditions allow firstly accepting a vaguely realizable decision in multiple iterations, then more clear and grounded decision. There are also some restrictions $C_x \subseteq C$ to allow the choice of alternatives depending on the vision of surrounding conditions $x \in X$. Process dynamics in subject's surrounding is out of reach for direct perception, therefore its representations are formed using identification with the idea of choosing a vision alternative depending on the observed conditions. Here there are restrictions $X_s \subseteq X$ to allow representations as identification alternatives depending on the observed conditions $s \in S$.

Proceeding from these assumptions, according to [1], we introduce the strategy determinations.

A single-valued transformation $\lambda : X \to C$ so that $\lambda(x) \in C_x$, $x \in X$, is called a choice or control function; an ordered set $(\lambda_1, ..., \lambda_n) \equiv \lambda_1^n$ is called a choice strategy on the length horizon $n < \infty$; $\lim\{\lambda_1^n\} = \lambda_1^{n\infty}$ when $n \to \infty$ is a strategy directed to achieving a local ideal, which determines subject's reason for existence.

A single-valued monotone transformation $\xi : S \to X$ so that $\xi(s) \in X_s$, $s \in S$, is called an identification function; an ordered set $(\xi_1, ..., \xi_n) \equiv \xi_1^n$ is called an identification strategy on the length horizon $n < \infty$; a consecutive order $\{\xi_1^n, n = 1,2,...\}$ is called an identification strategy on the limited horizon. Due to the fact, that the subject tends to forming useful vision, so there is $\lim\{\xi_1^n\} = \xi^\infty$ при $n \to \infty$.

As the sets S and X meet the condition $|S| > |X|$, therefore a single-valued transformation ξ: $S \to X$ causes separation of the set S on subsets

$$\xi^{-1}(x) = \bigcup\{s \in S : \xi(s) = x\} \subset S, \ x \in X.$$

Subsets $\xi^{-1}(x) \subset S$, $x \in X$ are associated sets, i.e. any element $s \in \xi^{-1}(x)$ uniquely determines appropriate representation $x \in X$. Therefore, it can be said that subsets $\xi^{-1}(x) \subset S$, $x \in X$ form classes of equivalent representations. It allows using the methods of the fuzzy sets theory to formalize subject's representations, e.g. as described in [4].

A structure alternative $\gamma_n \in G$ chosen at the moment n is *a structure choice at* n-th step of decision-making; an ordered set $(\gamma_n, ..., \gamma_1) \equiv \gamma_1^n$ is a *structure choice* strategy on a decision-making horizon of the length $n < \infty$; a consecutive order $\{\gamma_1^n, n = 1, 2, ...\}$ is a structure choice strategy on the limited horizon. As the

subject tends to the correspondence between its structure of interests and requirements of the accepted ethical system, so there is $\lim\{\gamma_1^n\} = \gamma^{\infty}$ when $n \to \infty$.

3 A decision-making model with changing preference structure

According to [4] a selection criteria for a control strategy have the meaning of a desired specific value of a purposeful condition based on the result with formalization, which has the formula for the utility function $E\varphi^g(C \times S \times X)$ that depends on the structure alternative $g \in G_s$ on a parameter. As the control process starts with a certain situation $x \in X$, then the criterion $E\varphi_n(\lambda_1^n \mid \gamma_1^n)$ will also depend on the situation $x \in X$ as from the initial condition. A number of situation X is finite, so the criterion $E\varphi_n(\lambda_1^n \mid \gamma_1^n)$ will be definitely represented by a vector in the space R^X of the dimension |X|. Its parts will be written as $E\varphi_n(\lambda_1^n \mid \gamma_1^n)(x)$, $x \in X$. According to the choice result, the subjects uffers emotionally, so the quality of the structure choice strategy γ_1^n should be described as a criterion meaning "choice results satisfaction". Therefore, the quality of the structure γ_1^n is natural to describe by a folding of an expected utility vector $E\varphi_n(\lambda_n \mid \gamma_1^n) \in R^X$ in a certain composite function $\mu : R^X \to R^1$. Then a strategy quality criterion γ_1^n might be written as $\mu_n(\lambda_1^n \mid \gamma_1^n) = \mu(E\varphi_n(\lambda_1^n \mid \gamma_1^n)) \in R^1$.

The subject associates its representation quality with evaluation of possibilities of achieving desired conditions when controlling $c \in C$, as well as with the possibility of extending the number of $C\uparrow$ by including efficient alternatives. The paper [6] uses the terms of the linguistic variable "utility", which are based on the values $E\varphi_n(\lambda_1^n \mid \gamma_1^n)$, as a representation estimation criterion. In these conditions utility estimates will depend on control strategies λ_1^n, a structure choice γ_1^n as on postulated conditions. The "utility" criterion will be labeled as follows $\psi_n(\xi_1^n \mid \lambda_1^n, \gamma_1^n)$. As identification starts with a certain state $s \in S$, this criterion will depend on the state $s \in S$, assigned as the initial condition. Here the set of states S is finite, so the identification criterion will be represented by a vector $\psi_n(\xi_1^n \mid \lambda_1^n, \gamma_1^n)$ in the space R^S of the dimension $|S|$.

In a goal-seeking state situation of the quality of control strategies and structure choice is described by criteria $E\varphi_n(\lambda_1^n \mid \gamma_1^n) \in R^X$ and $\mu_n(\gamma_1^n \mid \lambda_1^n) \in R^1$ respectively. They have a meaning of specific value by a result and satisfaction with choice results. The identification strategy quality is described by the criterion $\psi_n(\xi_1^n \mid \lambda_1^n, \gamma_1^n) \in R^S$, which has a meaning of representation utility to achieve desired states. The use of introduced criteria assumes determination of corresponding information structures or models that allow making an appropriate choice.

Let us assume the existence of an information structure of representations I, which reflects subject's knowledge and experience on: modes of action (control), his own interests and preferences, dynamics of surrounding transition into different states. Therefore, it is likely that there is a structural transformation of this structure into an information structure, which enables creating a specific value criterion $E\varphi_n(\lambda_1^n \mid \gamma_1^n)$ and a domain model. Let us call such transformation a *"specific value transformation"*, and the induced information structure will be called "information structure of a specific value of a goal-seeking state situation by a result" and designated as $U = U(I)$.

In a similar way, if there is a structural transformation of the structure I into an information structure, which enables creating an identification criterion $\psi_n(\xi_1^n \mid \lambda_1^n, \gamma_1^n)$ and identification procedure models, then we will call such transformation "identification transformation" and designate it as R, an induced information structure will be called "*identification information structure*" and designated as $R = R(I)$.

Subject's representation about a goal-seeking state situation is subjective and qualitative, based on observations and analysis of the surrounding transition process affected by control $c \in C$ into various states $s \in S$. Let us indicate the rule of such transition using $q^g(S \mid S \times C)$ from $S \times C$ into S. Actually, the subject for estimation of possible result value uses the model $Q^g(X \mid X \times C)$ from $X \times C$ into X constructed by identification strategy results ξ_1^n. When constructing we take in to account control strategies λ_1^n, a structure choice γ_1^n, or it is defined by such strategies. It means that a transformation of the operational function $q^g(S \mid S \times C)$ into function of subject's understanding of the surrounding processes $Q^g(X \mid X \times Y)$ is possible only in the aposterior mode depending on used strategies $(\lambda_1^n, \gamma_1^n, \xi_1^n)$. Such transformation and construction of the desired specific value criterion $E\varphi_n(\lambda \mid \gamma_1^n)$ is possible when "utility" information structures are formed successively depending on used strategies. This condition is written as $U_n = U(\lambda_1^n, \gamma_1^n, \xi_1^n)(I)$, $n = 1,2,...$ As this condition is necessary for forming the desired utility criterion and a domain model, so it should be pointed out every time it is used. Note that the criterion $E\varphi_n(\lambda_1^n \mid \gamma_1^n)$ tacitly depends on the identification strategy ξ_1^n due to introduction an induced structure U_n into the choice model. As it was stated above, the criterion $\mu_n(\gamma_1^n \mid \lambda_1^n) \in R^1$ of the structure choice quality is determined by a criterion folding $E\varphi_n(\lambda_1^n \mid \tau_1^n) \in R^X$. Generality of information structure of their formation allows writing

$$\begin{cases} E\varphi_n(\lambda_1^n \mid \xi_1^n) \\ \mu_n(E\varphi_n(\xi_1^n \mid \lambda_1^n)) \\ U_n = U(\lambda_1^n, \gamma_1^n, \xi_1^n)(I). \end{cases}$$

To construct identification criterion we need to use a specific function, which would have a meaning of "utility". For this purpose it is necessary to construct verbal estimates on the values of a function $E\varphi^g(S \times X \times Y)$. A required transformation exists and might be performed in the aprior mode (i.e. before choosing decisions). Such transformation is determined by the subject as regard to a fuzzy measure, which might be constructed when the defined function is $q^g(S \mid S \times C)$ from $S \times C$ into S. As its analogue has a form of $Q^g(X \mid X \times C)$ in subject's mind, and he can define it in a unique manner in the information structure I, therefore, there is no need in additional transformations. "Representation utility" function construction depletes a necessary structural transformation. We will call it identification structural

transformation and define as *R*. The induced information structure will be called "Representation utility" information structure and defined as $R = R(I)$.

Taking into account these reasons an identification criterion is written as follows:

$$\begin{cases} \psi_n(\xi_1^n \mid \lambda_1^n, \gamma_1^n) \\ R = R(I) \end{cases}$$

The induced definitions and constructions show that quality criteria for strategies are different and interdependent. Therefore the choice problem has game meaning and is reduced to searching as table compromise between aiming at maximizing desired specific value of a goal-seeking state by a result and minimizing possible loss due to wrong actions. Such compromise is called *balance*.

It should be noted that the information structure $U_n = U(\lambda_1^n, \gamma_1^n, \xi_1^n)(I)$, which is a base for the criterion $\mu_n(E\varphi_n(\gamma_1^n \mid \lambda_1^n))$, must be formed consequently depending on used strategies. Thus, required balances will be interdependent not only at each step $n = 1, 2, \ldots$ of forming decisions, but they will also depend on the decisions chosen at the previous steps. Considering this fact, it is natural to call balances *dynamic*.

The triple of strategies $\{\overset{o}{\lambda}{}_1^n, \overset{o}{\gamma}{}_1^n, \overset{o}{\xi}{}_1^n\}$, which meet the conditions

$$\begin{cases} \begin{cases} E\varphi_n(\overset{o}{\lambda}{}_1^n \mid \overset{o}{\gamma}{}_1^n) \geq E\varphi_n(\lambda_1^n \mid \overset{o}{\gamma}{}_1^n) \quad \forall \lambda_1^n, \\ \mu_n(\overset{o}{\gamma}{}_1^n \mid \overset{o}{\lambda}{}_1^n) \geq \mu_n(\gamma_1^n \mid \overset{o}{\lambda}{}_1^n) \quad \forall \gamma_1^n, \\ U_n = U(\overset{o}{\lambda}{}_1^n, \overset{o}{\gamma}{}_1^n, \overset{o}{\xi}{}_1^n)(I) \end{cases} \\ \begin{cases} \psi_n(\overset{o}{\xi}{}_1^n \mid \overset{o}{\gamma}{}_1^n, \overset{o}{\lambda}{}_1^n) \geq \mu_n(\xi_1^n \mid \overset{o}{\gamma}{}_1^n, \overset{o}{\lambda}{}_1^n) \quad \forall \xi_1^n, \\ R = R(I), n = 1,2,..... \end{cases} \end{cases}$$

are called *dynamic balances*.

According to the abovementioned assumptions, the number of cycles that form decisions is unlimited. Therefore, dynamic balances must be meaningful, including the situation when $n \to \infty$.

For this purpose, it is natural to require fulfillment of the following additional conditions:

1) when $n \to \infty$ strategy quality criteria must tend to specific limits;

2) such limits cannot depend on the initial conditions.

As criteria are not assigned in an explicit form, then realization of these properties is not explicit. It requires as signing necessary properties and then indicating criteria in the explicit form, which satisfies these properties.

According to the induced assumptions, quality criteria of stationary strategies $\lambda^n, \gamma^n, \xi^n$ when $n \to \infty$ have limits. Therefore the triple of stationary strategies $(\overset{o}{\lambda}{}^{\infty}, \overset{o}{\gamma}{}^{\infty}, \overset{o}{\xi}{}^{\infty})$ is called stationary balances if there are limits that meet the conditions:

$$\begin{cases} \begin{cases} \varphi(\overset{o}{\lambda}{}^{\infty} \mid \overset{o}{\gamma}{}^{\infty}) \geq \varphi_n(\lambda^{\infty} \mid \overset{o}{\gamma}{}^{\infty}), \forall \lambda^{\infty} \\ \mu(\overset{o}{\gamma}{}^{\infty} \mid \overset{o}{\lambda}{}^{\infty}) \geq \mu_n(\gamma^{\infty} \mid \overset{o}{\lambda}{}^{\infty}), \forall \gamma^{\infty} \\ U = U(\overset{o}{\lambda}{}^{\infty}, \overset{o}{\gamma}{}^{\infty}, \overset{o}{\xi}{}^{\infty})(I) \end{cases} \\ \begin{cases} \psi(\overset{o}{\xi}{}^{\infty} \mid \overset{o}{\lambda}{}^{\infty}, \overset{o}{\gamma}{}^{\infty}) \geq \psi(\xi^{\infty} \mid \overset{o}{\lambda}{}^{\infty} \overset{o}{\gamma}{}^{\infty}, \forall \xi^{\infty}; \\ R = R(I) \end{cases} \end{cases}$$

As a result, the content of the choice modeling problem consists in finding a compromise between aiming at achieving a maximal desired specific value by a result and minimal loss from wrong representations taking into account their mutual dependence. According to the equilibrium solution principle, compromise must be "un improvable" equally by all parts of interests.

When achieving such compromise it is fair to say that subject's interests are materialized with "the best result". Provided that dynamic balances meet the requirements of asymptotic stationary, it is also fair to say that subject's interests are materialized with "the best result" on the unlimited horizon, including $n \to \infty$. It follows that dynamic balances determine the meaning and the method of interests materializing with "the best result". Thus, dynamic balances naturally determine *internal aim* when making decisions.

4 Information structures in decision-making

The formal descriptions introduced determine not only the conditions of decision-making, but also proper a priori information carriers. Together, they form a set of the following formal properties:

S is an environment state set; $\beta(S)$ is a priori possibility distribution for a state set; *X* is a situation set; $X_S \cap X \neq \varnothing$ stands for the limitations determining the presence of the "right" ideas as diagnostics alternatives depending on $s \in S$ states; *C* is a control alternative set; $C_x \subseteq C$ means control alternative feasibility limitation depending on $x \in X$ situations; *G* is a structural alternative set; $q^g(S \mid S \times C)$ is a transitional function of $S \times C$ to *S*; $E\varphi^g(C \times (S \times X))$ stands for the utility function representing a priori preferences for $c \in C$ alternatives depending on $s \in S$ states, $x \in X$ situations and $g \in G$ structural alternatives.

This set defines an a priori information structure which is to be set according to decision-making rules.

The peculiarity of conditions of an information structure is that it is supposed to include a task of both states and situations, the choice of control actions depending on situations which, being qualitative characteristics, representing relationship to a state, are inaccessible for direct observation and need preventive maintenance.

Under these conditions, the regularity of situation dynamics cannot be set a priori. Therefore, decision-making rules suggest posing the laws of dynamics states only, defined by the $q^g(S \mid S \times C)$ transition function from $S \times C$ to *S*. In this sense, information given a priori is minimal.

In the conditions of a priori information deficiency, the minimum structure can be incomplete. Then it is necessary to introduce plausible assumptions (in the form of hypothesis set Γ) which would allow formulating a problem definition as some approach of the initial task [2]. Let us assume, for example, that in the basic information structure transfer function $q^g(S \mid S \times C)$ is not given, but there is set of hypotheses Γ of it. Then technically it can be assumed that transfer function $q^{(g,\gamma)}(S \mid S \times X)$ depends on some γ parameter getting values from given set Γ, but the true value of the parameter is unknown.

It is obvious that it will also demand the choice of the, in a sense, “best” hypothesis of a transitional function. At the same time expanded information structure completeness can be observed only according to final results of the problem research.

5 Game approach to formalization of the choice problem

Assumptions of the choice specify the existence of two aspects of an agent's interests, one of which is determined by the bias in the management of a desirable object evolution, and the other - by the choice of a preference structure. The purposeful control concept defines the third aspect of interests associated with the need for the diagnostics of the situation depending on the observed condition. In compliance with these three aspects three sets of alternatives are to be assigned: set *C*– action modes, set *G* - structural alternatives and set *X* - diagnostic alternatives. It is also assumed to assign the utility function $E\varphi^{g}(C\times S\times X)$ and the transient function $q^{g}(S\mid S\times C)$ of $S\times C$ to *S*. Assigning these objects suggests the possibility of forming the qualitative mode choice criterion as postulated by the situational control concept that makes sense of expected utility and the purposeful state situation model choice quality criterion that concerns the risk. These criteria are obviously different and in a way interdependent. The natural presumption is that in order to choose structural alternatives the corresponding quality criterion may be introduced. It differs from the rest of the criteria and in some way is dependent on the choice of other alternatives. It is commonly known that in similar conditions the problem of mode choice has a gaming intension [2]. Then each set of alternatives can be formally linked with a party concerned (a player), whose interests are related to the choice of alternatives from the corresponding set of alternatives according to their individual quality criterion. Within a set of alternatives every party has the freedom of choice. Since interests of each party represent a certain component of agent’s interests, parties are to comply with the common to them agent’s interests, when selecting alternatives. Therefore the problem of mode choice acquires a gaming content in relation to corporate interests [6], and the subject of interest plays the role of a center. He can accept the proposed trade-off alternative if it is hardly possible to improve it without infringing at least one component of interests. The compromise that meets this requirement will be called a "corporate stable equilibrium."

Conclusions

The paper considers a decision-making model for an agent, who can form internal aim and uses subjective representations on a choice situation.

It is shown that the aim of choice is to maximize a specific value of the choice situation by a result. The choice result is determined by agent’s representations on the choice situation and on his own interests. When making a decision the agent uses three sets of alternatives: controlling *C* (mode so faction), structural *G* and identification *X*. Therefore, it is possible to assume the existence of three virtual sides, which make a choice of corresponding alternatives that are balanced strategies.

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