

STOCHASTIC COMPUTER SIMULATION OF THE IONIC DIFFUSION THROUGH BIOLOGICAL TISSUES UNDER THE EFFECT OF DIRECT ELECTRIC FIELD

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Abstract: The effect of the exposure of biological tissues to external electric fields is still a potent source of controversy. This work addresses the diffusion of ions under the effect of an external DC electric field. This was done by studying the diffusion coefficient D as an indicating parameter for such effects. The work was based on a stochastic computer simulation in which the tissue was considered as a matrix containing the elements under study. The size of the matrix was up to $30,000 \times 30,000$. A two-dimensional honey comb cellular pattern was simulated such that it allowed six maximum possible element-to-element communications. The effect of vacancy concentration and annealing time were tested firstly in the absence of electric field. Then different values of the electric field were applied. Moreover, different vacancy concentrations were studied under the effect of the electric field. The results showed that the ionic penetration increases proportionally with the strength of the electric field as well as the percentage of the available vacancies in the host medium.

Keywords: IONIC DIFFUSION, BIOLOGICAL TISSUES, STOCHASTIC SIMULATIONS

1. Introduction

Many theoretical efforts have been devoted to studying diffusion in different media. Electric fields enhance the diffusion of charge carriers in disordered materials. It was shown that the diffusion coefficient in one-dimensional hopping depends linearly on the electric field, while in three dimensions the dependence is quadratic [1].

Diffusion takes place in biological tissues as well. The effective diffusion coefficient, D_{eff} , of salt into a biological tissue due to the application of an electric field at different temperatures showed an increase of ion transport [2]. The effective diffusion coefficients of K^+ and Cl^- ions are appreciably reduced in narrowing the channel in the cell membrane when subjected to an external field. The extent of the reduction is similar for both the anionic and cationic species [3].

Diffusion in the extracellular space (ECS) is constrained by the volume fraction, hence a modified diffusion equation was proposed to govern the transport behavior of many molecules in the brain [4].

Liu and Shi [5] developed two-dimensional and three-dimensional FEM model to study the transport of ionic species in an externally applied electric field. They found that more chlorides are driven out of samples with increasing direct current density and treatment time.

In the present work we introduce a stochastic model to follow and determine the diffusion coefficient of an ionic tracer through a biological tissue. Hence, the effect of the application of an external DC field on the diffusion and the penetration depth of these ions in biological tissue is studied.

2. Computer Model

The present model simulates the ionic diffusion under the following assumptions: A part of the biological tissue is represented as a 2D matrix of sizes up to $30,000 \times 30,000$ elements. Each element represents either a host particle, a vacancy or a tracer ion. The diffusants (tracers), are considered as positive ions.

At zero time, the diffusants occupy the first row of the matrix and are in a continuous flow; each particle that leaves the surface

into the matrix is replaced by another one. The rest of the matrix is either occupied by the host elements (biological cells) or with vacancies that are randomly distributed throughout the matrix. The characteristics of each element are represented by one byte which contains information of the type (host, vacancy or tracer), spatial location and the time elapsed since diffusion starts. The biological tissue could be simply modeled as a close-packed spherical array of cells as shown in Fig. 1.

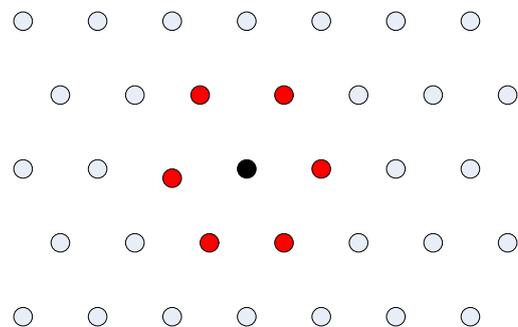


Fig. 1. The six neighbors' structure in a hexagonal matrix modelling a biological tissue. The middle black element is the tracer with six neighboring vacancies represented by red circles.

Since the diffusion process follows a random walk procedure, in the present model the tracer ion diffuses through the vacancies available in the nearest sites such that the jump follows a random choice of the accessible vacancies. The effect of the electric field is represented as a controlled bias in the jump direction. The field direction affects the randomization process of jumping so that along the direction of the field there is a higher probability than other directions. Fig. 2 shows a visualization of the system under consideration, for both cases of free diffusion and the diffusion under the effect of a DC electric field.

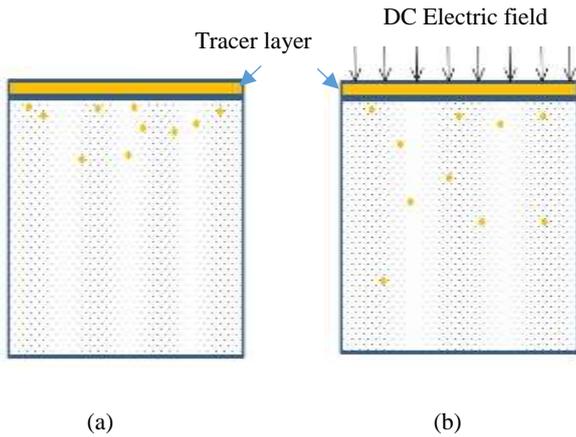


Fig. 2. The diffusion process (a) in the absence of external field and (b) under the effect of DC electric field. The traces are represented by the larger circles.

Hence the concentration $c(x,t)$ of the ions/tracer after a certain time t and a location x is obtained from the following

$$c(x,t) = constant \times e^{\frac{-x^2}{4Dt}}$$

The mean squared radius of the diffusion pattern is represented by:

$$\langle R^2 \rangle = \frac{\sum_{i=1}^N R_i^2}{N}$$

where N is the number of jumps and R_i is the individual displacement.

The diffusion coefficient D can be calculated from the slope of $\langle R^2 \rangle$ versus the annealing time t :

$$D = \langle R^2 \rangle / 4t$$

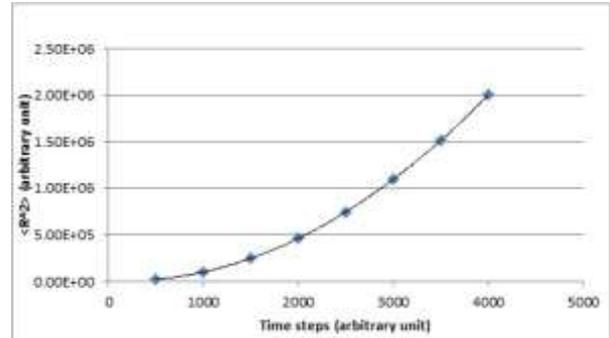
The concentration is calculated by sectioning the matrix to a certain number of rows and calculating the number of the tracer ions in each section. As the thickness of the layers becomes smaller, the accuracy of the penetration profile which describes the diffusion increases. Plotting the logarithm of the concentration $c(x,t)$ against x^2 for different time steps and different vacancy concentrations is used to obtain the diffusion coefficient D according the equation

$$Slope = -1/4Dt$$

The annealing time is taken as the total number of iterations which is varied up to 500,000 time steps. The vacancies are randomly distributed, and their concentration is varied from 5% to 80%.

3. Results

Firstly, the diffusion pattern is investigated in the absence of an external field and assuming that all the sites are vacant. Figs. 3 and



4 show the relation of the mean squared displacement, $\langle R^2 \rangle$, and the diffusion coefficient, D , as a function of time.

Fig. 3. Variation of the mean squared radius and time steps in the case of free diffusion.

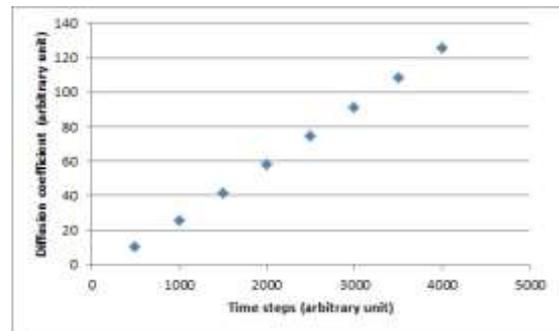


Fig. 4. The diffusion coefficient with time steps.

In both figures, the classical pattern of the random walk diffusion in the absence of external field is preserved. The diffusion coefficient increases linearly with annealing time. The penetration depth $\langle R^2 \rangle$ of the tracers is studied in the presence of randomly distributed vacancies of different ratios in a biological tissue. Fig. 5 shows the increase in the penetration distances of the tracers with annealing time at a vacancy concentration of 50%. In this figure the time steps are increased up to 1.2 millions so that the asymptotic level of the curve is obtained; at this level the penetration distance is about to reach the boundaries of the matrix.

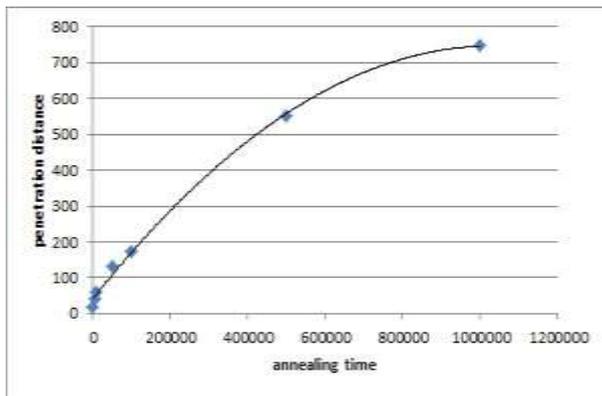
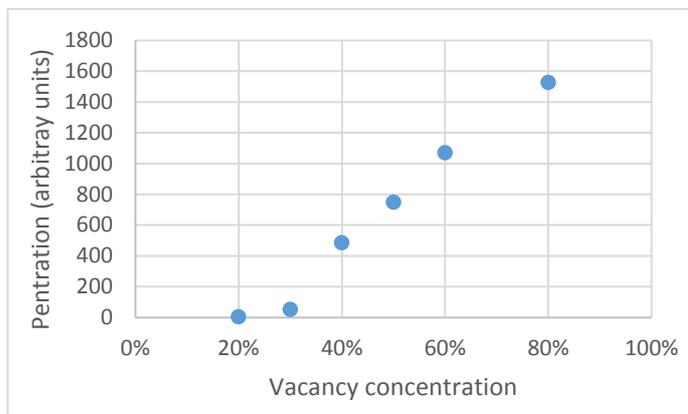


Fig. 5. The variation of penetration distance (arbitrary units) with the annealing time.

Fig. 6 show the mean penetration distance with vacancy concentration. It is obvious that at low vacancy percentage up to 20% , the tracers hardly invade the host matrix. Then, $\langle R^2 \rangle$ increases almost linearly. Similar behavior is observed when the diffusion coefficient is plotted versus the vacancy concentration. Fig. 7 illustrates this relation in absence of external field and for a



fixed annealing time.

Fig. 6. The variation of the penetration with vacancy concentration.

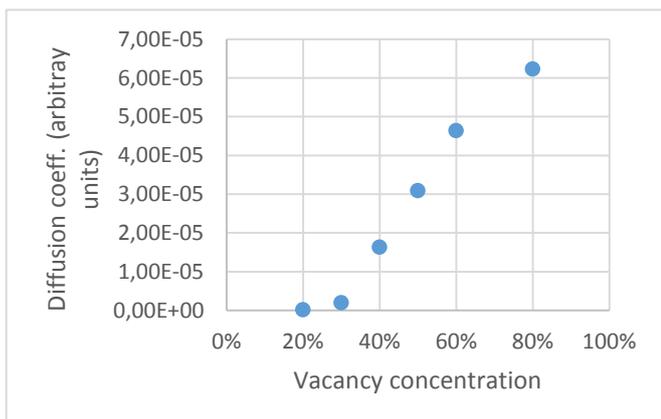
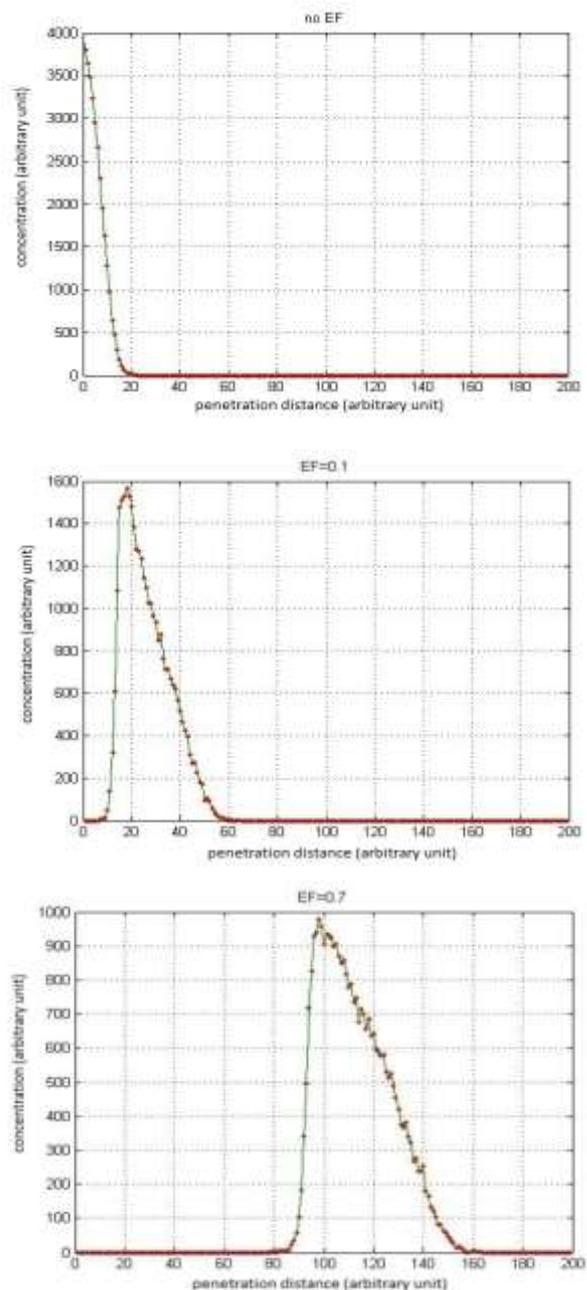


Fig. 7. The variation diffusion coefficient with vacancy concentration.

The diffusion of positive ions in a biological host under the effect of DC electric field is then examined for different field strengths, different annealing times and varying vacancy concentrations. Considering the matrix having a 90% vacancy and the DC electric field, EF, is increasing gradually (EF=0% , 10%, and 70%), the diffusion pattern of the tracer in the host matrix is illustrated in Fig. 8. We infer that the concentration of the diffused



ions travels deeper as a result of increasing the electric field strength.

Fig. 8. Penetration profiles for ions in a 90% vacant matrix under the effect of direct electric field for different strengths, namely, EF=0% , 10%, and 70% and for constant annealing time.

Fig. 9 illustrates the increase in the diffusion coefficient with the strength of the external field at a constant vacant percentage and for the annealing time.

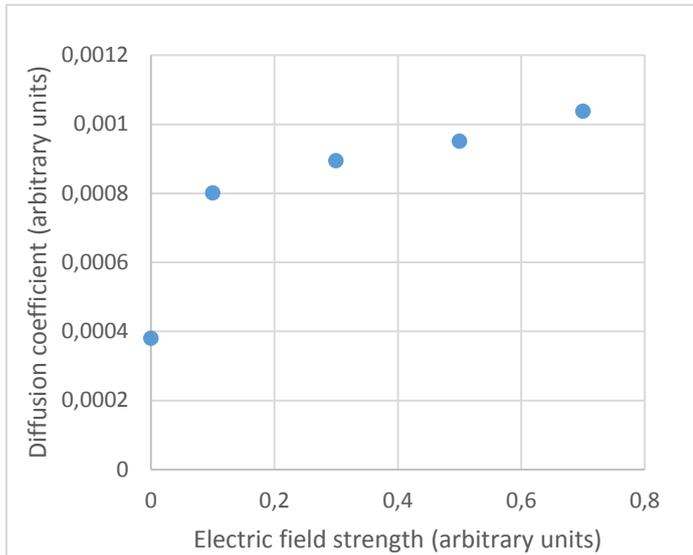


Fig. 9. Variation of diffusion coefficient with electric field strength.

4. Discussion and Conclusion

In the present work we have considered the profile of ionic diffusion in a biological tissue under the effect of DC electric fields. Initially, in the absence of an electric field and the vacancies occupy 0-30% of the host tissue, we found that the penetration of diffusing ions and the diffusion coefficient doesn't increase much. As the percentage of vacancies increases more than 30%, the penetration of the diffusing ions and the diffusion coefficient increase linearly with the vacancies concentration.

Preliminary results showed that when the matrix is 50% vacant, as the EF increases the penetration increases up to a point after which it decreases again. This happens because there is no much space for the movement of the ions, they are hindered by the matrix structure. As the percentage of vacancies increases the penetration of the diffusants ions increases. This increase is accelerated with the applications of an external field EF. The positive ions have more space and more probability to jump forwarded aided by the applied field.

In conclusion the present work introduces a stochastic model that tackles the problem of ionic diffusion in a biological tissue. Emphasis is given to the effect of both the existence of different percentage of vacancies available for random jumps and the effect of an external DC field of different strengths.

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