STATISTICAL METHODS FOR THE ANALYSIS OF THE MONTE CARLO SIMULATION RESULTS IN VISION SYSTEMS

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Abstract: *In this paper, we present some results of statistical processing of the results of numerical simulation of the characteristics of vision systems through the atmosphere obtained with the help of a special software package created by us (Gendrina I.Yu., Kvach A.S.).*
Keywords*: Vision system, angular brightness distribution, Monte Carlo method, regression.*

Introduction

Various methods of statistical research such as correlation-regression analysis, dynamic series, variance analysis, etc. in the study of vision systems are used for studying and subsequent prediction of the patterns of radiation transfer. We have conducted Monte Carlo experiments to calculate the Point Spread Function for linear system "underlying surface – atmosphere". The one is defined as the system response to the input signal, representing a point mass, located at a certain point, and can be determined as the angular brightness distribution of surface-based point source measured with receiving device at the top of the atmosphere. Then we have attempted to apply elements of correlation-regression analysis for the study of the influence of various optical and geometrical parameters on the Point Spread Function.

Vision system (VS) is understood as an observation scheme including the underlying surface, a "cloudy environment" (atmosphere), and an optical device that captures incoming radiation. To study radiation transfer in such systems, the theory of systems and the theory of radiation transfer are traditionally used (Zuev V.E., Belov V.V., Veretennikov V.V.).

The main system characteristic for VS is the point spread function (PSF); it is defined as the response L of linear system to the input signal, representing a point mass $\delta(x-x_1)\delta(y-y_1)$, located at a certain point (x_1,y_1): $L[\delta(x-x_1)\delta(y-y_1)]=h(x,y;x_1,y_1)$.

An arbitrary object (function) $f(x,y)$ can be considered as a set of point masses. For instance: $f(x,y)=\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} f(x_1,y_1)\delta(x-x_1,y-y_1)dx_1dy_1$. Then, a result of the system impact (image) can be represented in the form:

$$g(x,y)=L[f(x,y)]=\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} f(x_1,y_1)h(x,y;x_1,y_1)dx_1dy_1 .$$

Obviously, regularities of the image distortion due to impact of any system can be studied by analyzing the effect of this system on the point spread function.

The model of the atmosphere includes the following characteristics: the total attenuation coefficient $\sigma(\lambda,\vec{r})=\sigma_{sc}(\lambda,\vec{r})+\sigma_a(\lambda,\vec{r})$, where σ_{sc} – the scattering coefficient, σ_a – the absorption coefficient; $g(\lambda,\mu,\vec{r})$ – aerosol phase function. Here $\vec{r}=(x,y,z)$ – radius-vector of the current point in space, $\mu(\vec{\omega}',\vec{\omega})$ – cosine of the scattering angle of radiation coming from direction $\vec{\omega}'$, in the direction $\vec{\omega}$, λ – is the wavelength of incident radiation.

The paper considers two models of the atmosphere:

1. vertically bounded plane-parallel layer-homogeneous aerosol-molecular;
2. vertically bounded plane-parallel aerosol-molecular, including overcast layer. For the cloud layer, different characteristics from those of the first model is assumed: coefficients of attenuation, absorption, scattering, and the aerosol phase function.

1. Problem Statement and methods

The geometric scheme of calculations is as follows: at the lower boundary of the atmosphere (on the underlying surface $z=0$) there is a point source of unit capacity. At the upper boundary of the atmosphere ($z=30\,km$) there is an optical receiver that can receive scattered radiation coming from different directions (observation angles). The brightness of the scattered radiation is solution of the integro-differential transport equation (Marchuk G.I., Mikhailiov G.A., Nazaraliev M.A., Darbinjan, Kargin B.A., Elepov B.S.), which can be practically solved only by approximate or numerical methods.

One of the most universal methods for solving this problem is the simulation method, or the Monte Carlo method. The basis of the Monte Carlo method is the integral transport equation of the second kind with a generalized kernel for the density of particles' collisions (Marchuk G.I., Mikhailiov G.A., Nazaraliev M.A., Darbinjan, Kargin B.A., Elepov B.S.):

$$f(\vec{x})=\int\limits_X k(\vec{x}',\vec{x})f(\vec{x}')d\vec{x}'+\psi(\vec{x}),\quad f=Kf+\psi$$

Here $\vec{x}=(\vec{r},\vec{\omega})$ – is the point of the phase space of coordinates and directions, $\psi(\vec{x})$ – source function, K – integral operator with kernel $k(\vec{x}',\vec{x})$:

$$k(\vec{x}',\vec{x})=\frac{\sigma_{sc}(\vec{r})\cdot g(\mu)exp(-\tau(\vec{r}',\vec{r}))\sigma(\vec{r})}{\sigma(\vec{r}')2\pi|\vec{r}-\vec{r}'|^2}\cdot\delta\left(\vec{\omega}-\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}\right).$$

In this paper, one of the algorithms of the Monte Carlo method was used - the method of local estimation (Marchuk G.I., Mikhailiov G.A., Nazaraliev M.A., Darbinjan, Kargin B.A., Elepov B.S.).

Algorithm for local estimation consists in calculating following functional:

$$J(\Omega_i)=\int\limits_{\Omega_i}\Phi(\vec{r}^*,\vec{\omega}^*)d\vec{\omega}^*=\int\limits_X l_i(\vec{x}',\vec{x}^*)f(\vec{x}')d\vec{x}'= \quad (1)$$
$$=M\sum_{n=0}^{N}Q_n\cdot l_i(\vec{x}_n,\vec{x}^*)$$

$$l_i(\vec{x},\vec{x}^*)=\frac{exp(-\tau(\vec{r},\vec{r}^*))\cdot g(\mu^*)}{2\pi|\vec{r}-\vec{r}^*|^2}\Delta_i(\vec{s}^*). \quad (2)$$

Here $\vec{s}^*=\dfrac{\vec{r}^*-\vec{r}}{|\vec{r}^*-\vec{r}|}$, $\mu^*=(\vec{\omega},\vec{s}^*)$, $\Delta_i(\vec{s})$ – is the indicator of region Ω_i. Φ – flux of particles at given point $\vec{x}^*$. Q_n – weight of the particle, $f(\vec{x})$ – density of collisions.

2. Initial data

We will consider the process of radiative transfer through aerosol-molecular atmosphere, which comprises a layer overcast, by neglecting the reflection from underlying surface. We used the following data (Gendrina I.Yu., Kvach A.S.):

1. Wavelength (mkm) in transparent windows: 0.347; 0.530; 0.694; 0.860; 1,060; 3,390; 10.60.
2. Lower boundary of atmosphere 0 km above Earth's surface, upper boundary H of the atmosphere 30 km above the Earth's surface.
3. Optical thickness for a cloudless atmosphere are presented in Table 1.

Table 1. *Optical thickness of the cloudless atmosphere*

Wavelength, mkm	Optical thickness
0,347	0,228
0,53	0,158
0,694	0,124
0,86	0,098
1,06	0,092
3,39	0,067
10,6	0,041

4. Lower boundary of the cloud layer - 1 km above the Earth's surface, upper boundary - 2 km above the Earth's surface. The optical models of the cloud layer – “haze H” and “cloud C1” (Deirmendjian D.)
5. In this work, we considered Lambertian model of sources of radiation. In this case the density of the initial areas looks like: $p(\vec{\omega})=\frac{\mu}{\pi}$, where $\mu = arccos\,\theta$, θ - zenith angle of initial direction.

3. Simulation results

Quantitative values brightness of scattered radiation for the cloudless atmosphere are presented in our previous publication (Gendrina I.Yu., Alekseenko M. A.). Similar values for various models of the cloud atmosphere are given in Tables 2,3.

Table 2 contains the results for the brightness of scattered radiation for the atmosphere with a cloud layer of the "Haze H" type.

Table 3 contains similar data for the atmosphere with a cloud layer of the "Cloud C1" type.

These types vary in value of the average cosine of scattering phase function:

$$\bar{\mu} = \frac{1}{2}\int_{-1}^{1}\mu g(\mu)d\mu.$$

It is known that this parameter characterizes the elongation of aerosol phase function. For example in case $\lambda = 0{,}694$ mkm the average cosine amounts to 0,745 for type “Haze H” and 0,857 for type “Cloud C1”.

Table 2. *Brightness of scattered radiation for the atmosphere with a cloud layer of the "Haze H" type, W/mkm $\cdot$ m^2*

Angles of reception, grad	Wavelength, mkm			
	λ=0,374	λ=0,530	λ=0,694	λ=0,860
4,5	2,24E-05	1,67E-05	1,38E-05	1,13E-05
13,5	1,09E-06	7,65E-07	6,60E-07	5,30E-07
22,5	2,23E-07	1,83E-07	1,55E-07	1,39E-07
31,5	8,44E-08	5,63E-08	5,54E-08	4,73E-08
40,5	3,49E-08	2,73E-08	2,81E-08	6,65E-08
49,5	1,86E-08	5,15E-08	1,54E-08	1,18E-08
58,5	1,30E-08	1,10E-08	1,71E-08	7,75E-09
67,5	1,94E-08	1,12E-08	7,95E-09	2,11E-08
76,5	1,58E-07	2,13E-08	4,96E-09	3,68E-09
85,5	1,20E-08	6,37E-09	1,23E-09	1,03E-09

Table 3. *Brightness of scattered radiation for the atmosphere with a cloud layer of the "Cloud C1" type, isotropic source, W/mkm $\cdot$ m^2*

Angles of reception, grad	Wavelength, mkm			
	λ=0,374	λ=0,530	λ=0,694	λ=0,860
4,5	4,24E-07	5,01E-07	5,45E-07	5,69E-07
13,5	1,32E-07	1,36E-07	1,01E-07	9,47E-08
22,5	5,06E-08	4,30E-08	4,23E-08	3,50E-08
31,5	2,29E-08	1,84E-08	1,20E-08	1,19E-08
40,5	7,00E-09	8,43E-09	6,45E-09	5,16E-09
49,5	3,29E-09	2,44E-09	3,10E-09	2,69E-09
58,5	1,84E-09	1,59E-09	1,36E-09	1,39E-09
67,5	1,17E-09	1,13E-09	7,77E-10	8,25E-10
76,5	8,77E-10	9,73E-10	7,45E-10	5,32E-10
85,5	1,70E-10	5,77E-10	8,95E-11	1,78E-10

4. Statistical analysis of simulation results

To establish functional relationship between the angular distribution of brightness and optical parameters, regression analysis was used, which is widely used to restore aerosol and cloud characteristics, and also to assess their effect on climate (Khayer M. M. et al.). The regression equation for the angular distribution of brightness in the aerosol-molecular and cloud atmosphere relatively wavelength of the incident radiation was obtained in the form $y=\frac{b_1}{x}+b_0$ for all reception angles. Regression coefficients for the cloudless atmosphere are given in Table 4. The regression coefficients for the cloudy atmosphere are given in Table 5. The tables also shown coefficients of determination R^2. This coefficient indicates the proportion of the total variation in the dependent variable y due to variability of x and characterizes the total quality of regression. The statistical significance of determination coefficient can be confirmed with the help of Fisher statistics: $F=\frac{R^2}{1-R^2}\cdot(n-2)$. Here n is the number of observations. The value F is compared to value $F_{\alpha;k_1,k_2}$ from Fisher table. Here α - the given level of significance, $k_1=1$ and $k_2=n-2$.

Table 4. *Coefficients of the regression equation for the cloudless atmosphere*

Angles of reception, grad	b_0	b_1	R^2
4,5	1,19E-05	3,25E-05	0,993
13,5	2,31E-07	5,05E-07	0,976
22,5	6,98E-08	1,19E-07	0,927
31,5	3,92E-08	4,04E-08	0,784
40,5	2,44E-08	1,51E-08	0,570
49,5	1,39E-08	6,51E-09*	0,482
58,5	6,82E-09	3,64-09	0,609
67,5	2,74E-09	2,35E-09	0,771
76,5	5,21E-10	1,23E-09	0,907
85,5	3,00E-11	1,76E-10	0,929

Table 5 *Coefficients of the regression equation for the cloud atmosphere*

Angles of reception, grad	b_0	b_1	R^2
4,5	3,52E-06	7,12E-06	0,985
13,5	1,08E-07	3,63E-07	0,997
22,5	3,63E-08	7,53E-08	0,973
31,5	1,26E-08	2,66E-08	0,976
58,5	3,68E-09*	4,34E-09	0,598
76,5	-3,34E-08*	5,04E-09	0,606
85,5	-2,37E-09*	4,50E-09	0,809

We would like to present also the brightness dependence relatively the upper boundary of cloud. The regression equation in this case was obtained in a simple form: $y = b_1 x + b_0$ for all reception angles. Regression coefficients and determination coefficients for the case of isotropic source, wavelength of $\lambda = 0{,}347$ mkm, cloud layer type of the "Haze H" and “Cloud C1” are given in Table 6,7.

Table 6. *Coefficients of the regression equation for the cloud atmosphere. Model «Haze H»*

Angles of reception, grad	b_0	b_1	R^2
4,5	5,14E-07	-3,07E-08	0,990
13,5	1,53E-07	-7,70E-09	0,975
22,5	5,92E-08	-2,33E-09	0,851
31,5	2,65E-08	-6,54E-10*	0,654
40,5	7,36E-09	-1,75E-10	0,946
49,5	3,60E-09	-6,36E-11	0,625
58,5	2,11E-09	-5,13E-11	0,840
67,5	1,25E-09	-2,96E-11	0,886

Table 7. *Coefficients of the regression equation for the cloud atmosphere*
Model «Cloud C1»

Angles of reception, grad	b_0	b_1	R^2
4,5	4,98E-07	-2,61E-08	0,966
13,5	1,44E-07	-4,88E-09	0,953
22,5	5,10E-08	-1,48E-10	0,158
31,5	2,14E-08	7,33E-10	0,976
40,5	6,30E-09	1,55E-10*	0,631
49,5	3,15E-09	4,54E-11	0,882
58,5	1,89E-09	-8,70E-12*	0,379
67,5	1,18E-09	-7,32E-12	0,802
76,5	1,11E-09	2,66E-11*	0,151

Note. * - insignificant coefficient.

Conclusions

Statistical estimation of regression equations for significance was carried out on the basis of the F-test and estimation of the determination coefficient. With 90% confidence, it can be argued that the considered dependence is statistically significant.

The analysis shows that between obtained angular distributions of intensity and wavelength) in transparent windows for both cloudless and cloudy for the atmosphere, there is a link that can be with a good degree of accuracy to describe hyperbolic regression equation.

Between the brightness and the upper boundary of cloud, there is a link that can be with a degree of accuracy to describe linear regression equation.

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