ELECTRIC POWER SYSTEMS MODELING AND EDUCATION: THE CONTROLLABLE LOADS IN A SHORT TERM SYSTEM BALANCE

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Abstract: *In this paper we try to build a framework in active power optimization model building when controllable loads are available for balancing purposes. A certain classification of non-fixed loads is given as well as the respective variables and constraints that have to be introduced in the mixed-integer linear programming model. A numerical example is also given illustrating the modeling approach. Some analysis on the presented numerical data is done in order to show sensitivity to certain environmental issues.*

Keywords: OPTIMIZATION MODELING, ELECTRIC POWER SYSTEMS OPTIMIZATION, CONTROLLABLE LOADS

1. Introduction

When a load can be switched on or off when needed for system balance purposes, it is considered as a controllable load. An example of a controllable load in an EPS from the System's Operator point of view is the pumping capacities of an PHPS or a group of consumers whose power supply can be interrupted (and / or restricted) during peak load periods. For a micro-grid, a controllable load can be any consumer whose power can be managed and / or limited or its performance may be postponed (dispatched) over time. There are many loads whose operation extends over several time intervals. In this case for those loads is said that they have a working cycle. Since there are many possibilities in the handling of the controllable loads with a work cycle, it is necessary to introduce a certain classification for modeling purposes of such loads.

2. Loads classification and modeling framework

Type I. Controllable loads over time with an interruptible work cycle: These loads are assumed to have a fixed work cycle duration d_{l1*} and a fixed power level P_{l1*} in a single interval. The operation modeling of these loads requires the introduction of binary variables $v_{l1,j}$ having *true* value ($v_{l1,j} = 1$) when the load is switched to power level P_{l1*} in the interval j. Additions of the form $v_{l1,j}P_{l1*}$ must be introduced in the balance constraints. A constraint (1) handles the duration of the work cycle:

$$\sum_j v_{l1,j} = d_{l1*} \tag{1}$$

Type II. Controllable loads over time and consumption with an interruptible work cycle: The power consumed by this load $P_{l2,j}$ in the single interval j depends on the duration of the operating cycle $d_{l2}^{\min} \le d_{l2} \le d_{l2}^{\max}$. The total load consumption P_{l2}^{Σ} is known for a full work cycle. So the hour consumption $P_{l2,j}$ and the duration of the duty cycle d_{l2} are optimized introducing binary variables $v_{l2,j} = 1$ when the load is switched to power $P_{l2}^{\min} \le P_{l2,j} \le P_{l2}^{\max}$. In the latter the interval's limits are derived via: $P_{l2}^{\min} = P_{l2}^{\Sigma} : d_{l2}^{\max}$ and $P_{l2}^{\max} = P_{l2}^{\Sigma} : d_{l2}^{\min}$. Only the load consumption $P_{l2,j}$ is included in the balance constraint. For modeling the rest of the requirements, inslucidng the dependence between the cycle duration and the power level in the unit interval the following constraints are added:

$$P_{l2,j} - v_{l2,j}P_{l2}^{\max} \le 0 \text{ and } P_{l2,j} - v_{l2,j}P_{l2}^{\min} \ge 0 \tag{2}$$

$$\sum_j v_{l2,j} = d_{l2} \tag{3}$$

$$\sum_j P_{l2,j} = P_{l2}^{\Sigma} \tag{4}$$

$$d_{l2}^{\min} \le d_{l2} \le d_{l2}^{\max} \text{ are integer and } v_{l2,j} \text{ are binary} \tag{5}$$

Type III. Controllable loads over time with a non-interruptible work cycle: These are loads that that once started can not be switched off until the whole fixed-duration work cycle is completed. It is assumed that the load consumption in the unit interval of the work cycle with a fixed duration d_{l3*} is P_{l3*}. Once the work cycle has begun, it can not be interrupted. To ensure this requirement, 3 sets of binary variables are introduced:

$s_{l3,j} = 1$ if the load starts a cycle at the beginning of j

$v_{l3,j} = 1$ if the load is running at power P_{l3*} in the interval j

$f_{l3,j} = 1$ if the load finishes its work cycle at the beginning of the interval j

Because the hour consumption of the load P_{l3*} is a constant additions of the form $v_{l3,j}P_{l3*}$ must be introduced in the balance constraints.

The constraint handling the duration of the work cycle remains unchanged:

$$\sum_j v_{l3,j} = d_{l3*} \tag{6}$$

Constraints ensuring the un-interruptible requirement for the working cycle are introduced:

$$s_{l3,j} - v_{l3,j} \le 0 \tag{7}$$

$$s_{l3,j} - f_{l3,j} = v_{l3,j} - v_{l3,j-1} \tag{8}$$

$$f_{l3,j} + v_{l3,j} \le 1 \tag{9}$$

$$s_{l3,j} + \sum_{k=j+1}^{j+d_{l3*}-1} f_{l3,k} \le 1 \tag{10}$$

Type IV. Controllable loads over time and power level with a non-interruptible work cycle: In this case both the duration $d_{l4}^{\min} \le d_{l4} \le d_{l4}^{\max}$ of the work cycle is optimized and the power level at a unit interval $P_{l4}^{\Sigma} / d_{l4}^{\max} \le P_{l4,j} \le P_{l4}^{\Sigma} / d_{l4}^{\min}$. Constraints ensuring the un-interruptible requirement for the working cycle must be added using the three sets of binary variables:

$s_{l4,j} = 1$ if the load starts a cycle at the beginning of j

$v_{l4,j} = 1$ if the load is running at power $P_{l4,j}$ in the interval j

$f_{l4,j} = 1$ if the load finishes its work cycle at the beginning of the interval j

In the balance constraints only the power of the load $P_{l4,j}$ is added being and optimization variable. A constraint for the work cycle duration is present with respect to the simple bounds $d_{l4}^{\min} \le d_{l4} \le d_{l4}^{\max}$:

$$\sum_j v_{l4,j} = d_{l4} \qquad (11)$$

The following constraints ensure the un-interruptible work cycle and the power levels:

$$s_{l4,j} - v_{l4,j} \le 0 \qquad (12)$$

$$s_{l4,j} - f_{l4,j} = v_{l4,j} - v_{l4,j-1} \qquad (13)$$

$$f_{l4,j} + v_{l4,j} \le 1 \qquad (14)$$

$$\sum_j P_{l4,j} = P_{l4}^{\Sigma}\ ,\ P_{l4,j} - v_{l4,j} P_{l4}^{\max} \le 0 \ \text{and}$$

$$P_{l4,j} - v_{l4,j} P_{l4}^{\min} \ge 0 \qquad (15)$$

3. A numerical example

To illustrate the controllable loads with a work cycle modeling techniques the following nomenclature and presumptions might be used. A semi-autonomous building has available controllable and uncontrollable loads and generators and can also purchase or sell power to the external grid. Forecasts for its own uncontrollable load and generation are available as well as prices for buying and selling power.

In the model formulation '*P*' stands for '*Power*', indices '*n*' represent '*non*', '*f*' stands for '*fixed*', '*l*' stands for '*load*', '*G*' stands for '*Generation*' and '*a*' stands for '*accumulation*'. The scenario under consideration includes equal hourly pricing for selling and purchasing power from the network ($c_{buy,t} = c_{sell,t}$) and relatively high price for the own generation units ($c_{nfG} > \max\{c_{buy,t}\}$). The numerical example uses a dataset (given $P_{fG,j}$ and $P_{fl,j}$) aiming at maximization of total profit:

$$\max J = \sum c_{sell,j}P_{sell,j} - \sum c_{buy,j}P_{buy,j} - \sum c_{nfG,j}P_{nfG,j} \qquad (16)$$

The power of the controllable loads consists of total of seven members divided in four groups of loads according to the classification given in the beginning.

Two loads are from Type I with fixed work cycle duration that can be interrupted and power:

$d_{l1,1*}$ = 3 hours, $P_{l1,1*}$ = 2,2 kWh

$d_{l1,2*}$ = 4 hours, $P_{l1,2*}$ = 2,5 kWh

Two loads are from Type II with non-fixed power and duration of the work cycle and it can also be interrupted::

$d_{l2,1}$ might be between 2 up to 4 hours, $P^{\Sigma}_{l2,1}$ = 8.2 kWh

$d_{l2,2}$ might be between 3 up to 7 hours, $P^{\Sigma}_{l2,2}$ = 14 kWh

Two loads from Type III with fixed work cycle duration and power in a unit interval but the continuity of the cycle can not be interrupted:

$d_{l3,1*}$ = 2 hours and $P_{l3,1*}$ = 1,8 kWh

$d_{l3,2*}$ = 4 hours and $P_{l3,2*}$ = 1,4 kWh

A single load from the last Type IV is considered and its work cycle is un-interruptible. Its work cycle duration is optimized as well as its power in the unit time interval:

d_{l4} may be 3 up to 6 hours with P^{Σ}_{l4} = 12 kWh

The balance constraint includes all controllable loads additional variables:

$$P_{sell,j} + P_{a,j} + \sum_{l_1=1}^{L_1} v_{l1,j}P_{l1*} + \sum_{l_2=1}^{L_2} P_{l2,j} + \sum_{l_3=1}^{L_3} v_{l3,j}P_{l3*} + \\ + \sum_{l_4=1}^{L_4} P_{l4,j} + L_j = P_{buy,j} + P_{R,j} + P_{Ga,j} + P_{G,j} \qquad (17)$$

The total power consumed by the controllable loads for the whole optimization period must be not less than the technological minima:

$$\sum_{l_1=1}^{L_1} v_{l1,j}P_{l1*} + \sum_{l_2=1}^{L_2} P_{l2,j} + \sum_{l_3=1}^{L_3} v_{l3,j}P_{l3*} + \sum_{l_4=1}^{L_4} P_{l4,j} \ge E_T \qquad (17)$$

For each load the respective additional constraints (1-15) must be added: two constraints of type (2) for each controllable load $l_1 \in L_1$ (Type I), for each controllable load $l_2 \in L_2$ (Type II) a set of constraints (2) to (5), for each controllable load $l_3 \in L_3$ (Type III) a set of constraints (6) to (10) and for the single controllable load $l_4 \in L_4$ (Type IV) the constraints (11) to (15).

Table 1: *Optimal values of the variables for accumulation, generation, buying and selling and own controllable generator*

Hour *j*	$P_{a,j}$	$P_{Ga,j}$	$P_{buy,j}$	$P_{sell,j}$	$P_{G,j}$
Col. №	**1**	**2**	**3**	**4**	**5**
1	0	0	0	3,3	50
2	0	33,2	33,2	0	0
3	0	31,4	31,4	0	0
4	0	33,9	33,9	0	0
5	0	43,5	43,5	0	0
6	0	39,4	39,4	0	0
7	0	0	0	33,8	0
8	0	0	0	41,2	0
9	10	0	0	51,1	0
10	16	0	0	71	0
11	16	0	0	60	0
12	16	0	0	54,7	0
13	16	0	0	66	0
14	0	0	0	58	0
15	0	0	0	39,1	0
16	0	0	0	76,6	0
17	0	0	0	59,2	0
18	16	0	0	53,5	0
19	16	0	0	37,2	0
20	16	0	0	6,3	0
21	16	0	0	8	0
22	16	0	0	8	0
23	0	28	28	0	0
24	0	28	28	0	0

The constraint (19) stands for the accumulation-generation efficiency coefficient η. The fact that no purchasing and selling in a time interval is allowed, as well as accumulation and generation from accumulating units is not possible two sets of binary variables are introduced in the model (v_j = 1 when buying power from the external network and u_j = 1 when generating power from the accumulation units) to construct the mutually exclusive alternatives. Constraints (20) assure that no purchase and selling power onto the external grid will occur in a same time interval. Constraints (21) assure that no accumulation and generation will occur in a same time interval. The power of the controllable generator is fixed above (22).

$$\eta \sum P_{a,j} = \sum P_{Ga,j} \qquad (19)$$

$$P_{sell,j} \le (1 - v_j)\, P_{sell}^{\max} \quad \text{and} \quad P_{buy,j} \le v_j P_{buy}^{\max} \qquad (20)$$

$$P_{a,j} \le (1 - u_j)\, P_a^{\max} \quad \text{and} \quad P_{Ga,j} \le u_j P_{Ga}^{\max} \qquad (21)$$

$$P_{nfG,j} \le P_{nfG}^{\max} \qquad (22)$$

The optimal values of the variables are given in Tables 1 and 2. The optimal working cycle durations for the loads of Groups II and

IV loads are 2 hours, 3 hours and 6 hours respectively. Since best prices are night, all controllable loads under consideration work at night (Table 2). With such levels of the fixed generation the building can not function autonomously, i.e. without buying and selling from the external grid, because the sum load from the controllables (18.7 kWh) and the maximum possible P_a^{Max} value can not equal the estimated difference between fixed generation and load (76.6 kWh at 16h).

Table 2: *Optimal values for the controllable loads consumption over the optimization horizon*

j	$P_{l1,1,j}$	$P_{l1,2,j}$	$P_{l2,1,j}$	$P_{l2,2,j}$	$P_{l3,1,j}$	$P_{l3,2,j}$	$P_{l4,1,j}$
№	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	2,2	2,5	4,1	4,7	1,8	1,4	2
2	0	0	0	0	1,8	1,4	2
3	0	0	0	0	0	1,4	2
4	0	2,5	0	0	0	1,4	2
5	2,2	2,5	4,1	4,7	0	0	2
6	2,2	2,5	0	4,7	0	0	2
7	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0

This building can operate in a mode without buying from the external network ($P_{buy,j} = 0$) but just selling ($0 \le P_{sell,j}$). With such a realization of the building's operation, the optimal timetable for the operation of the controllable loads shifts to the afternoon hours (Tables 3 and 4).

Table 3: *Optimal values of the variables for accumulation, generation, buying and selling and own controllable generator when only selling is allowed*

Hour j	$P_{a,j}$	$P_{Ga,j}$	$P_{buy,j}$	$P_{sell,j}$	$P_{G,j}$
Col. №	**1**	**2**	**3**	**4**	**5**
1	20	0	0	8,5	50
2	0	8	0	0	0
3	0	8	0	0	0
4	0	8	0	0	0
5	0	8	0	0	0
6	0	8	0	0	0
7	20	0	0	14,5	0
8	20	0	0	26,0	0
9	20	0	0	17,1	0
10	0	0	0	55	0
11	0	0	0	44	0
12	0	0	0	38,7	0
13	0	0	0	50	0
14	20	0	0	36,6	0
15	20	0	0	35,2	0
16	20	0	0	55,2	0
17	20	0	0	37,8	0
18	0	0	0	37,5	0
19	0	16	0	37,2	0
20	0	16	0	6,3	0
21	0	16	0	8	0
22	0	8	0	0	0
23	0	8	0	0	0
24	0	8	0	0	0

The accumulation and generations schedule changes as well as the optimal duration of the single load from the fourth group (L4) as it becomes 3 hours.

Table 4: *Optimal values for the controllable loads consumption over the optimization horizon when only selling is allowed*

j	$P_{l1,1,j}$	$P_{l1,2,j}$	$P_{l2,1,j}$	$P_{l2,2,j}$	$P_{l3,1,j}$	$P_{l3,2,j}$	$P_{l4,1,j}$
№	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	2,2	2,5	4,1	4,7	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0
7	2,2	2,5	4,1	4,7	1,8	0	4
8	2,2	2,5	0	4,7	1,8	0	4
9	0	0	0	0	0	0	4
10	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0
14	0	0	0	0	0	1,4	0
15	0	2,5	0	0	0	1,4	0
16	0	0	0	0	0	1,4	0
17	0	0	0	0	0	1,4	0
18	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0

4. Conclusion

An approach to controllable loads with a working cycle consisting of more than one unit interval is proposed. The same modeling technique is used for power generating units start-up and stopping cycles in problems for optimal thermal units maintenance medium-term planning. The latter is a modification of the unit commitment problem but it's result influence the UC solution as availability. The numerical example shows how binary and integer values handle the physical requirements of the controllable loads. It also shows the sensitivity towards simple bounds over one set of optimization variables and its interpretation.

5. Bibliography

[1] Saravanan, B., Das, S., Sikri, S. et al., A solution to the unit commitment problem - a review, Frontiers in Energy (2013) Vol 7: pp. 223-226, https://doi.org/10.1007/s11708-013-0240-3

[2] Стоилов Д. Г., Янев К. И., Избор на състава на работещите агрегати в електроенергийните системи чрез смесено-целочислено линейно програмиране, списание Енергетика, брой 5, 2000 г., стр 21-23

[3] Стоилов Д., Ваковски Д., Алгоритми за оптимално разместване на планираните престои на агрегатите от КЕЦ в условия на дефицит, Енергиен форум’2007, Варна, юни 2007г., Сборник доклади том II, стр. 221-226.

[4] Стоилов Д., Кънева М., Ваковски Д., Математически модел за оптимално разпределяне на планираните престои на кондензационните агрегати в условия на дефицит, Списание Енергетика, брой 6, 2007г.

[5] 122 Viana A., Pedroso J. P., A new MILP-based approach for Unit Commitment in power production planning, International Journal of Electrical Power & Energy Systems, Volume 44, Issue 1, January 2013, p. 997-1005