

# ROBUST BI-CRITERIA APPROACH TO OPTIMIZE THE COMPOSITION AND PROPERTIES OF MAGNESIUM ALLOY

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**Abstract.** The paper presents standard statistical robust bi-criteria procedure for determining expert assessments for the influence of magnesium alloy components on the controlled mechanical properties: tensile strength and relative elongation. There are obtained regression models describing mechanical characteristics from the amount of aluminum, manganese, nickel and silicon directly related to the exploitation properties of the product. The applied bi-criteria approach makes it possible to determine of compositions ensuring relatively optimal values of the explored quality indicators.

**KEY WORDS.** SIMULATION, MAGNESIUM ALLOYS, MODELING, OPTIMIZATION, METALLURGICAL DESIGN.

## I. Introduction

Modern industry has to improve the parameters of creating products taking into account environmental and economic constraints. In this aspect competing magnesium companies need to have software tools and approaches to assist their activity in finding rational decisions related to the impact of composition and processing parameters on the final properties of the manufactured products. The possession of such tools helps to monitor and control production and technology change with reaching various technological properties of the final product. The creation of mathematical models to analyze the objects of metallurgical and casting process under examination is an important stage in achieving this goal. These models contribute to improve the set of properties and the final product quality. It is confirmed that it is possible to meet the requirements of the current market by implementation of such models. The wide range of problems, which Taguchi method has been applied to, is shown in [1].

The core of Taguchi approach consists of the method for reducing the influence of factors called noise (disturbing) that impair the quality parameters of the product/process. It is where the radical difference from the traditional technique of quality, which provides identification of existing sources and conduction of measurements that are often costly due to their control. The parametric design of Taguchi ensures non-sensitivity to noise along the way to the proper selection of certain parameters called controllable factors. The centerpiece of this approach is the method of reducing the impact of factors called noise that reduce the product/process quality parameters. It is where the radical difference from the traditional technique of quality, which provides identification of existing sources and conducting measurements often costly due to their control, lies. Taguchi parametric design ensures non-sensitivity to noise through a proper selection of certain parameters called controllable factors.

The aim of this paper is to present a robust approach for determining the influence of alloying elements on the properties of magnesium-based casing alloys that ensures better results than the input ones used to obtain a mathematical model.

The proposed approach facilitates the optimization of the magnesium alloy chemical composition improving the properties of the final product. These requirements generally are followed according to the standards but also may be associated with certain additional requirements claimed by users. All these pre-imposed conditions lead to a set of constraints that must be satisfied by acceptable solutions. Some restrictions can be defined as relations with true quantitative nature. This is especially important to restrictions on mechanical properties of the final product. Their proper formula is based on good mathematical models describing the effect of alloy composition and processing parameters on the

final properties of casting magnesium alloy. The statistical analysis of industrial data is an important and supporting alternative in such cases. That is why we have limited the field of study only to the influence of the chemical composition of the heat-treated alloys on the set of properties. In the context of the analysis of metallurgical processes, different methods to study the data described in the references can be found [2],[3] and [4].

The statistical analysis presented in this paper is based on of data collected during the real production process described in [5] and [6].

The ranges of change of the used alloying elements of ferrous alloys are listed in Table 1.

**Table 1. Minimum and maximum values of alloying components**

Input parameter	Chemical symbol	min [%]	max [%]
x <sub>1</sub>	Al	0.0	10.0
x <sub>2</sub>	Mn	0.0	1.5
x <sub>3</sub>	Zn	0.0	6.5
x <sub>4</sub>	Cu	0.0	2.7
x <sub>5</sub>	Ni	0.0	0.3
x <sub>6</sub>	Si	0.0	1.0

Regardless of that, the proposed optimization approach for modeling the final mechanical properties of alloys can be applied to any production process with steel manufacturing.

## II. General description of the approach

The analysis presented in this paper is related to the analysis of mechanical properties of magnesium specimens described by the following parameters: tensile strength -  $R_m$  [MPa] and relative elongation -  $A$  [%]. The limitations connected with these parameters are due to magnesium grade characteristics and customer's specifications. However, the main problem is that these parameters cannot be under direct observation during the manufacturing process, so any limitations associated with them can not be clearly defined in the optimization model. That means that we must develop models linking the final mechanical properties of the specimen/sample of the steel chemical composition as all as the parameters of the production process.

The regression analysis allows describing the relation between the variables of input and output, without going into the phenomenon nature during the process.

The regression models presented below have been created based on the data collected during the industrial production process.

The statistical analysis described in this section is based on a data set of 53 records extracted from the whole database.

The Least Squares method, LS is used to estimate the regression parameters. The estimated models of parameters *Rm* and *A* obtained in the examinations are given below.

In respect to the problem under examination, nonlinear regression dependencies have been identified for each of the mechanical properties of magnesium alloys. The regression dependencies are of the following kind:

$$f_i(x) = b_0^i + \sum_{j=1}^6 b_j^i x_j + \sum_{j=1l=l=j+1}^6 \sum_{j+1}^6 b_{jl}^i x_j x_l + \sum_{j=1}^6 b_{jj}^i x_j^2 \quad (1)$$

Here  $b_{ij}$  are the regression model parameters. The coefficients in equations are defined in Table 2. The models can be used for prediction if the check-up  $F > F(0.5, v_1, v_2)$  described in details has been made.

**Relation**  $(S/N) = \frac{\text{signal}}{\text{noise}}$  The effects of the factors are determined for each row, using the formula to minimize performance characteristics:

$$\frac{S}{N} = -10 \log \left( \frac{1}{n} \sum_{j=1}^n y_{ij}^2 \right) \quad (2),$$

for maximize

$$\frac{S}{N} = -10 \log \left( \frac{1}{n} \sum_{j=1}^n \frac{1}{y_{ij}^2} \right) \quad (3)$$

**Table 2. Coefficients of regression models of the examined target parameters.**

No	Coefficient	Rm [MPa]	A [MPa]
1	Free member	114.255	16.33366
2	X <sub>1</sub>	25.97015	0.6716988
3	X <sub>2</sub>	9.704941	-18.22966
4	X <sub>3</sub>	74.42215	-3.222518
5	X <sub>4</sub>	66.06575	12.28882
6	X <sub>5</sub>	3114.254	101.5687
7	X <sub>6</sub>	140.6771	12.65238
8	X <sub>1</sub> X <sub>2</sub>	-0.4041829	-0.1963183
9	X <sub>1</sub> X <sub>3</sub>	-1.084408	-0.006829805
10	X <sub>1</sub> X <sub>4</sub>	-72.2078	-0.6545856
11	X <sub>1</sub> X <sub>5</sub>	237.9817	9.555618
12	X <sub>1</sub> X <sub>6</sub>	-0.7780583	0.1666764
13	X <sub>2</sub> X <sub>3</sub>	-44.53689	2.326275
14	X <sub>2</sub> X <sub>4</sub>	101.9566	-15.04205
15	X <sub>2</sub> X <sub>5</sub>	1768.658	336.508
16	X <sub>2</sub> X <sub>6</sub>	-36.39456	21.76996
17	X <sub>3</sub> X <sub>4</sub>	0.7895336	0.1440411
18	X <sub>3</sub> X <sub>5</sub>	-414.173	18.49442
19	X <sub>3</sub> X <sub>6</sub>	-99.81348	0.05190802
20	X <sub>4</sub> X <sub>5</sub>	2369.462	150.6452
21	X <sub>4</sub> X <sub>6</sub>	138.7343	-61.84769
22	X <sub>5</sub> X <sub>6</sub>	-2435.761	-121.6928
23	X <sub>1</sub> <sup>2</sup>	-1.798813	-0.1700993
24	X <sub>2</sub> <sup>2</sup>	53.00094	7.873145
25	X <sub>3</sub> <sup>2</sup>	-5.917203	0.3912327
26	X <sub>4</sub> <sup>2</sup>	-43.78463	-2.977206
27	X <sub>5</sub> <sup>2</sup>	-74966.48	-4711.342
28	X <sub>6</sub> <sup>2</sup>	-105.1666	-24.45902
	R	0.888	0.914
	F	3.449	4.684

The composition optimization is applied only in respect to yield strength *Rm* and respective elongation *A*.

**Table 3. Orthogonal matrix I (27,13) developed by Taguchi with factors at three levels**

Run	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	1	3	3	3	3	3	3	3	3	3
4	1	2	2	2	1	1	1	2	2	2	3	3	3
5	1	2	2	2	2	2	2	3	3	3	1	1	1
6	1	2	2	2	3	3	3	1	1	1	2	2	2
7	1	3	3	3	1	1	1	3	3	3	2	2	2
8	1	3	3	3	2	2	2	1	1	1	3	3	3
9	1	3	3	3	3	3	3	2	2	2	1	1	1

10	2	1	2	3	1	2	3	1	2	3	1	2	3
11	2	1	2	3	2	3	1	2	3	1	2	3	1
12	2	1	2	3	3	1	2	3	1	2	3	1	2
13	2	2	3	1	1	2	3	2	3	1	3	1	2
14	2	2	3	1	2	3	1	3	1	2	1	2	3
15	2	2	3	1	3	1	2	1	2	3	2	3	1
16	2	3	1	2	1	2	3	3	1	2	2	3	1
17	2	3	1	2	2	3	1	1	2	3	3	1	2
18	2	3	1	2	3	1	2	2	3	1	1	2	3
19	3	1	3	2	1	3	2	1	3	2	1	3	2
20	3	1	3	2	2	1	3	2	1	3	2	1	3
21	3	1	3	2	3	2	1	3	2	1	3	2	1
22	3	2	1	3	1	3	2	2	1	3	3	2	1
23	3	2	1	3	2	1	3	3	2	1	1	3	2
24	3	2	1	3	3	2	1	1	3	2	2	1	3
25	3	3	2	1	1	3	2	3	2	1	2	1	3
26	3	3	2	1	2	1	3	1	3	2	3	2	1
27	3	3	2	1	3	2	1	2	1	3	1	3	2

By Taguchi methodology (Khosrow Dehnad, 1989) an experiment modeled on orthogonal matrices developed by him is carried out. The experiment can be accomplished in two ways by:

- a real experiment leading to obtaining results for processing;
- a numerical experiment with the presence of adequate regression models.

The availability of the described model coefficients, which can be used to predict, give a possibility to make a numerical experiment involving Taguchi method. The noise matrix is selected from orthogonal matrix I (27,13) with 27 rows and 13 columns developed by Taguchi. The matrix is worked out with factors at three levels – Table 3.

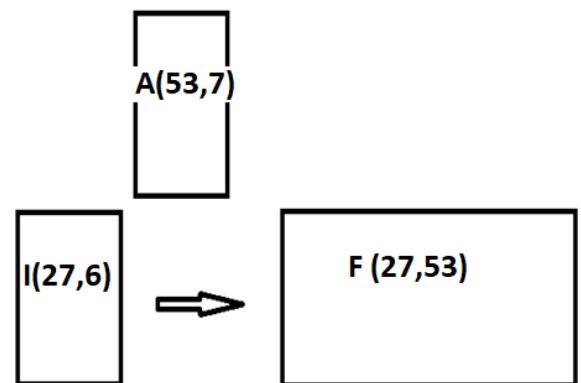
The methodology proposed is implemented for tensile strength Rm and relative elongation A. To take out the models of these two target functions, 53 experiments that form the data matrix A (53, 6 +1) have been used. Here the added column "1" is for the output target function Rm or A stored compactly in the matrix.

To optimize the computing process, the scheme, which having been processed for the particular case takes the following kind, is selected.

In numerical experiments that use models based on the chemical composition the noise can be expressed only in the change of the respective components. It is assumed to express noise  $\Delta$  in the following way

$$\Delta_i = \frac{\bar{x}_i}{k} \text{ where further calculations are made for } k \text{ equal to } 100 \text{ and } 70.$$

Here  $\bar{x}_i$  is the mean value of relevant variable "i".



**Fig.1. Organizing experiments with parametric planning with matrices I, A and F**

For level "1" of I (27,6) noise is subtracted from relevant  $x_i$  taking the value of  $x_i - \Delta_i$ . With level "2" no correction is applied, the value of  $x_i$  is preserved. With level "3" noise is added to relevant  $x_i$  taking the value of  $x_i + \Delta_i$ . In numerical experiments where models based on chemical composition are used, noise can be expressed only in the change of the respective components. Noise  $\Delta$  is

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and 70. Here  $\bar{x}_i$  is the average value of the respective variable "i". In level "1" noise is subtracted from respective  $x_i$  taking the value of  $x_i - \Delta_i$ . In level "2" no correction is applied, the value of  $x_i$  is preserved. In level "3" noise is added to respective  $x_i$  taking the value of  $x_i + \Delta_i$ .

Thus, noise is expressed in the change of chemical composition. The calculation process is organized as follows:

A row of matrix I (27,6) is taken (for example, row 1 - I (1,6)). In this row level "1" is assigned for each  $x_i$ , i.e. noise will be taken out from each value  $x_i$ .

Thus F (1,1) of the matrix F (27,53) is obtained from the first row of A (53,7). The same rule is applied to the rest of the series F (53,6) and it forms F (27,53).

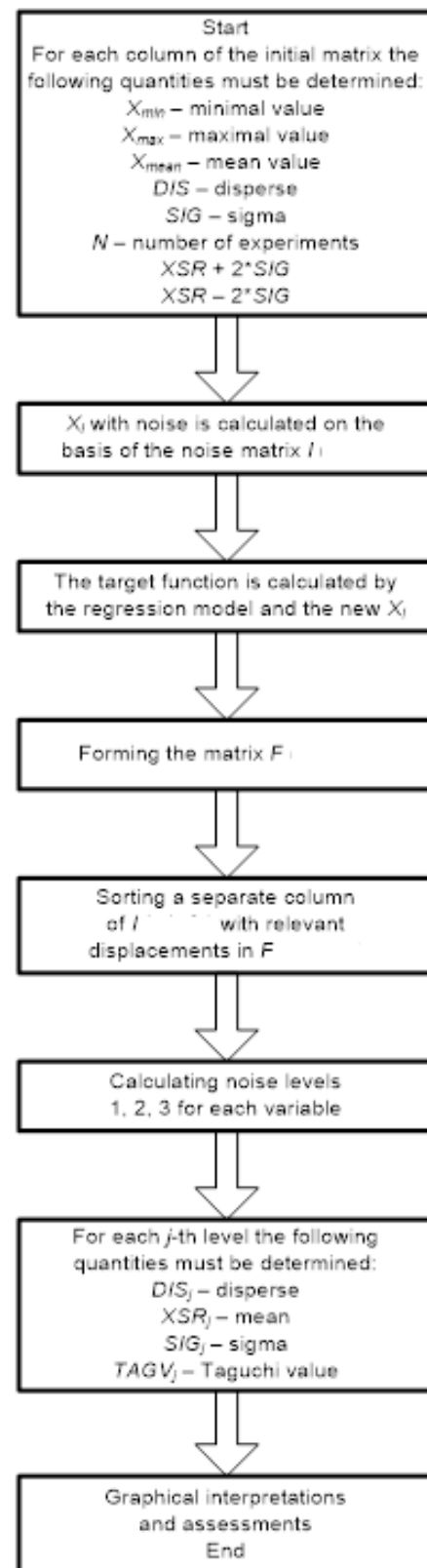
It is continued with the next row of matrix I (27,6) performing the following sequence. Each row of matrix I (27,6) forms a relevant row of matrix F (27,53).

Calculations are performed according to the following algorithm.

If we take the first column of matrix I (27,6) relevant to  $X_1$ , it is evident that the first nine rows correspond to level "1" of noise, the second nine lines correspond to level "2" and the third nine rows correspond to level "3" of noise. This makes possible to use the values of the first nine rows of matrix F (27,53) to calculate level "1", to use the second nine rows to calculate level of "2" and the third nine rows for calculation at level "3" for  $X_1$ . For other columns from 2 to 8 it is necessary to sort in ascending order  $X_i$  from I(27,6). After sorting the column obtains the kind of the first column.

After sorting of the respective variable, calculations for different levels can be made. It is continued with the next matrix row I (27,6) performing the following sequence. Each row of matrix I (27,6) forms a corresponding row of matrix F (27,53). If we take the first column of matrix I (27,6) corresponding to the  $X_1$ , one can see that the first nine rows correspond to noise level "1" of noise, the second nine rows correspond to level "2" and the third nine rows correspond to noise level "3". That allows using the values of the first nine rows of matrix F (27,90) to calculate level "1", the second nine rows to calculate level "2" and the third nine rows to calculate level "3" for  $X_1$ . For the rest columns from 2 to 6 it is necessary to sort by ascending order of  $X_i$  of I(27,6). After sorting the column takes the kind of the first column. With sorting, if shifts are made, they are reflected in matrix F (27,53). After sorting the corresponding variable it is possible to make calculations for different levels.

In the numerical experiment noise was first determined with  $K=70$ . The analysis of the graphics shows low sensitivity for both  $R_m$  and  $A$ . In these calculations, as shown in the Table 4.



**Fig. 2. Computational algorithm**

The conclusion that can be made for the tensile strength –  $R_m$  is that all the factors have a significant effect on aluminum, manganese, zinc and nickel and it is expected they to change in the direction of decreasing values, and copper and silicon to increasing values. About the results for the relative elongation –  $A$ , from all six variables two of the variables - nickel and silicon – should not be changed, and the rest of the variables – aluminum, manganese, zinc and copper – need to change in the direction of increasing their

value. As the experiment is numerical, it is possible to perform numerical optimization with the mathematical models obtained as the values of  $X_i$  are remained to change within the limits defined by the output data (Table 1). That some of the variables remain unchanged, i.e. keep their initial values it imposes the necessity to separately carry out optimization for the chemical composition of each alloy. As a method of optimization, the method of Hook and Jives was chosen. This method is characterized as one of the best to solve problems with different parameters of the goal functions. Specifically, the tensile strength  $R_m$  is changed from  $X_1$ ,  $X_2$ ,  $X_3$ ,  $X_4$ ,  $X_5$  and  $X_6$ , and the relative elongation is varied from  $X_1$ ,  $X_2$ ,  $X_3$  and  $X_4$ .

**Table 4. Levels of noise factors for the research parameters**

Variable	Element of composition	Noise level	
		Rm	A
$X_1$	Al	1	3
$X_2$	Mn	1	3
$X_3$	Zn	1	3
$X_4$	Cu	3	3
$X_5$	Ni	1	2
$X_6$	Si	3	2

The fact that one of the variables does not change, i.e. they preserve their initial values requires optimization to be performed separately with chemical composition for each alloy. The ones mentioned,  $X_5$  and  $X_6$ , are maintained at their level, but are held by changing the rest. In this way, 53 optimizations are performed, with each case obtaining a separate value of the extremum. Then all maxima are sorted in ascending order and the largest is selected. With the values of the variables of the relative elongation, the value of the tensile strength  $R_m$  is calculated. Thus, the two-criteria approach is implemented. The optimal composition is shown in the table.

Al	Mn	Zn	Cu	Ni	Si
10.0%	1.5%	6.5%	2.7%	0.3%	1%

Such an approach is justified because the task, if viewed from the point of view of technology, is that individual optimization is the refinement of a separate actual alloy.

Optimization in this way coincides with the approach of searching for a global extremum from a set of starting points.

This outcome indicates that the task is feasible and the approach applied can result in improvement of the alloy composition.

### III. Conclusion

The numerical experiment has proved the ability to improve the quality of magnesium alloy of a certain class. Mathematical models suitable for forecasting and optimization have been derived. The approach of Taguchi applied has led to a desired result, to separate variables  $X_i$  for the examined parameters that do not influence significantly on the final result. With this limit, the numerical optimization for maximum search has been conducted with each chemical composition. That allows improving it. Relative elongation  $A$  turned to be less variable index and tensile strength  $R_e$  requires caution with extreme selecting. The decision of bi-criteria problem set has been defined thus proving that the Taguchi approach is applicable to a similar class of problems.

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