

## SOLIDIFICATION ON SURFACE

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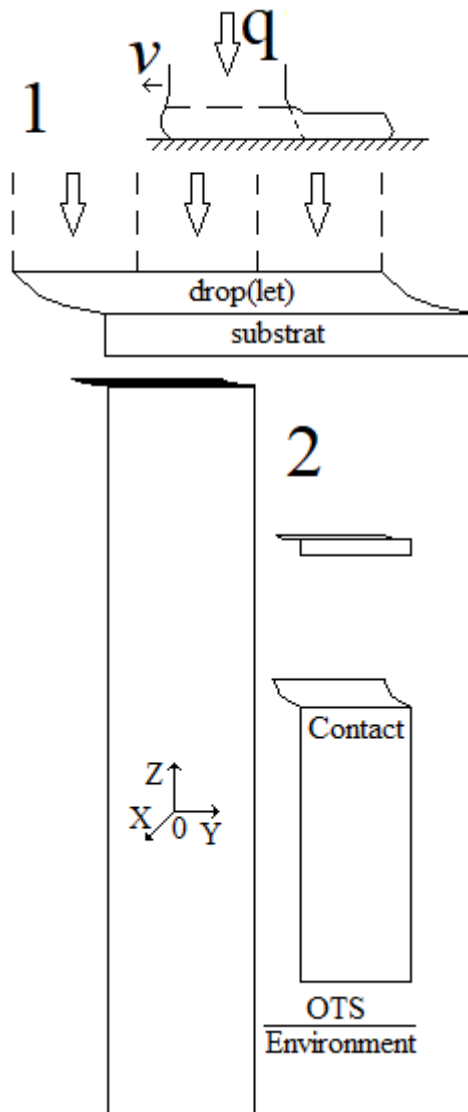
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**Abstract:** This article uses mathematical mathematical models of tasks by Stefan and Stefan-Schwarz describing the technologies of IMSCHA "Acad. A. Balevski ". Described are processes for solidifying a drop (droplet) over a surface of a metal substrate. Processes of solidifying of metal melts in the form of spheres having a radius of 50 nm are described. The temperature fields of the open thermodynamic system drop / substrate system are presented. The influence of the change of specific parameters from the hardening process is represented by the type of the temperature field of the OTS.

**Keywords:** Stefan and Stefan-Schwarz problem, drop (droplet) on the surface of substrate, temperatures curves

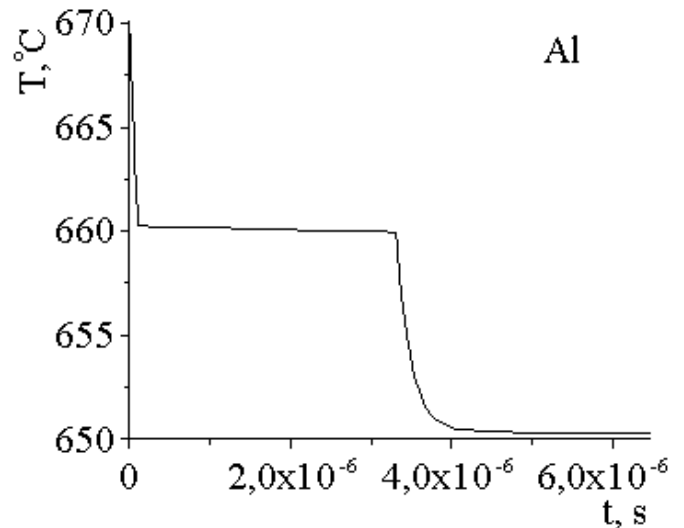
### 1. Introduction – основен процес

In Fig. 1 is shown the solidification on surface process:

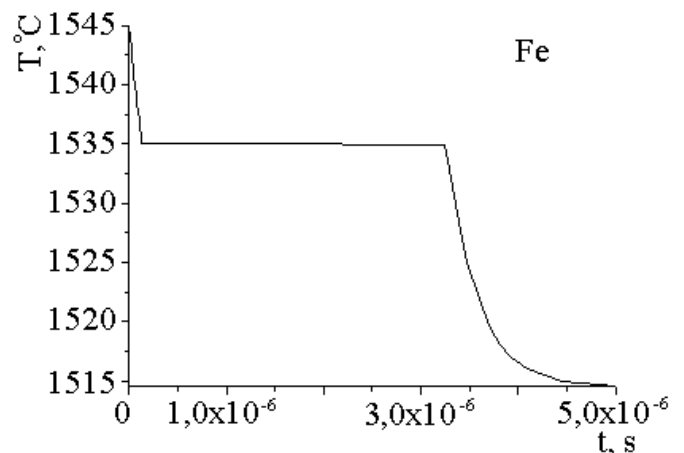


**Fig. 1** Geometric idea by cross section of the open thermodynamic system (OTS) – drop (let) solidification on surface of the substrate: 1 – Physics: drop and flow of melt,  $v$  and  $q$  are velocity and stream; 2 – different size of system.

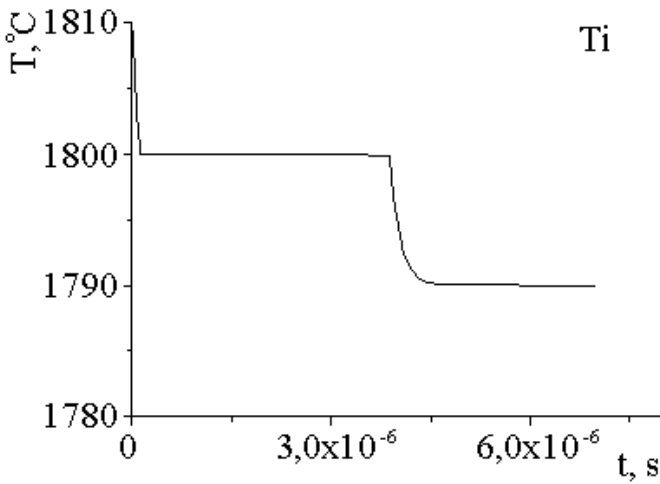
Figure 2 presents our results [20], supplemented by new ones. These are time-temperature curves of solidification of spheres of different materials with a radius  $R = 50$  nm:



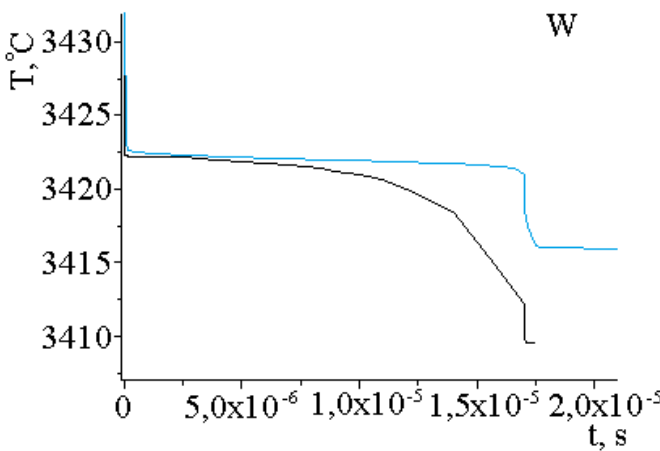
Pure Al:  $T_m = 660, 1^\circ \text{C}$ , latent heat of fusion at  $T_m$  is  $Q_m = 401,819 \text{ kJ/m}^3$ , heat conductivity  $\lambda_S = 209 \text{ w/(m}^3 \cdot \text{K)}$   $\lambda_L = 104,675 \text{ w/(m}^3 \cdot \text{K)}$ , heat capacity  $c_S = 2540 \text{ kJ/(kg} \cdot \text{K)}$   $c_L = 2380 \text{ kJ/(kg} \cdot \text{K)}$ , density  $\rho_S = 2540 \text{ kg/m}^3$ ,  $\rho_L = 2380 \text{ kg/m}^3$ , index S – solid, index L – liquid;



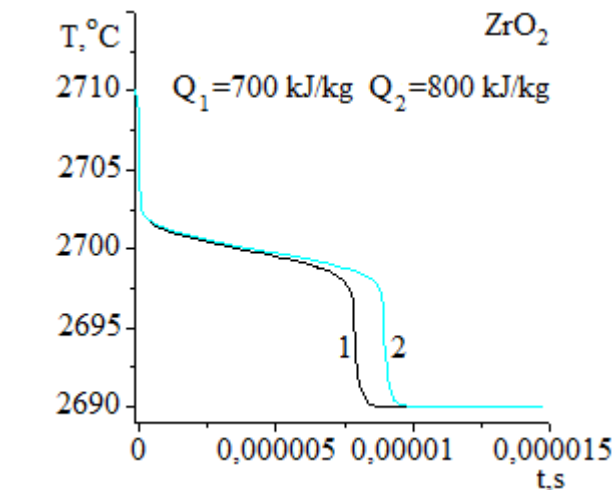
Pure Fe:  $T_m = 1535^\circ \text{C}$ ,  $Q_m = 272 \text{ kJ/m}^3$ ,  $\lambda_S = 33 \text{ w/(m}^3 \cdot \text{K)}$   $\lambda_L = 29 \text{ w/(m}^3 \cdot \text{K)}$ ,  $c_S = 447 \text{ kJ/(kg} \cdot \text{K)}$   $c_L = 800 \text{ kJ/(kg} \cdot \text{K)}$ ,  $\rho_S = 7400 \text{ kg/m}^3$ ,  $\rho_L = 7145 \text{ kg/m}^3$ ;



Pure Ti:  $T_m = 1800^\circ\text{C}$ ,  $Q_m = 376,810\text{ kJ/m}^3$ , heat conductivity  $\lambda_S = 22.5\text{ w/(m}^3\cdot\text{K)}$   $\lambda_L = 18\text{ w/(m}^3\cdot\text{K)}$ , heat capacity  $c_S = 523\text{ kJ/(kg}\cdot\text{K)}$   $c_L = 628\text{ kJ/(kg}\cdot\text{K)}$ , density  $\rho_S = 4540,6\text{ kg/m}^3$ ,  $\rho_L = 4500\text{ kg/m}^3$ , index S – solid, index L – liquid;



Pure W:  $T_m = 3422^\circ\text{C}$ ,  $Q_m = 255,390\text{ kJ/m}^3$ ,  $\lambda_S = 196\text{ w/(m}^3\cdot\text{K)}$   $\lambda_L = 76,2\text{ w/(m}^3\cdot\text{K)}$ ,  $c_S = 150,7\text{ kJ/(kg}\cdot\text{K)}$   $c_L = 152,8\text{ kJ/(kg}\cdot\text{K)}$ ,  $\rho_S = 19300\text{ kg/m}^3$ ,  $\rho_L = 19200\text{ kg/m}^3$ . Important: the difference between the two curves is due to the precision of defining the parameters of the mathematical model of the heat source in Stephen's task [].



Material ZrO:  $T_m = 2700^\circ\text{C}$ ,  $Q_m = 700 \div 800\text{ kJ/m}^3$ ,  $\lambda_S = 1.95\text{ w/(m}^3\cdot\text{K)}$   $\lambda_L = 2.44\text{ w/(m}^3\cdot\text{K)}$ ,  $c_S = 730\text{ kJ/(kg}\cdot\text{K)}$   $c_L = 730\text{ kJ/(kg}\cdot\text{K)}$ ,  $\rho_S = 3300\text{ kg/m}^3$ ,  $\rho_L = 3200\text{ kg/m}^3$ ; curve 1 -  $Q_m = 700\text{ kJ/m}^3$ ; 2 -  $Q_m = 800\text{ kJ/m}^3$  [14].

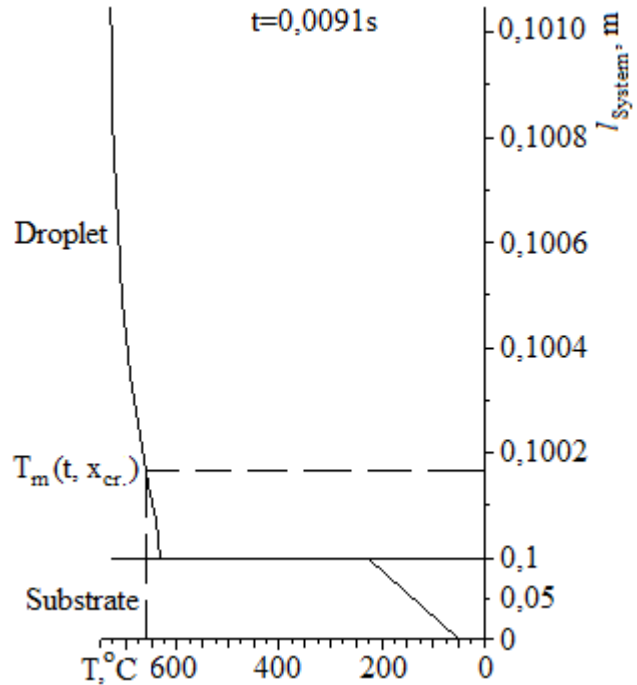
Fig. 2 Solidification of different materials – spheres of different materials with a radius  $R = 50\text{ nm}$ .

Mathematical models are based on the tasks of Stefan and Stefan-Schwarz, created and realized through non-commercial software products at IMSETCHA "Acad. A. Balevski "BAS [2, 3, 4,

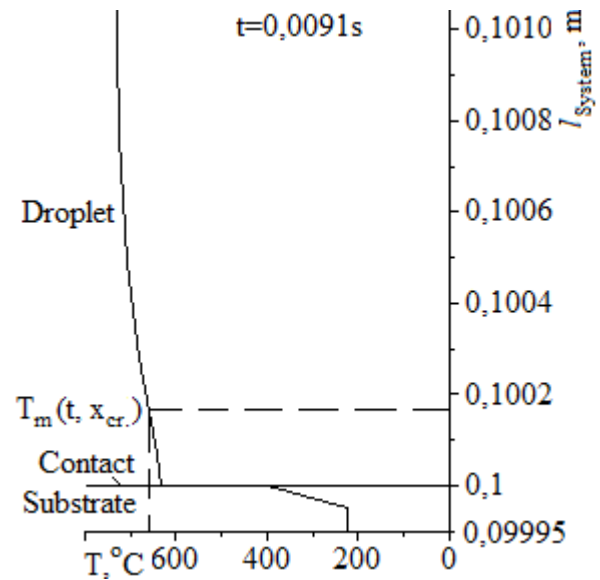
5 and 6]. The process of solidification (phase transition of first order) is fundamental in material science for obtaining structures bearing different working properties of the respective materials [1, 6]. The aim of this work is the calculation process of solidification a drop on different sizes on the surface of a metal substrate.

2. Numerical solutions - Stefan-Schwarz tasks

In Fig. 3 are introduced solidification temperature field by Finite elements method (FEM) of this task (see Fig.1):



Stefan-Schwarz temperature field: The geometry space of FEM is 3D but the temperature task in the direction of axis OZ is 1D;



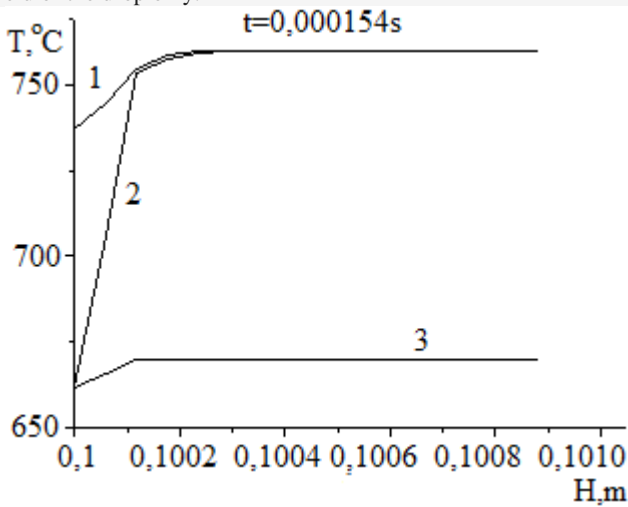
Important: ccontact heat exchange with the temperature difference at the surface of the drop-substrate contact;

Fig. 3 Stefan-Schwarz temperature great OTS with size (substrate  $x_{max}=0,02, y_{max}=0,02, z_{max}=0,1$ ) [m] and (drop  $x_{max}=0,02, y_{max}=0,02, z_{max}=0,00105$ ) [m].

The results of Fig. 3 present the possibilities of computational mathematics and physics of non-stationary field of solidification (phase transition of first order) on a macro-scale. The technological parameters of the phase transition of first order i.e. the process of formation of the polycrystalline structure on a macro-scale.

The technological parameters of the hardening process are known for a long time or for the phase transition of the first

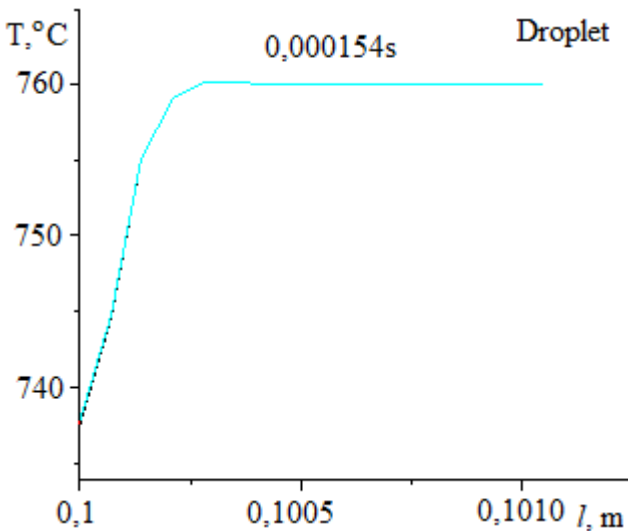
generation. We choose the parameter: the initial temperature field of the corresponding OTS  $T(x, y, z, t = 0)$  and the coefficient of heat transfer at the contact surface  $\alpha_c$  (see Fig. 1 and Fig. 2). In Fig. 4 we present the influence of these two parameters on the temperature field of the drop only:



**Fig.4** Comparative analysis by the type of temperature field only at the drop by changing the initial melt temperature and by changing the coefficient of contact heat transfer: 1  $T_{drop}(x, y, z, 0) = 760, 1^\circ C, \alpha_c = 56000 \text{ w/m}^2 \text{ s}^\circ C$ ; 2  $T(x, y, z, 0) = 760, 1^\circ C, \alpha_c = 560000 \text{ w/m}^2 \text{ s}^\circ C$ ; 3  $T(x, y, z, t=0) = 670, 1^\circ C, \alpha_c = 56000 \text{ w/m}^2 \text{ s}^\circ C$ .

From Fig. 4 clearly shows the influence on the type of the temperature field at time  $t = 0.0091s$ .

The thermophysical properties of the substrate have a definite effect on the phase transition of first order. For this reason, the following figures shows the influence of the temperature field in the droplet and the substrate in a small OTC with dimensions ( $x_{max} = 0,00002 \text{ m}, y_{max} = 0,00002 \text{ m}, z_{substrate \ max} = 0,1$  and  $z_{droplet} = 0,00105 \text{ m}$ ):

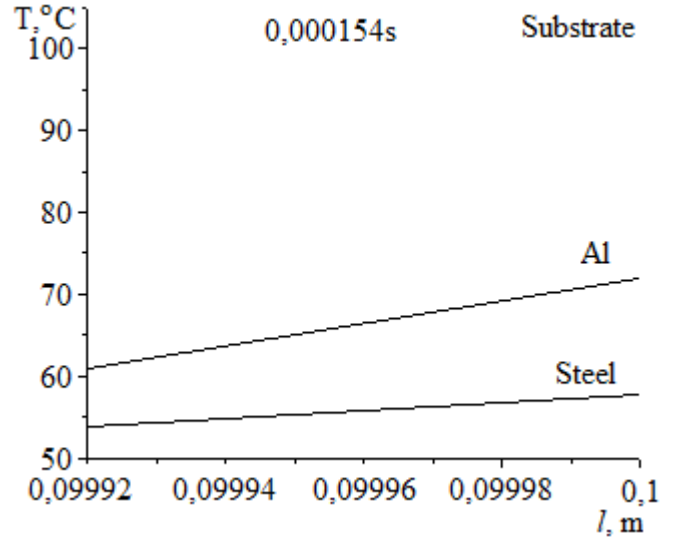


**Fig. 5** Temperature fields of the droplet at time  $t = 0.000154s$  steel (black line) and aluminum (blue line) substrates. Initial conditions are:  $T_{drop}(x, y, z, 0) = 760, 1^\circ C, \alpha_c = 56000 \text{ w/m}^2 \text{ s}^\circ C$ ; 2  $T(x, y, z, 0) = 760, 1^\circ C$ . The minimal effect is very clearly seen even at start time  $t = 0,000154s$ .

Here, we note that the thermophysical parameters of the substrate are part of the macro-level influence parameters.

**Important: Your own substrate is related to modern technology called 3DPrinter.**

On the following figure 6 presents a comparative analysis of the temperature fields of the OTC in substrates with different heat-physical properties (aluminum and steel substrates):



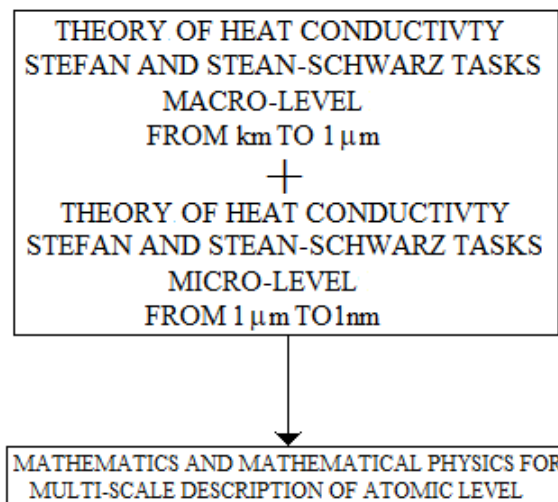
**Fig. 6** Comparative analysis of temperature fields in substrates with different thermophysical properties in the case of Al and Steel. Initial conditions are:  $T_{drop}(x, y, z, 0) = 760, 1^\circ C, \alpha_c = 56000 \text{ w/m}^2 \text{ s}^\circ C$ ; 2  $T(x, y, z, 0) = 760, 1^\circ C$ . The minimal effect is very clearly seen even at start time  $t = 0,000154s$ .

From Fig. 6 shows the significant difference with the direct relationship to the contact heat exchange, namely the influence of the surface temperature of the contact substrate/droplet.

The results obtained relate to different technological processes. Stefan Schwarz's main task allows him to develop as a connecting task with mathematical and mathematical physics tasks. The latter are related to lower levels. These ideas have been developed in various studies by many authors.

Stephen-Schwartz's task is the natural aggregate mathematical approach to describe technological processes in phase transitions of first and secondary order. Material science has evolved as an interdisciplinary field of study [7, 8, 9 and 10]. The next figure presents the multi-scale linking possibility of heat conduction theory and mathematical modeling [2, 3, 4, 5, 6, 11, 12 and 13]:

**LEVELS OF DESCRIPTION OF PHASE TRANSITION OF FIRST AND SECONDARY ORDERS**



**Fig. 7** Multi-scales description of phase transition of first and secondary orders by Stefan-Schwartz tasks + Mathematics and mathematical physics.

According to the principles of material science [7, 8, 9 and 10] and mathematical modeling [2, 3, 4, 5, 6, 11, 12 and 13], a methodology for Basic Knowledge is proposed: Methodology for Basic Knowledge is proposed: 1. General evaluation of the structure (on the meso-level) and the properties of the material; 2. Select a section of mathematics and mathematical physics for multi-scale

description; 3. A specific mathematical model of the structure, with the working properties, the technological process, the work and the life of the material. A very important point is the use of this methodological frame work for the additive capabilities of the 3Dprinter [15, 16 and 17] technology i.e. *3Dprinter interaction with die casting and heat treatment*. 3Dprinter + die casting + heat treatment are an essential task of Industry 4.0 [1]. Mathematical modeling in Industry4.0, in our view, is close to pure mathematics [6]. Such an assumption canal so be accepted on the basis of works [2, 3, 4, 5, 6, 11, 12, 13, 18 and 19]. The 3Dprinter technology has a great application (from food, medicine and weapons) and is an incentive for its mathematical modeling [17 and 18].

Additive manufacturing AM [15] is a concept that covers: casting, separation of metal turning, milling, plastic deformation, welding, forging and electroerosity processing heat treatment. But at the end of the 20th century new technologies for depositing materials such as micro-casting [21] and spraying were developed at Stanford University and Carnegie Mellon University [22]. 3DPrinter modifies the AM concept by expanding its idea of sequentially adding and linking material sequentially, following a three-dimensional profile of the blank, with automated control [15].

Significant results have been achieved with 3D printer technologies [23, 24 and 25]. The reason for the powerful entry of a 3D printer is the good price that falls great convenience to work with full control of the technology. This makes the 3D printer actually leading the Industry 4.0 revolution [1].

### 3. Conclusions

The obtained results represent the broad application of Stefanov type tasks in macro and nano-scale areas.

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