CALIBRATION OF AN ARTICULATED VEHICLE MODEL

Prof. dr hab. n.t. Adamiec-Wójcik I. [1], Prof. dr hab. n.t. Wojciech S. [1]
Faculty of Management and Transport – University of Bielsko-Biala, Poland [1]
i.adamiec@ath.bielsko.pl

Abstract: *A model of an articulated vehicle (tractor with a trailer and/or semitrailer) formulated using joint coordinates and homogenous transformations is presented. Experimental measurements of yawing velocities of the vehicle units have been carried out for a sharp turn manoeuvre. These results are used to calibrate the mathematical models. Using optimisation methods the parameters of tires for the Dugoff-Uffelman model are chosen in such a way that the results of calculations and measurements are compatible.*

Keywords: ARTICULATED VEHICLE, MULTIBODY MODEL, MODEL CALIBRATION, OPTIMISATION

1. Introduction

Articulated vehicles are vehicles which consist of two or more units, the first one of which is a tractor and the others are trailers connected by pivot joints, which enable the vehicle to perform a sharp turn. Due to the trends in the world economy the use of articulated vehicles plays a significant role in transport systems. Development of unmanned and automotive transportation systems requires much research in control and analysis of dynamics of articulated vehicles.

Safety is one of the main issues in analysis of behavior of the articulated vehicles especially in respect of stability of motion. There is a considerable amount of research devoted to the analysis of rollover and jack-knifing problems [1-4]. Control strategies are usually proposed on the basis of simplified dynamic models [5,6]. On the other hand the dynamic model has to take into account as many parameters as possible in order to reflect real motion but numerical efficiency is also a very important factor.

In this paper the dynamic model of an articulated vehicle is derived using multibody methods [7]. Joint coordinates are used to describe kinematics of the vehicle which makes the model to be derived with the smallest number of generalized coordinates. In order to define the geometry of the system we use homogenous transformations which are very popular in robotics [8].

Tire models play an important role in every model of a vehicle. The most popular models are Pacejka's magic formula [9] and the Dugoff-Uffelman model [10]. Both of them depend on parameters which have to be determined experimentally and the results of numerical simulations strongly depend on the values chosen. In this paper the Dugoff-Uffelman model is used and in order to choose the parameters of the model calibration procedure based on solution of an optimization problem is proposed. The optimization problem is defined so that the parameters of the tire are chosen in such a way that the results of experimental measurements are compatible with those from numerical simulations.

2. Mathematical model of an articulated vehicle

Mathematical models of vehicles are derived with a different level of detail depending on the purpose of the model. Very often the equations of motion are formulated analytically. Multibody methods are useful especially in the description of articulated vehicles consisting of n vehicle units, where each unit is treated as a separate rigid body connected with others in the kinematic chain by means of rotary joints. When joint coordinates and homogenous transformations developed in robotics are applied, the motion of each link (unit) in the chain is described with respect to the preceding link. The main difference, as far as vehicles are concerned, is that the additive units (suspension, wheels, steering system) are coupled to the link (vehicle unit) and thus a tree shape of the whole system is obtained (Fig.1).

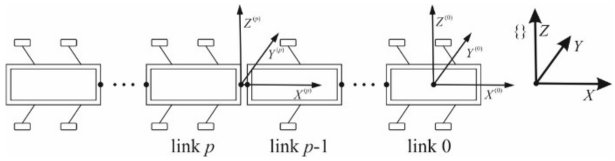

Fig. 1 S*ystem of vehicle units (links) in the tree like shape.*

The procedure of generating the equations of motion is general and a single vehicle can be considered ($n = 1$) as a special case. In order to describe kinematics of the articulated vehicle, the coordinate systems are assigned to each vehicle unit. If the first link (tractor or a single vehicle) is considered, the respective coordinate system is placed in the center of mass of the unit and its motion is described by six coordinates which are three displacements $x^{(1)}, y^{(1)}, z^{(1)}$ and three ZYX Euler angles $\psi^{(1)}, \theta^{(1)}, \varphi^{(1)}$. The motion of vehicle unit p ($p = 1, \ldots, n$) is described with respect to preceding unit p-1 in the kinematic chain by means of one to three rotary degrees of freedom depending on the kind of the coupling between those units.

For the purpose of the paper let us consider a truck with a trailer (Fig.2).

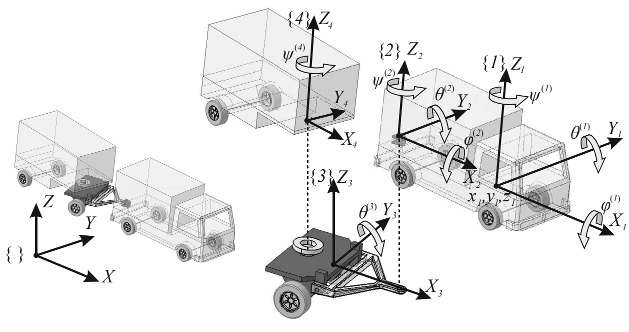

Fig. 2 *Truck with a trailer.*

In this case the whole vehicle is treated as a system of four vehicle units: a truck, a drawbar, a dolly and a trailer. Some of the units have wheels and thus the generalized coordinates describing the motion of the unit consist of the main unit generalized coordinates and rotation angles of wheels. A simplified model of suspension which reduces its flexibility to the contact point between the tire and the road is considered. Thus the model of the truck with a trailer shown in Fig.2 is described by the following generalized coordinates:

1) tractor with four wheels:

$$\mathbf{q}^{(1)} = \tilde{\mathbf{q}}^{(1)} = \left[\tilde{\mathbf{q}}_{\mathbf{b}}^{(1)\mathbf{T}} \quad \tilde{\mathbf{q}}_{\mathbf{s}}^{(1)\mathbf{T}} \right]^{\mathbf{T}} \tag{1.1}$$

where $\tilde{\mathbf{q}}_b^{(1)} = \left[x^{(1)}\ y^{(1)}\ z^{(1)}\ \psi^{(1)}\ \theta^{(1)}\ \varphi^{(1)}\right]^T$,

$\tilde{\mathbf{q}}_s^{(1)} = \left[\tilde{\varphi}_1^{(1)}\ \tilde{\varphi}_2^{(1)}\ \tilde{\varphi}_3^{(1)}\ \tilde{\varphi}_4^{(1)}\right]^T$

2) drawbar:

$$\mathbf{q}^{(2)} = \left[\mathbf{q}^{(1)^T} \quad \tilde{\mathbf{q}}^{(2)^T}\right]^T = \left[\mathbf{q}^{(1)^T} \ \psi^{(2)} \ \theta^{(2)} \ \varphi^{(2)}\right]^T \tag{1.2}$$

3) dolly with two wheels:

$$\mathbf{q}^{(3)} = \left[\mathbf{q}^{(2)^T} \quad \tilde{\mathbf{q}}^{(3)^T}\right]^T = \left[\mathbf{q}^{(2)^T} \quad \tilde{\mathbf{q}}_b^{(3)^T} \quad \tilde{\mathbf{q}}_s^{(3)^T}\right]^T \tag{1.3}$$

where $\tilde{\mathbf{q}}_b^{(3)} = [\theta^{(3)}]$, $\tilde{\mathbf{q}}_s^{(3)} = \left[\tilde{\varphi}_1^{(3)} \ \tilde{\varphi}_2^{(3)}\right]^T$

4) trailer with two wheels:

$$\mathbf{q}^{(4)} = \left[\mathbf{q}^{(3)^T} \quad \tilde{\mathbf{q}}^{(4)^T}\right]^T = \left[\mathbf{q}^{(3)^T} \quad \tilde{\mathbf{q}}_b^{(4)^T} \quad \tilde{\mathbf{q}}_s^{(4)^T}\right]^{\mathbf{T}} \tag{1.4}$$

where $\tilde{\mathbf{q}}_b^{(4)} = [\psi^{(4)}]$, $\tilde{\mathbf{q}}_s^{(4)} = \left[\tilde{\varphi}_1^{(4)} \ \tilde{\varphi}_2^{(4)}\right]^T$

The equations of motion of the whole vehicle can be formulated in the partitioned form as follows:

$$\mathbf{A}\ddot{\mathbf{q}} = \mathbf{f} \tag{2}$$

where $\mathbf{A} =$

$$\begin{bmatrix}
\mathbf{A}_{1,1}^{(1)} + \mathbf{A}_{1,1}^{(2)} + \mathbf{A}_{1,1}^{(3)} + \mathbf{A}_{1,1}^{(4)} & \mathbf{A}_{1,2}^{(2)} + \mathbf{A}_{1,2}^{(3)} + \mathbf{A}_{1,2}^{(4)} & \mathbf{A}_{1,3}^{(3)} + \mathbf{A}_{1,3}^{(4)} & \mathbf{A}_{1,4}^{(4)} & \mathbf{A}_{1,s}^{(1)} & \mathbf{A}_{1,s}^{(3)} & \mathbf{A}_{1,s}^{(4)} \\
\mathbf{A}_{2,1}^{(2)} + \mathbf{A}_{2,1}^{(3)} + \mathbf{A}_{2,1}^{(4)} & \mathbf{A}_{2,2}^{(2)} + \mathbf{A}_{2,2}^{(3)} + \mathbf{A}_{2,2}^{(4)} & \mathbf{A}_{2,3}^{(3)} + \mathbf{A}_{2,3}^{(4)} & \mathbf{A}_{2,4}^{(4)} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{A}_{3,1}^{(3)} + \mathbf{A}_{3,1}^{(4)} & \mathbf{A}_{3,2}^{(3)} + \mathbf{A}_{3,2}^{(4)} & \mathbf{A}_{3,3}^{(3)} + \mathbf{A}_{3,3}^{(4)} & \mathbf{A}_{3,4}^{(4)} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{3,s}^{(3)} \\
\mathbf{A}_{4,1}^{(4)} & \mathbf{A}_{4,2}^{(4)} & \mathbf{A}_{4,3}^{(4)} & \mathbf{A}_{4,4}^{(4)} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{4,s}^{(4)} \\
\mathbf{A}_{s,1}^{(1)} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{s,s}^{(1)} & \mathbf{0} & \mathbf{0} \\
\mathbf{A}_{s,1}^{(3)} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{s,s}^{(3)} & \mathbf{0} \\
\mathbf{A}_{s,1}^{(4)} & \mathbf{0} & \mathbf{A}_{s,3}^{(1)} & \mathbf{A}_{3,s}^{(3)} & \mathbf{0} & & \mathbf{A}_{s,s}^{(4)}
\end{bmatrix}$$

$$\mathbf{q} = \begin{bmatrix} \tilde{\mathbf{q}}^{(1)} \\ \tilde{\mathbf{q}}^{(2)} \\ \tilde{\mathbf{q}}^{(3)} \\ \tilde{\mathbf{q}}^{(4)} \\ \tilde{\mathbf{q}}_s^{(1)} \\ \tilde{\mathbf{q}}_s^{(3)} \\ \tilde{\mathbf{q}}_s^{(4)} \end{bmatrix}, \mathbf{f} = \begin{bmatrix} \mathbf{f}_1^{(1)} + \mathbf{f}_1^{(2)} + \mathbf{f}_1^{(3)} + \mathbf{f}_1^{(4)} \\ \mathbf{f}_2^{(2)} + \mathbf{f}_2^{(3)} + \mathbf{f}_2^{(4)} \\ \mathbf{f}_3^{(3)} + \mathbf{f}_3^{(4)} \\ \mathbf{f}_4^{(4)} \\ \mathbf{f}_s^{(1)} \\ \mathbf{f}_s^{(3)} \\ \mathbf{f}_s^{(4)} \end{bmatrix}.$$

The number of degrees of freedom of the tractor with a trailer is:

$$n = \tilde{n}_b^{(1)} + \tilde{n}_s^{(1)} + \tilde{n}_b^{(2)} + \tilde{n}_b^{(3)} + \tilde{n}_s^{(3)} + \tilde{n}_b^{(4)} + \tilde{n}_s^{(4)} = 6 + 4 + 3 + 1 + 2 + 1 + 2 = 19 \tag{3}$$

The above is just an example of generating the equations of motion of an articulated vehicle. Using this procedure the model can be easily extended with more bodies such as other trailers. In the formulae presented the general notation is assumed in which sign ~ above a coordinate means that the coordinate is defined in the local coordinate system while a coordinate without this sign is defined in global coordinate system.

3. Tire model

In order to consider forces acting between the tire and the road we use the Dugoff-Uffelman tire model. It is simpler and requires smaller number of coefficients than the most popular model called 'Pacejka magic formula'. The forces and moments acting at wheel k of unit p shown in Fig.3 can be calculated as functions of normal force $F_{z,k}^{(p)}$ according to the formulae:

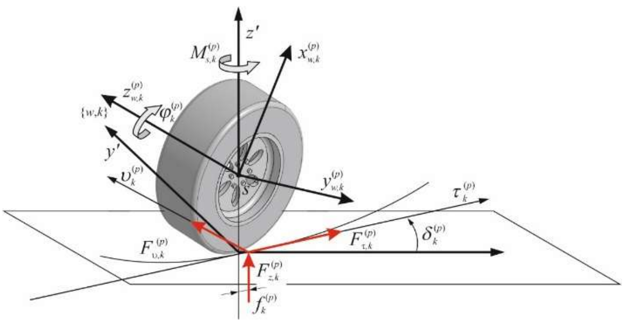

Fig. 3 Forces acting on wheel j of unit p.

$$F_{\tau,k}^{(p)} = \chi_{x,j,k}^{(p)} F_{z,k}^{(p)} \tag{4.1}$$

$$F_{\upsilon,k}^{(p)} = \chi_{y,j,k}^{(p)} F_{z,k}^{(p)} \tag{4.2}$$

$$M_{s,k}^{(p)} = M_{s,k}^{(p)}\left(F_{z,k}^{(p)}\right) \tag{4.3}$$

where $k = 1, \dots, n_s^{(p)}$is the number of a wheel in unit$p$ $\chi_{x,k}^{(p)}, \chi_{y,k}^{(p)}$ are coefficients.

Coefficients $\chi_{x,k}^{(p)}, \chi_{y,k}^{(p)}$ and function $M_{s,k}^{(p)}$ depend on tire and road parameters (stiffness, material characteristics, geometry) and vehicle motion. One of those parameters is basic lateral stiffness coefficient $L_{0,k}^{(p)}$, which will be used in the optimization problem described in the next section. Formulae for calculation of coefficients $\chi_{x,k}^{(p)}, \chi_{y,k}^{(p)}$ and moment $M_{s,k}^{(p)}$ are presented in [11].

4. Optimisation problem

Validation is an important stage of development of a model. In order to validate the model presented in section 2 experimental measurements have been carried out for a truck with a trailer shown in Fig.4.

Fig. 4 *Articulated vehicle used in experiments.*

Measurements were taken during the motion of the vehicle with constant speed v =60 km/h which was performed for a sharp turn of the steering wheel. Both the steering angle of the steering wheel and yawing velocity of the tractor (value $\dot{\psi}^{(1)}$) and the trailer ($\dot{\psi}^{(1)} + \dot{\psi}^{(2)} + \dot{\psi}^{(4)}$) were measured. When parameters for the tire were assumed equal to those given in literature the results of numerical simulations for yawing velocities differed from experimental measurements. Thus a dynamic optimization problem has been formulated as minimization of the functional:

$$\Omega = \Omega(p_1, \dots, p_s) = c_1 \left[\frac{1}{T} \int_0^T \left[\dot{\psi}_e^{(1)} - \dot{\psi}_c^{(1)} \right]^2 dt \right]^{\frac{1}{2}} + c_2 \left[\frac{1}{T} \int_0^T \left[\dot{\psi}_e^{(m)} - \dot{\psi}_c^{(m)} \right]^2 dt \right]^{\frac{1}{2}} \quad (5)$$

where: c_1, c_2 are assumed constants, T is the simulation time, $\dot{\psi}_e^{(1)}, \dot{\psi}_e^{(m)}$ are measured yawing velocity of the tractor and trailer respectively and $\dot{\psi}_c^{(1)}, \dot{\psi}_c^{(m)}$ are calculated yawing velocity of the tractor and the trailer respectively. The parameters $p_1, \dots, p_4$ of the minimized functional represent values of basic lateral stiffness coefficient $L_{0,k}^{(p)}$ for all the tires. It is assumed that these stiffness coefficients are the same for the right and left wheels and thus the following is assumed:

$$\begin{aligned} &p_1 = L_{0,1}^{(1)} = L_{0,2}^{(1)} \text{ for front wheels of the tractor,} \\ &p_2 = L_{0,3}^{(1)} = L_{0,4}^{(1)} \text{ for rear wheels of the tractor,} \\ &p_3 = L_{0,1}^{(3)} = L_{0,2}^{(3)} \text{ for the wheels of the dolly,} \\ &p_4 = L_{0,1}^{(4)} = L_{0,2}^{(4)} \text{ for the trailer wheels.} \end{aligned} \quad (6)$$

It has to be noted that when solving this task in order to calculate the value of Ω for a given combination of parameters $p_1, \dots, p_4$, it is necessary to integrate the equations of motion (3), which now depend not only on the generalized coordinates but also on parameters $p_1, \dots, p_4$ over the interval $< 0, T >$; this means that the equations of motion (3) have to be integrated at each optimization step. The boundary conditions for the task are formulated in the form:

$$p_{i,min} \le p_i \le p_{i,max} \qquad for\ i = 1, \dots, 4 \quad (7)$$

where $p_{i,\min}, p_{i,\max}$ are minimum and maximum admissible values of parameter p_i.

The nonlinear optimization task (5), (7) was solved using the downhill simplex method. Table 1 presents the values of $L_{0,1}^{(p)} = L_{0,2}^{(p)}$ obtained as a solution of this task.

Table 1: *Initial (before optimization) and calculated (after optimization) values of parameters $p_1, \dots, p_4$..*

	Before optimization	After optimization
p_1	15	20.96
p_2	15	16.29
p_3	13	14.39
p_4	13	14.97

These values are obtained when it is assumed that there is viscous friction in the connection between the dolly and the trailer and the friction coefficient equals 0.04. Figures 5 and 6 present yawing velocity of the tractor and trailer respectively.

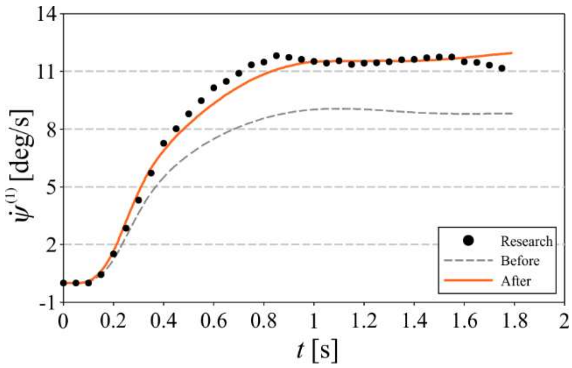

Fig. 5 *Yawing velocity of the tractor*

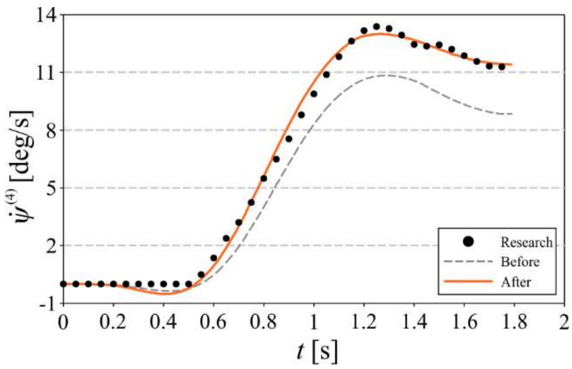

Fig. 6 *Yawing velocity of the trailer*

It can be seen that the courses of yawing velocities obtained for values of the lateral stiffness coefficient taken from literature (broken line) differ from those from the experiment although the form of the course is similar. When tire parameters are calculated as a result of optimization procedure the results of experimental measurements and numerical simulations are compatible.

5. Final remarks

This paper presents a procedure which enables us to improve the accuracy of the articulated vehicle model elaborated. The tire parameters obtained on the test stands should be adjusted by comparing of the results of the real tests of truck behavior with the results of the calculations. The application of dynamic optimization enables the modified tire parameters to be quickly determined. The method of modelling by means of joint coordinates and homogenous transformations seems very advantageous for mathematical modeling of a truck combination.

Finally, it can be concluded that calibration of a dynamic model of a truck combination ensures that we achieve good qualitative and quantitative compatibility between the results of the real test and results of calculations.

References

1. Bouteldja M., V.Cerezo. Jackknifing warning for articulated vehicles based on a detection and prediction system, in Proceedings of the 3rd International Conference on Road Safety and Simulation, Indianapolis, 2011.
2. Fencher P.S., Winkler C., Ervin R., Zhang H., Using braking to control the lateral motion of full trailers Supplement to Vehicle System Dynamics 29, 462-468, 1998
3. Hussain K., Stein W., Day A. J. Modelling commercial vehicle handling and rolling stability Proc. IMechE Part K: J. Multi-body Dynamics 219, pp. 357-369, 2005.
4. Kaneko T., Kageyama I., A study on the braking stability of articulated heavy vehicles, JSAE Review 24, 157-164, 2003.
5. Bolzern P., DeSantis R.M., Locatelli A. An input-output linearization approach to the control of an n-body articulated vehicle. Journal of Dynamic Systems, Measurement, and Control. 123, 309-316, 2001.
6. Yang X., Song J., Gao J., Fuzzy Logic Based Control of the Lateral Stability of Tractor Semitrailer, Mathematical Problems in Engineering Volume 2015, Article ID 692912
7. Adamiec-Wójcik I., Awrejcewicz J., Grzegożek W., Wojciech S., Dynamics of articulated vehicles by means of multibody methods, in J. Awrejcewicz, M. Kaźmierczak, J. Mrozowski, P. Olejnik editors: Dynamical systems: mathematical and numerical approaches, Łódź, 2015.
8. Craig J.J. Introduction to robotics, Massachusetts, Addison-Wesley 1988.
9. Pacejka H.B., Bakker E.,The Magic Formula Tyre Model. Suppl. to Vehicle System Dynamics, vol.21, pp.1-18, 1993.
10. Dugoff H., Fancher P.S., Segel L., An Analysis of Tire Traction Properties and Their Influence on Vehicle Dynamics Performance. SAE Technical Paper 700377, 1970.
11. Grzegożek W., Adamiec-Wójcik I., Wojciech S., Komputerowe modelowanie dynamiki pojazdów samochodowych (Computer modelling of vehicle dynamics) Kraków: Cracow University of Technology Press, 2003.