MODELLING AND SIMULATION
OF HIGH-FREQUENCY AUTONOMOUS PUBLIC-TRANSPORT SERVICE

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Abstract: *In this paper, a simulation model of the novel concept of autonomous vehicles is developed for public transport (PT) service. The model aims to serve PT planners as a tool enabling the simulation of a metro-like, high-frequency service with punctual autonomous vehicles running on exclusive lanes. The model was applied in a numerical example of a passenger bus, with different scenarios employed. The outcome of the simulation shows that characteristics of the modelled system are reproduced, with the following results: (i) high-frequency, metro-like PT service, with reduced passenger waiting time, (ii) reduced vehicle capacity with reduced average dwell time; (iii) the least number of unserved passengers, across all scenarios, and (iv) better utilisation of vehicle capacity (with fewer unserved passengers) achieved through a small decrease in vehicle frequency, which led to a negligible increase of the average passenger waiting times. In addition, the model allows us to examine the system's behaviour under uncertainty, considering passenger arrivals and vehicle headways as random input variables with predefined probability distributions. The proposed model enables modellers not only to evaluate a system's performance, but also to examine different working conditions and possible future scenarios.*

Keywords: PUBLIC TRANSPORT, PUBLIC TRANSPORT LINE, SIMULATION SYSTEM, GPSS, MODEL, SCENARIO, VEHICLE CAPACITY, VEHICLE FREQUENCY, AVERAGE WAITING TIME

1. Introduction

One of the most significant factors influencing people's lives nowadays is time. When doing their daily activities, such as going to work, school, or shopping, people aim to save time, especially when travelling in private cars or on PT. Therefore, in order to make PT more competitive, public operators and authorities should fulfil transport demand [1,2,3] by providing passengers with adequate transport supply [4] and reliable and regular transport service. This can be done either by running large-capacity vehicles along a PT route or small-size vehicles with increased frequency (reduced headways), which reduces the uncertainty and waiting times, thus resulting in PT service with greater punctuality.

A small number of large-size vehicles running with lower frequency is less expensive for the carrier in terms of operating costs but leads to increased waiting times for passengers at PT stops (stations). On the other hand, more frequent public transport service decreases passenger waiting times at PT stations along a route, but requires more vehicles and drivers, thus leading to increased operating costs, including wages to drivers, as well as to increased vehicular traffic and CO_2 emissions.

In some towns around the world [5] PT is offered free to riders along some routes. Therefore, regardless of the kilometers travelled by the vehicles and without taking into account their operating cost and the cost of driver wages, the question that needs to be answered in this case would be: *Which PT control strategy would provide the adequate PT supply (with reduced waiting times at stops) to address the PT demand, by serving as many passengers as possible, regardless of the cost to the operator?* The main priority, therefore, is the level of service and reliability. One of the possible and modern ways to achieve this is to implement a PT service operating autonomous vehicles capable of providing riders with increased reliability and enough flexibility to accommodate the fluctuations of passenger demand along the different routes and throughout the different hours of the day.

1.1. Objective and tasks

The main objective of this study is to improve the reliability of PT service by reducing the average waiting time experienced by passengers. As part of the study, the following tasks were employed: (i) conduct a review of articles discussing simulation systems and tools that could be used to build a model for the problem examined; (ii) develop a simulation model enabling the modelling of passengers' transit process; (iii) apply the model calibrated on a numerical example using different simulation scenarios; (iv) analyse and discuss the simulation output results; and (v) draw reasonable conclusions and outline further research work.

1.2. Scope of work

The scope of work is to accurately model a single line of a PT service and precisely simulate the passengers' transit process. This was attained by examining a variety of simulation modelling systems, simulation programming languages, and simulation tools.

1.3. Significance of work

This work resulted in a flexible simulation model developed that uses the novel concept of autonomous vehicles. This concept was applied to the PT field through a numerical example of an autonomous-bus service. Through the application of the proposed simulation model, which utilises autonomous vehicles that run on isolated (exclusive) lanes, our paper answered the question "*Which operational strategy has the potential to improve the reliability of PT service?*" To do so, we performed simulations under different scenarios and examined the combined impact of parameters such as (i) rate of passenger arrivals at bus stops; (ii) vehicle headway (considered to be a deterministic or random variable); and (iii) vehicle capacity on a) the average waiting time (the most significant measure of transit service reliability) and b) the unserved demand (expressed in terms of the number of passengers who were not transported to their destination point by the end of the simulation). The achieved outcome of the presented example is metro-like, high-frequency (short headways), punctual autonomous buses running on isolated lanes (no intersections, no traffic lights, and no congestion) and arriving at bus stops on time.

2. Simulation Modelling Framework

2.1. Choice of a simulation tool

Of the rich variety of the existing simulation systems, programming languages, products, packages and tools reviewed in [6] and [7] – general-purpose and specialised – the modern and powerful system for discrete-event and continuous simulation GPSS (General Purpose Simulation System) "World" version for *Windows* (also known as GPSS W [8]) was chosen and used as a modelling tool to create a model of a PT line route. An alternative of the GPSS/H [9] version, GPSS W, falls into the group of simulation systems for modelling discrete systems. In GPSS the simulation model is built by means of consecutively linked (connected) standard blocks in a block diagram (flowchart) showing the logical structure of the modelled system. The dynamic components in the simulation model are represented by transactions. According to the algorithm's logic, these transactions flow in a sequence (sometimes simultaneously) from one block into another. Each block in the

model implements specific functionality such as generating passenger arrivals, simulating queues, servicing facilities (servers), processing times, etc. In addition to the existing functionality, GPSS allows a user, through the built-in programming language (PLUS – Programming Language Under Simulation), to create and execute his/her own (customised) procedures.

In addition to the above, it is worth mentioning that the GPSS World simulation system can be distinguished due to the following advantages:

• the student version of GPSS World is free to use;

• GPSS W has no high system requirements in terms of hardware. The installation file is small in size, which makes it easy and quick to install on the Windows operating system;

• the Graphical User Interface of GPSS is user-friendly;

• GPSS W is accompanied by two manuals, the Reference manual [10] and Tutorial manual [11], which provide readers with solid theoretical knowledge and contain plenty of practically-oriented sample simulation models, which accelerates the learning process and makes the work with the system more effective;

• GPSS W provides users with the possibility to write, save, and keep the program code in files that are easy to read with simple text editors, such as "Notepad";

• the program code of the model compiles very quickly – it takes only a couple of seconds to compile and execute even complex simulation models;

• as already mentioned, GPSS W is equipped with a built-in programming language (PLUS) allowing users to use functions and probability distributions as well as enable them to create their own customised procedures, thus extending the capability of the systems and making the simulation model that is developed more powerful;

• GPSS W provides graphical tools and reports enabling users to represent the simulation output results and further analyse them;

• GPSS W also enables users to:

➢ generate arbitrary sequence of random numbers by means of the built-in random-number generators;

➢ perform repetitive realisations (implementations) with the model;

➢ check (debug) the program source code for errors;

➢ observe the flow of the dynamic components (transactions) from one block to another at run-time (during the simulation) in accordance with program's logic as well as to perform stepwise simulation in a sequence of steps;

➢ automatically build a journal (log file) showing the history of the events;

➢ produce detailed reports containing the simulation output results including values of the system's parameters evaluated during the simulation as well as descriptive statistics and representation of the simulation results in tables;

➢ build histograms representing the probability distributions of the random variables modelled.

2.2. Simulation model description

The simulation model developed is composed of the following four segments:

• the "Declaration segment" is used to declare all the constants,

variables, saved (accumulated) values, functions, procedures, queuing systems' servers capacities, matrices containing in-vehicle travel times, and number of passengers who board/alight buses going through each bus stop;

• the "Passenger arrivals" segment models passenger arrival at each bus stop. This is the segment that models and implements the main logic of the passenger transportation process.

Each of the bus stops along the route has been modelled from two different perspectives: that of passengers and that of vehicles:

➢ from the passengers' point of view, the bus stops have been modelled with a queue characterised by an incoming flow of passengers whose inter-arrival times follow a specific probability distribution (Uniform, but Poisson arrivals can also be modelled), which remains unchanged during peak hour. The average time in the queue represents the average time a passenger spends waiting at the bus stop for a vehicle to arrive;

➢ from the point of view of the vehicles, since in theory more than one vehicle could arrive at the bus stop at a time, bus stops were modelled as multi-server devices capable of "accommodating" a predefined number of vehicles. Since vehicles are not "allowed" to stay outside the bus stop, the "bus bunching" phenomenon and waiting delays in waiting for other buses to leave the stop can be avoided;

• the "Simulation clock" segment controls the duration of the simulation and number of the realisations of the model;

• the "Procedures" segment is where two procedures are called during the simulation when a bus arrives at a bus stop with the purpose:

➢ of checking for the maximum possible number of passengers (if there are any at all) who can board the bus depending on the vehicle's capacity, thus making sure that the bus capacity is utilised as much as possible and ensure that the number of passengers who board the bus does not exceed the available vehicle's capacity (depending on how many passengers are already on the bus);

➢ of checking for the number of passengers getting off the bus at each bus stop, thus making sure that the number of passengers who get off the vehicle does not exceed the number of passengers in the vehicle.

Each vehicle that has a unique number has been modelled as a multi-server queuing system for which the server's capacity represents the vehicle's passenger capacity, and the servicing time is the in-vehicle travel time. The interesting factor is that in this case the server (i.e., the bus) is a dynamic element that travels through the route while servicing (transporting) the passengers.

An illustrative scheme of the system modelled is shown in Fig. 1 and a flowchart of the algorithm of the simulation model developed is shown in Fig. 2.
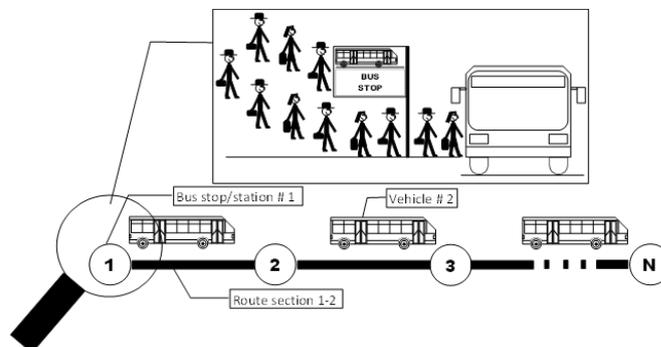

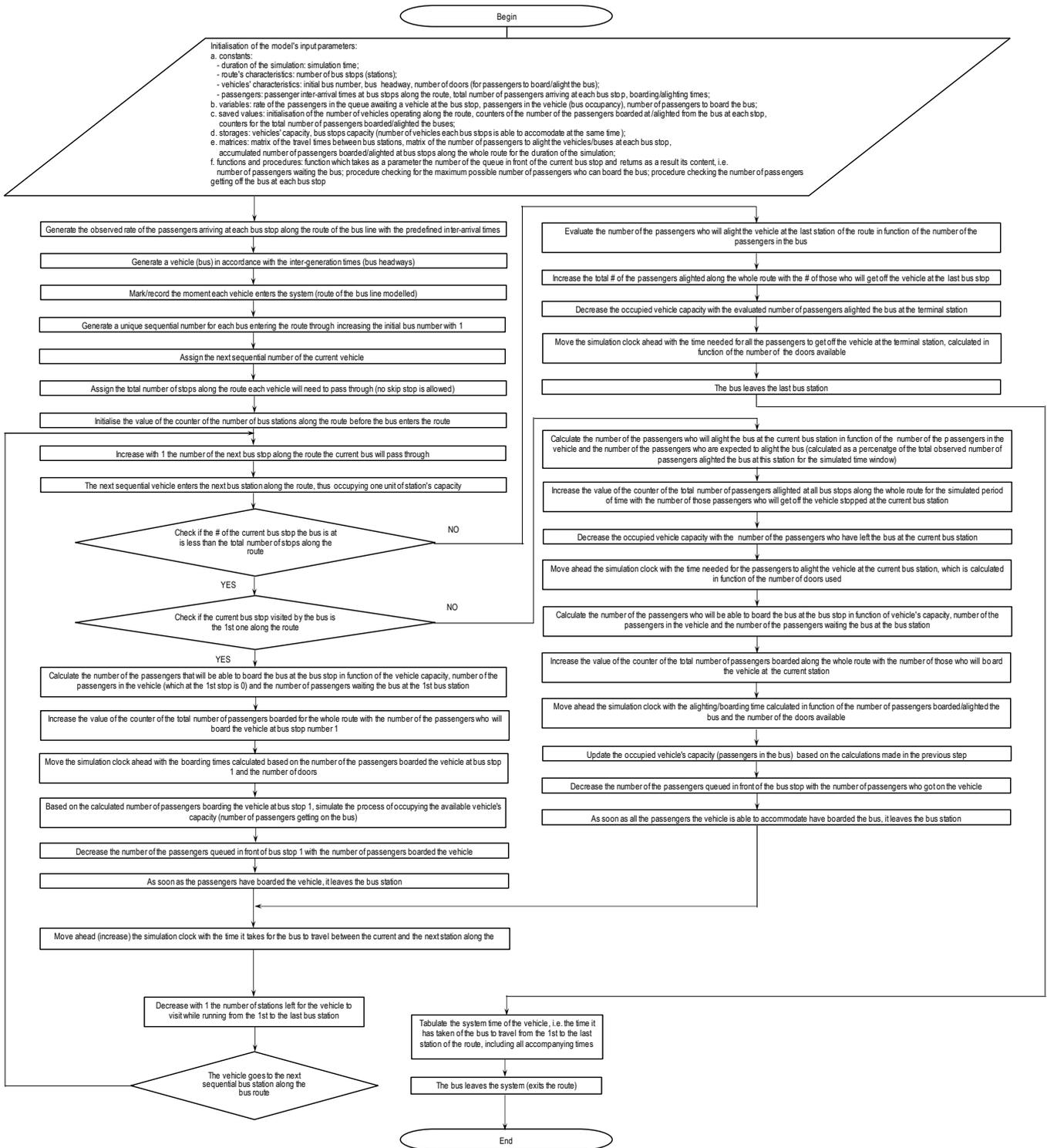*Fig. 1. Illustrative scheme of the simulation model*

Begin

Initialisation of the model's input parameters:
a. constants:
  - duration of the simulation: simulation time;
  - route's characteristics: number of bus stops (stations);
  - vehicles' characteristics: initial bus number, bus headway, number of doors (for passengers to board/alight the bus);
  - passengers: passenger inter-arrival times at bus stops along the route, total number of passengers arriving at each bus stop, boarding/alighting times;
b. variables: rate of the passengers in the queue awaiting a vehicle at the bus stop, passengers in the vehicle (bus occupancy), number of passengers to board the bus;
c. saved values: initialisation of the number of vehicles operating along the route, counters of the number of the passengers boarded at/alighted from the bus at each bus stop,
  counters for the total number of passengers boarded/alighted the buses;
d. storages: vehicles' capacity, bus stops capacity (number of vehicles each bus stops is able to accommodate at the same time);
e. matrices: matrix of the travel times between bus stations, matrix of the number of passengers to alight the vehicles/buses at each bus stop,
  accumulated number of passengers boarded/alighted at bus stops along the whole route for the duration of the simulation;
f. functions and procedures: function which takes as a parameter the number of the queue in front of the current bus stop and returns as a result its content, i.e.
  number of passengers waiting the bus; procedure checking for the maximum possible number of passengers who can board the bus; procedure checking the number of passengers
  getting off the bus at each bus stop

Generate the observed rate of the passengers arriving at each bus stop along the route of the bus line with the predefined inter-arrival times

Generate a vehicle (bus) in accordance with the inter-generation times (bus headways)

Mark/record the moment each vehicle enters the system (route of the bus line modelled)

Generate a unique sequential number for each bus entering the route through increasing the initial bus number with 1

Assign the next sequential number of the current vehicle

Assign the total number of stops along the route each vehicle will need to pass through (no skip stop is allowed)

Initialise the value of the counter of the number of bus stations along the route before the bus enters the route

Increase with 1 the number of the next bus stop along the route the current bus will pass through

The next sequential vehicle enters the next bus station along the route, thus occupying one unit of station's capacity

Check if the # of the current bus stop the bus is at is less than the total number of stops along the route
NO

YES

Check if the current bus stop visited by the bus is the 1st one along the route
NO

YES

Calculate the number of the passengers that will be able to board the bus at the bus stop in function of the vehicle capacity, number of the passengers in the vehicle (which at the 1st stop is 0) and the number of passengers waiting the bus at the 1st bus station

Increase the value of the counter of the total number of passengers boarded for the whole route with the number of the passengers who will board the vehicle at bus stop number 1

Move the simulation clock ahead with the boarding times calculated based on the number of the passengers boarded the vehicle at bus stop 1 and the number of doors

Based on the calculated number of passengers boarding the vehicle at bus stop 1, simulate the process of occupying the available vehicle's capacity (number of passengers getting on the bus)

Decrease the number of the passengers queued in front of bus stop 1 with the number of passengers boarded the vehicle

As soon as the passengers have boarded the vehicle, it leaves the bus station

Evaluate the number of the passengers who will alight the vehicle at the last station of the route in function of the number of the passengers in the bus

Increase the total # of the passengers alighted along the whole route with the # of those who will get off the vehicle at the last bus stop

Decrease the occupied vehicle capacity with the evaluated number of passengers alighted the bus at the terminal station

Move the simulation clock ahead with the time needed for all the passengers to get off the vehicle at the terminal station, calculated in function of the number of the doors available

The bus leaves the last bus station

Calculate the number of the passengers who will alight the bus at the current bus station in function of the number of the passengers in the vehicle and the number of the passengers who are expected to alight the bus (calculated as a percentage of the total observed number of passengers alighted the bus at this station for the simulated time window)

Increase the value of the counter of the total number of passengers alighted at all bus stops along the whole route for the simulated period of time with the number of those passengers who will get off the vehicle stopped at the current bus station

Decrease the occupied vehicle capacity with the number of the passengers who have left the bus at the current bus station

Move ahead the simulation clock with the time needed for the passengers to alight the vehicle at the current bus station, which is calculated in function of the number of doors used

Calculate the number of the passengers who will be able to board the bus at the bus stop in function of vehicle's capacity, number of the passengers in the vehicle and the number of the passengers waiting the bus at the bus station

Increase the value of the counter of the total number of passengers boarded along the whole route with the number of those who will board the vehicle at the current station

Move ahead the simulation clock with the alighting/boarding time calculated in function of the number of passengers boarded/alighted the bus and the number of the doors available

Update the occupied vehicle's capacity (passengers in the bus) based on the calculations made in the previous step

Decrease the number of the passengers queued in front of the bus stop with the number of passengers who got on the vehicle

As soon as all the passengers the vehicle is able to accommodate have boarded the bus, it leaves the bus station

Move ahead (increase) the simulation clock with the time it takes for the bus to travel between the current and the next station along the

Decrease with 1 the number of stations left for the vehicle to visit while running from the 1st to the last bus station

The vehicle goes to the next sequential bus station along the bus route

Tabulate the system time of the vehicle, i.e. the time it has taken of the bus to travel from the 1st to the last station of the route, including all accompanying times

The bus leaves the system (exits the route)

End

Fig. 2. *Flowchart of the modelling algorithm of the simulation model*

3. Numerical Example

The simulation model was applied on a numerical example considering a PT (bus) line passing through six bus stops in direction from bus stop 1 to stop 6. The input data used to feed the model was:

- duration of the simulation, seconds;
- number of bus stops;
- in-vehicle travel times between stops calculated for predefined distances (sections lengths) and average travel speed, km/h;
- number of passengers boarding/alighting the vehicle at each stop during the simulated period of time (peak-hour);
- boarding and alighting times per passenger, seconds;
- bus headway, seconds;
- bus capacity, passengers;
- bus stations capacity expressed with the number of buses each stop can accommodate.

The output data comprises:

Stops:

- maximum and average number of passengers who waited for a vehicle at each stop as well as the number of passengers at each bus stop at the end of the simulation;
- total number of passengers who arrived at each bus stop during the simulated window of time according to the demand;
- average passenger waiting time at each bus stop;
- average time spent boarding/alighting the vehicle at each stop along the route.

Vehicles:

- maximum and average number of passengers in each vehicle (vehicle's load) during the simulation as well as utilisation of vehicle capacity;
- rate of unused passenger capacity at the end of the simulation;
- vehicle capacity that is in use at the end of simulation;
- total number of passengers who boarded each vehicle during the simulation;
- total number of passengers who boarded all the vehicles during the simulation (which does not necessarily equal the total number of passengers who arrived at the bus stops);
- total number of passengers who alighted all the vehicles during the simulation;
- total number of passenger who have not been served (transported) by the end of the simulation;
- total in-vehicle travel time along the whole route from bus stop 1 to bus stop 6;
- total dwell time accumulated along the whole route.

3.1. Scenarios

Four main scenarios have been created and implemented with the model to simulate the headway-based PT service provided to passengers:

Scenario 1: This scenario considers the passengers' transportation process along the examined bus route serviced by six buses, each with a capacity of 200 passengers, running with a 10-minute headway (frequency: 6 vehicles per hour).

Scenario 2: This scenario considers the passengers' transportation process along the examined bus route serviced by ten buses, each with a capacity of 120 passengers, running with a 6-minute headway (frequency: 10 vehicles per hour).

Scenario 3: This scenario considers the passengers' transportation process along the examined bus route serviced by twenty buses, each with a capacity of 60 passengers, running with a 3-minute headway (frequency: 20 vehicles per hour).

Scenario 4: This scenario considers the passengers' transportation process along the examined bus route serviced by thirty buses, each with a capacity of 40 passengers, running with a 2-minute headway (frequency: 30 vehicles per hour), thus operating

in a metro-like way corresponding to the "*think tram – drive bus*" concept [12].

3.2. Input data and assumptions

3.2.1. Assumptions regarding passengers' demand

Passenger arrivals

Uniformly distributed passenger inter-arrivals times at bus stops were modelled. In order to introduce uncertainty, the model proposed allows simulating passenger arrivals according to a predefined probability distribution such as Poisson distribution [13].

Passenger demand

The number of passengers boarding/alighting at each bus stop along the bus line route over the simulated period of time is shown in Table 1.

Table 1. *Passenger demand at bus stops (boarding/alighting passengers)*

Bus stops	Passengers boarding	Passengers alighting	Sections, i-j	Distance, km	In-vehicle travel time, seconds
1	500	0	-	-	-
2	400	100	1-2	2.0	180
3	300	200	2-3	2.0	180
4	200	300	3-4	2.0	180
5	100	400	4-5	2.0	180
6	0	500	5-6	2.0	180
Total:	1500	1500		10.00	900

Boarding/alighting times

Adebisi in [14] found that although alighting takes less time than boarding, the difference between the two times is less than half a second per passenger.

Assuming that parameters such as travel time, passenger demand, number of transferring passengers, and dwell time are deterministic, in compliance with Dueker et al. [15], Ang and McIvor in [16] set default values for the passenger boarding and alighting times to be equal to 3.5 and 2.2 seconds, respectively.

According to Cats et al. [17] dwell time at the stops along with departure time at origin stations and travel time between stops is one of the sources of uncertainty in public transit operations.

Liu and Ceder in [18] and [19] accepted values of 4.0 seconds and 2.0 seconds, respectively, for the marginal dwell times per each boarding and alighting passenger.

Nesheli and Ceder [20,21] applied models to case studies in which they set values of 2.5 and 1.5 seconds for the boarding and alighting of each passenger.

Tang et al. in [22] set values for boarding and alighting times per passenger to 2 and 1 seconds, respectively.

Taking into account the finding of Adebisi [14] and that the vehicles used in the model proposed in this paper have no steps, values of 2.0 seconds and 1.5 seconds, respectively, were assigned for boarding and alighting the bus.

Dwell time

In some research works dwell time is considered deterministic [18] or fixed [14], while in others it is considered a dependent variable in function of the number of boarding passengers and alighting passengers during different times of the day and along different route types [15].

Dueker et al. in [15] examined and analysed the determinants of dwell time – passenger activity, lift operations as well as low-floor bus, time of day, and route type. In order to evaluate dwell times under different operating conditions Dueker et al. used a model that enabled modeling dwell time at different times of the day, different route types, and various number of passengers boarding and alighting a vehicle.

As Liu and Ceder [18] solved the timetable-planning problem rather than an operations-related one, they considered the dwell time as deterministic, and not time-varying within the planning period.

Liu et al. in [23] concluded that "*bus dwell time contributes to travel time and headway variation*".

As defined in the Highway Capacity Manual cited by Dueker et al. in [15], dwell time is "*the time in seconds that a transit vehicle is stopped for the purpose of serving passengers. It includes the total passenger service time plus the time needed to open and close doors*", and therefore it is not constant. It depends on the number of passengers boarding/alighting and the time it takes for each passenger to get on/off the vehicle. Therefore, in contrast to some models in which the dwell time is constant, in the model proposed in this paper it is in function of the number of passengers wanting to board/alight the vehicle, i.e., the larger the number of the passengers alighting/boarding the vehicle, the larger the dwell time, which sometimes forces vehicles to stay at the bus stops longer than in the models with a deterministic dwell time, thereby making the model more realistic.

3.2.2. Operational characteristics

PT line and route characteristics

The PT route servicing the modelled bus line has total length of 10 km. There are six stops in total along the route – two terminal stations and four intermediate bus stops. The route sections connecting the stops along the route are each 2 km long.

Vehicles

Depending on the scenarios played (Table 2), the values for the frequency of the vehicles [1,2] and bus headway varies from 6, 10, 20, and 30 vehicles (buses) going one after another with headways of 10, 6, 3, and 2 minutes, respectively. For the purposes of the simulation, buses with total capacity of 200, 120, 60, and 40 passengers (including seats and standing room) were considered for scenarios 1-4. The buses running along the route while servicing the bus line, depending on the vehicle's capacity, are equipped with 4, 3, and 2 doors for scenarios 1, 2, and 3-4, respectively. The accepted discipline of servicing passengers is such that they can board the vehicle through all the doors available including the front, intermediate (if any), and rear doors.

Operating speed [3] and sections travel times

The in-vehicle travel times along the sections between the bus stops is calculated for a distance between the bus stops equal to 2.0 km and bus operating speed 40 km/h.

Based on the above, the main assumptions that have been made can be summarised as follows:

• vehicle headway is deterministic, i.e., it does not reflect any possible deviations due to traffic congestion (although the model enables modelling it as a random variable);

• passenger arrivals at bus stops are uniformly distributed throughout the modelled period of time;

• at bus stops passengers get on the first arriving vehicle. If the arriving vehicle is full, then passengers get on the next vehicle (which results in increased waiting time);

• a bus does not leave the bus station before all the passengers have boarded as long as the available vehicle capacity allows this to happen;

• since buses run along a lane considered isolated (separated) from the other vehicular traffic, there is no traffic congestion and traffic signals incurring delays;

• since buses go one after another according to the sequence by which they have entered the system (route), keeping a distance (one headway), one bus cannot bypass another.

A comparison of the observed (actual) and the simulated (reproduced) number of passengers who alighted the vehicles at each stop along the route of the examined bus line is shown in Fig. 3.

The simulated average waiting times were compared with those that were analytically calculated by the formula:

(1) $AWT = (AWT_{max} - AWT_{min})/2 = H/2$, seconds,

where:

H – vehicle (bus) headway, i.e., the time interval between two vehicles going along the route [1];

AWT_{min} – the minimum time (in seconds) passengers wait the vehicle at the bus stop, which is $AWT_{min} = 0$ seconds when a passenger comes at the time when bus arrives;

AWT_{max} – the maximum time (in seconds) passengers spend waiting when they arrive at the stop at the moment the bus departs the stop. In that case, passengers need to wait for the next bus, which arrives after time equal to H.
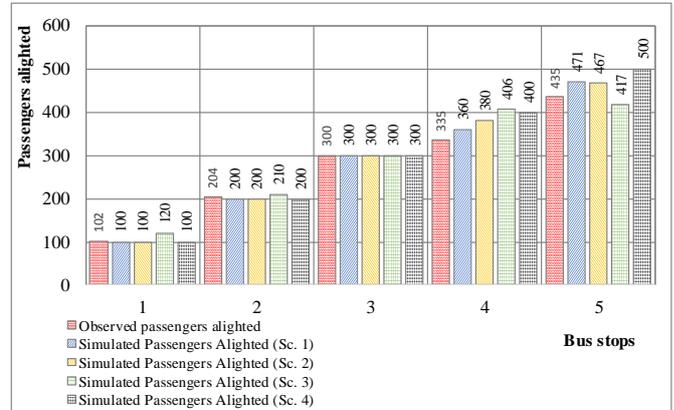

Fig. 3. Total observed vs. simulated number of passengers who alighted at each bus stop (in scenarios 1-4)

The simulated average waiting times at each bus stop, excluding stop 6, which is the last for the route, for each of the above four scenarios are shown in Fig. 4 below.
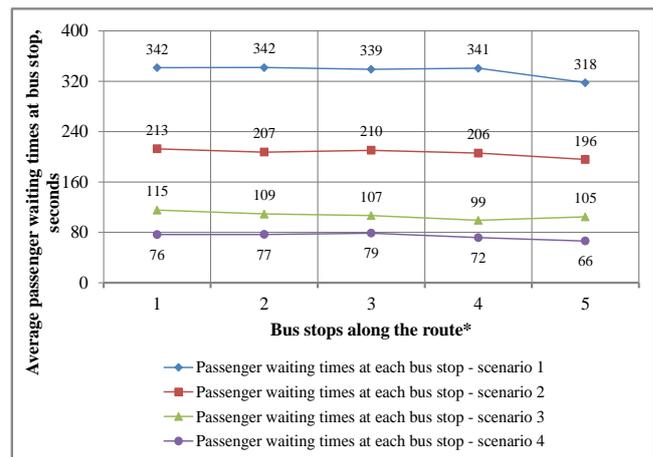

Fig. 4. Average passenger waiting times at the bus stops along the route
Bus stop number 6 is not displayed in the chart as it is the last stop for the route, and there are no passengers waiting there to board the vehicle.

Let us, for example, consider scenario 1 in which the bus headway is 600 seconds (10 minutes). Theoretically the minimum time each passenger will wait for a bus at the bus stop is $AWT_{min} = 0$ seconds and the maximum is $AWT_{max} = 600$ seconds; on average a passenger waits 300 seconds, i.e.:

$AWT = (600 - 0)/2 = 300$ seconds, which is almost equal to the average waiting time of 339 seconds evaluated as a result of the simulations performed as a weighted average of the passenger waiting times at the bus stops along the route. For scenarios 2-4, the analytical average waiting times have values:

$AWT = 180$ seconds for scenario 2, 90 seconds for scenario 3, and 60 seconds for scenario 4.

The above formula (1) does not take into account:

- any possible deviations due to bus delays or early arrivals;

- the time passengers spend due to a missed bus, i.e., if they have waited AWT_{max} for the bus, and after the bus has arrived if they did not board because the vehicle was full.

The average dwell time (ADT) shown in table 2 below accumulated along the whole route was calculated as a difference

between the average total travel time (ATTT) from the first to the last bus stop and the total in-vehicle travel time (TIVTT).

As can be seen in table 2, the comparison between the simulated weighted average waiting times and those times calculated via

formula (1) shows that the model satisfactorily reproduces the waiting times at all bus stops along the route.

Table 2. Scenarios played (Direction: From bus stop 1 to bus stop 6)

Scenario	Vehicle frequency, buses per hour	Bus headway, seconds	Vehicle capacity, number of passengers[1]	Number of doors	Average total travel time along the whole route (ATTT), seconds	Standard deviation of ATTT, seconds	Total In-vehicle travel time along the whole route (TIVTT), seconds	Estimated average dwell time (ADT) accumulated along the whole route, seconds	Simulated weighted average passengers waiting time at each bus stop[2], seconds	Analytically calculated average passengers waiting time at each bus stop, seconds	Number unserved passengers along the route[3]
1	6	600	200	4	1070	5.9	900	170	339	300	112
2	10	360	120	3	1035	4.9	900	135	209	180	63
3	20	180	60	2	1001	3.7	900	101	109	90	51
4	30	120	40	2	968	3.8	900	68	76	60	46

[1] *Includes the total passenger seats and the total number of standees*

[2] *Weighted average passenger waiting time calculated as a weighted sum of the simulated average waiting times at bus stops multiplied by the number of the passengers waiting these times*

[3] *Number of unserved passengers calculated as a difference between the total simulated passengers who boarded and alighted the bus*

Below are charts showing the relationship of the simulated average waiting time in function of vehicle frequency (Fig. 5) and the dwell time in function of bus capacity (Fig. 6) for scenarios 1-4.

Fig. 5. *Relationship between simulated average waiting time and vehicles' frequency*
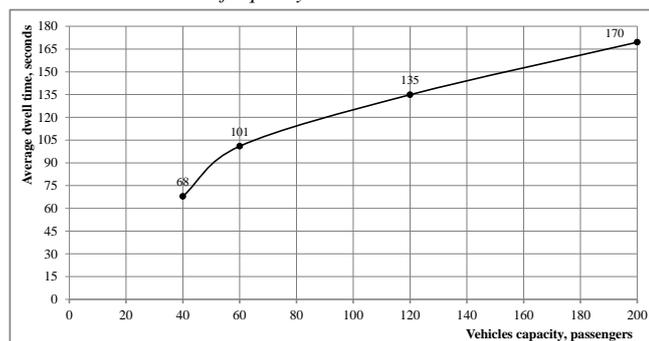
Fig. 6. *Relationship between dwell time and vehicles' passenger capacity*

Based on the results above, it can be concluded that variant 4 (with high-frequency, small-size vehicles) would not only provide PT users with decreased average waiting times due to the small bus headways, but also with reduced total travel times along the route due to the operation of small-capacity vehicles spending less dwell times at bus stops. Moreover, as can be seen in Fig. 7, in contrast to the three other scenarios, the highly frequent small-vehicle service (scenario 4) results in the lowest number of unserved passengers (those who have not alighted and are still traveling in the vehicles at the end of the simulation) (46 passengers), followed by scenario 3

with 51 passengers, and scenarios 2 and 1 with 63 and 112 passengers, respectively. In other words, compared to the scenario 1, in scenario 4 the number of the unserved passengers is more than 2 times less.

Fig. 7. *Number of the passengers who have not been served by the end of the simulation*

If we consider in detail the simulation output results for scenario 4, it can be seen in Fig. 8 that the PT operator satisfies PT demand entirely with a large number of small-size vehicles. The minimum and the maximum bus load along the sections are 24 and 36 passengers, which expressed as a percentage is 60% and 90% capacity utilisation, respectively. The average bus load is 29.3 passengers, which represents a capacity utilisation of 73.3%, indicating that around ¾ of the available passenger capacity is used. The model was used to evaluate and analyse the vehicles' capacity utilisation (Fig. 9) in a scenario with a reduced by 20% number of operating vehicles (from 30 to 24 buses) and reduced frequency in which the bus headway has been increased by 25% from 2 to 2.5 minutes. The purpose of this scenario was to answer the PT planners' question *What is the number of vehicles and the time interval between them that meet the existing PT demand, thus allowing PT planners to achieve improved bus utilisation and a small number of unserved passengers at the price of slightly increased average passenger waiting time?*
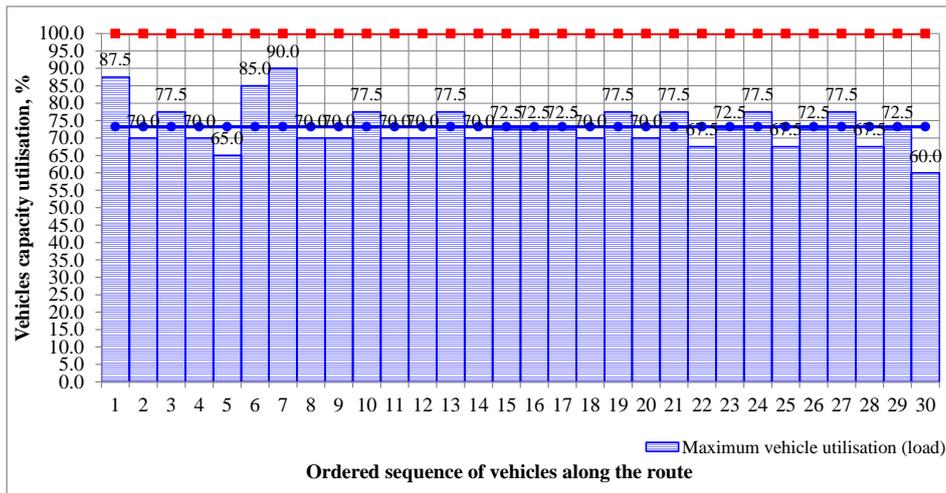
Fig. 8. *Maximum vehicle capacity utilisation reached for the vehicles along the bus route (scenario 4: 30 vehicles with capacity of 40 passengers going along the route with a 2.0-minute headway)*
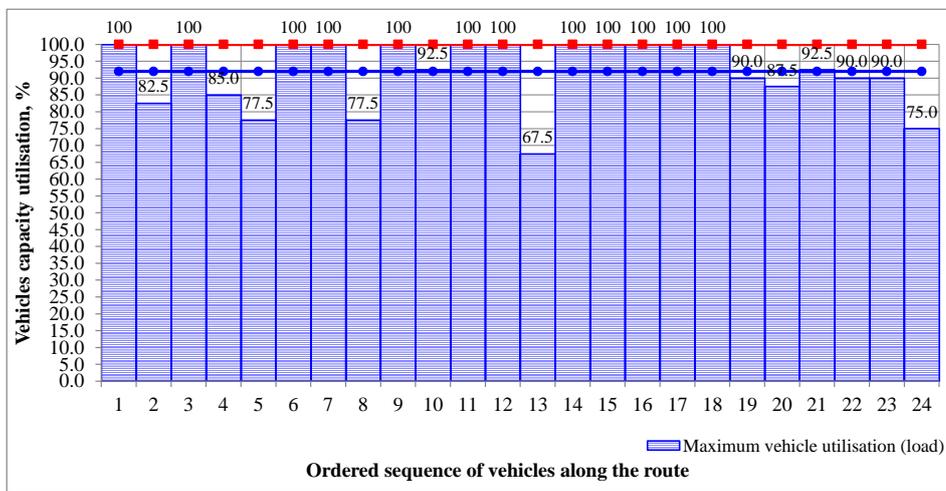

Fig. 9. *Maximum vehicle capacity utilisation reached for the vehicles along the bus route (scenario 4.1: 24 vehicles with capacity of 40 passengers going along the route with a 2.5-minute headway)*

The scenario with reduced number of vehicles (from 30 to 24, i.e., by 20%) and increased bus headway from 120 sec. to 150 sec. (25% increase) led to an increase in the average passenger capacity utilisation of the vehicles from 73.3% (29.3 passengers on average on the bus) to 92% (36.8 passengers on the bus), which is around a 20% increase at the cost of increased average passenger waiting time at the bus stops along the route from 65 sec. to 90 sec. (by 38.5%). The number of the unserved passengers has slightly decreased from 46 to 40 (13% decrease), which is insignificant and can be ignored. The average total travel time remains almost unchanged – it has been slightly increased from 968 to 985 seconds.

As can be seen in Fig. 10 and Fig. 11 representing the passenger load in vehicle # 7 (in scenario 4 with 30 buses running one after another with 2-minute headway) and vehicle # 3 (in scenario 4.1 with 24 buses running one after another with 2.5-minute headway) even for the most-loaded vehicle # 7 (Fig. 10) the maximum bus capacity utilisation has not been reached. The higher passenger load has been observed in 1 of the 5 route sections (# 3 between stops 3 and 4). In contrast, in the scenario with 6 fewer vehicles (24 rather than 30 buses) for vehicle 7, which is 1 out of 12 vehicles whose capacity has been reached (for half of the vehicles along the route, the maximum passenger capacity has been reached), is highly loaded in more than one section (section 3-4 between stations 3 and 4, followed by section 4-5 between bus stops 4 and 5). This shows how the proposed decreased number of buses running leads to better vehicle utilisation.
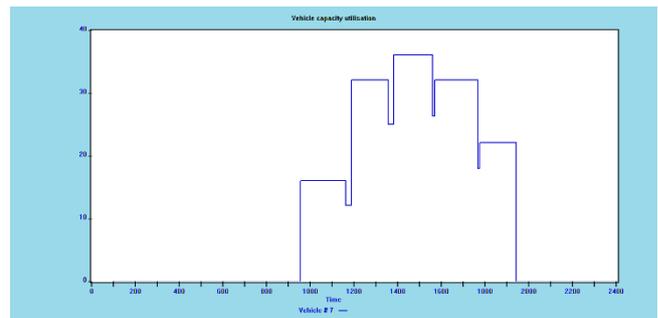

Fig. 10. *Passenger load by sections along the route for bus # 7 (scenario 4)*
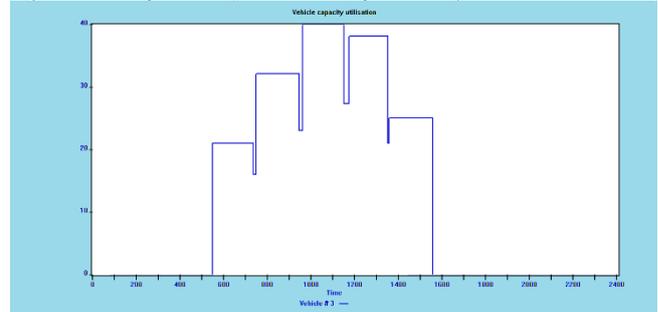

Fig. 11. *Passenger load by sections along the route for bus # 3 (scenario 4.1)*

The above example shows the power of the model that could be easily applied for planning the operations along a PT route. In the

example considered, the application of the model helped to better plan the PT service with reduced vehicle frequency, which resulted in better vehicle utilization and decreased number of the unserved passengers along the route.

4. Conclusion

Based on the analysis of the simulation output results, the following main conclusions can be drawn, and recommendations offered:

Although the choice of modelling framework depends on the purpose of the work, the characteristics of the system, and the existing constraints, models should:

- be easily implementable within at least some of the existing simulation systems, languages, and tools;

- be detailed enough to describe the system, reproduce the system's processes, and give accurate results;

- have a small number of input variables, which would make it possible to feed the model with data and directly apply it in practice;

- be easily maintainable, which would allow modellers to quickly adapt the model for different PT routes and transport conditions.

It turned out that among the large variety of existing simulation systems, programming languages, and tools, the GPSS World simulation system, used in this work, is a powerful tool enabling modellers to precisely design, easily build, and quickly implement a very flexible (in terms of the possibilities to be extended), and detailed enough model; after compiling for a couple of seconds, it provides modellers with output results and accumulated statistics in the form of generated reports that present the results in tables and graphs.

In light of the rich variety of simulation models and tools, this simulation model stands out because of:

- the small number of input variables, which makes the model's usage and subsequent application realistic;

- its flexibility, which enables, through minor modifications, relatively easily and quickly reworking the model and thus adapting it for modelling the passenger transportation process along a public-transport line characterised with specific parameters with regard to the route, the vehicles carrying passengers, and the variability of passenger demand;

- the ability of the model to enable evaluating the existing system's performance and efficiency along the route of a PT line under simulation, which would allow PT planners to effectively plan the PT service under different (existing and forecasted) conditions and scenarios.

The results from the simulation scenarios showed that the model satisfactorily reproduces the parameters of the system modelled, which allows us to summarise the simulation output results as follows:

- Among the scenarios, scenario 4 simulating a highly frequent, metro-like PT service, due to the small vehicle headways, provides passengers with a minimum average waiting time at bus stops;

- Due to the small capacity of the highly frequent vehicles used, which means less time for boarding/alighting (in contrast to the large-vehicle scenario), scenario 4 offers the lowest estimated average dwell time along the whole PT line;

- The highly frequent public-transit service provided in scenario 4 is the one that leaves the smallest number of unserved passengers along the route of the modelled PT line;

- A small decrease in the frequency of the vehicles (less operating vehicles) when a highly frequent, metro-like service is provided (scenario 4.1) leads to better utilisation of vehicle capacity (i.e. fewer unserved passengers) at the cost of an insignificant increase in the average passenger waiting time at bus stops.

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