COMPLEX SYSTEM, UTILITY AND DECISION CONTROL: A RISK PORTFOLIO OPTIMIZATION CASE

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Abstract: *The decision making is based on the objective preferences and starting from this position the incorporation of human preferences in complex systems is a contemporary trend in scientific investigations. In Complex system where a human participation is decisive for the final decision the human thinking, notions and preferences have cardinal significance and need analytical representation. Mathematical modeling of complex „human - process" systems and build mathematically well-founded control solution need analytical representation of quantitative information like preferences. This could be made by utility theory and stochastic approximation theory. The objective of the paper is to present a strict logical mathematical approach for modeling and estimation of human preferences as machine learning in the process of building of mathematical models of complex systems with human participation. The approach is demonstrated on a case study in the area of risk portfolios optimization and financial risk management with color noise. The objective of the paper is to present a mathematical approach for modeling and estimation of human preferences as machine learning in the process of building of mathematical models of complex systems with human participation.*

Keywords*: COMPLEX SYSTEM, VALUE BASED, UTILITY, DECISION, STOCHSTIC CONTROL, OPTIMAL PORTFOLIO*

1. Introduction

In the paper is demonstrated a system engineering value driven approach within determination of the investment policy in a continuous-time financial market modeled by a stochastic differential equation [10]. The problem consists in determination of investment policy, which represents the amount invested in a risky asset at any moment of a financial process. The investment policy solution presents an optimal portfolio allocation determined as optimal stochastic control at any moment. The inclusion of the decision maker (DM) expectation of the described financial process makes the investigated system a complex system**.** The construction of a mathematically grounded model of such a system is faced with the problem of shortage of mathematical precise information that presents the human activity. Often, in complex processes there is a lack of measurements or even clear scales for the basic heuristic information. At this level of investigations the decisions is close to the art to choose the right decision among great number of circumstances and often without associative examples of similar activity. The basic common source of information here are the human preferences. A solution of this problem is toseekinterpretationandexpressionofdifferentaspectsofthecomplexsystemthroughexpertanalysisanddescriptionoftheexpert's preferences as an element of the system. The paper presents an exploration of the decision-making process for complex situations in a complex environment in terms of laying the groundwork for decision making, understanding and exploring complex situations discussing human additive factors.

According to social-cognitive theories, people's strategies are guided both by internal expectations about their own capabilities of getting results, and by external feedback [7]. Probability theory, stochastic programming and stochastic optimization and utility theory address decision making under these conditions [4, 6]. The mathematical description or modeling on such a fundamental level requires basic mathematical terms and their gradual elaboration to more complex and specific terms like value and utility functions, operators on mathematically structured sets as well, and equivalency of these descriptions. Possible modeling is the so call value based modeling. Possible approach conception in this direction is the Utility theory. It is one of the most popular methodologies in Multi-Attribute Decision Making. The Utility theory permits development of complex models in which human participation is reflected analytically starting from marginal information as human preferences. In fact the Utility theory permits mathematical inclusion of the decision maker in the mathematical modeling and mathematical descriptions of complex processes with human participations [6, 10].This is a decision-driven strategy in modeling of complex processes where the decision aiding is considered in the context of personal decision analysis. The strategy realization requires a decision technology, which basis is the evaluation of human's objectively oriented preferences as utility function. The presentation of human preferences analytically with utility functions is a good possible approach for their mathematical description. It is the first step in realization of a human-centered value driven design process and decision making, whose objective is to avoid the contradictions in human decisions and to permit mathematical calculations in these fields. In this approach the human being has the role of decision-maker.

The process of modeling of a complex system by inclusion of the value or utility functions as objective functions bases on analytical description of the expert's preferences that allow mathematically the inclusion of the decision maker in the value based modeling of a complex system. If the subjective and probability uncertainty of DM preferences is interpreted as some stochastic noise, stochastic programming can be used for recurrent evaluation of the utility function, with noise (uncertainty) elimination. The utility evaluation is human-computer dialog between a decision-maker and computer-based evaluation tool. It concerns mathematically machine learning, since its basis is the axiomatic approach to decision making theory and stochastic approximation. Therefore knowledge is considered as capacity (potential and Actual) to take effective action.

2. Optimal Risk Portfolio Allocation

Dominant in scientific publications for portfolio management is that the objective function is out of touch with the concrete situation. The construction of an objective Decision maker's (DM's) function in accordance of his/her expectations is prevailing out of scientific discussion [12, 13]. But in the value based management of complex system the human participation is determinative and is necessary to be rendered in account as part of the mathematical model. This could be made by the use of decision making theory and his mathematical kernel -utility theory. In the paper the objective function is a utility DM's function which analytically represents DM's preferences. The polynomial approximation of a utility function, which bases on the evaluation of decision maker's preferences, is made by stochastic machine learning as pattern recognition of positive and negative answers over lotteries [10, 11]. In this way the portfolio management and control is in agreement with the DM's preferences.

We assume that the outcome set S (DM's amounts) is a two-attribute product set V×W, with generic element y= (v, w). The sets V and W are attribute sets where V designates the first attribute- π_t, (the amount $X_t\pi_t$, $\pi_t \in [0,1]$) invested in the risky process and W={X} designates the second attribute, the whole amount of money in BGN's (X). The aggregation of the two attributes in a multi-attribute utility function needs investigation of *Utility independence* in between the risky investment and the amount of money [6]. That is, DM's preferences on W do not depend on the particular deterministic level at which $\pi_t \in$V is fixed. A convenient implication of the described utility independence is that changing π does not affect rank-ordering in W (the set of amount X of money). A theorem of the multi-attribute utility theory determines the utility function of two attribute DM's U(v, w) within the form [6].

U(X, π) = f (π)p(X) + g(π) for some functions f, p, g with f positive function.

Let W be relevant over a range X^o to X^*, let V be relevant over a range v^o to v^* and assume that $U(X^*,\pi)>U(X^o,\pi)$ for all π and $U(X,\pi^*)>U(X,\pi^o,)$ for all X. We may rewrite this formula as follows [6]:

$$U(X,\pi) = U(X^o, \pi)+ [U(X^*,\pi)-U(X^o, \pi)].U(X,\pi^o).$$

The utility is decomposed to single attribute functions. Each one of these tree functions is evaluated as machine learning by stochastic approximation procedures. After 10^6 BGN's, is assumed that the process is sufficiently remote from the decision maker's expectations and as objective function is used a standard function namely the function (x^γ, γ = 0.321). The described objective Utility U(X,π) is shown on the Fig.1.

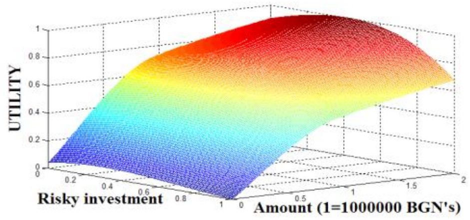


Fig. 1

The financial process is modeled with Black-Scholes stochastic differential equation. Let's take a non-risky asset S^0 and a risky one S. The Black-Scholes dynamic model represents stochastic differential equations described as:

$$dS_t^0 = S_t^0\, r dt \quad \text{and} \quad dS_t = S_t \mu dt + \sigma dW_t .$$

In the formula, μ and σ are constants (r=0.03, μ=0.05 and σ=0.3) and W is one dimensional Brownian motion. The investment policy is defined by a progressively adapted process $\pi=\{\pi_t,\ t\in[0,T]\}$ where π_t represents the amount of money ($X_t\pi_t$) ($\pi_t\in[0,1]$) invested in the risky process at moment t. The remaining amount (X_t- $\pi_t X_t$) is invested in the non-risky process at the same moment t. The time period T is 30 and 80 weeks. The dynamic of the liquidation value X_t of a self-financing strategy is given by the stochastic differential equation:

$$dX_t^\pi = \pi_t X_t^\pi \frac{dS_t}{S_t} + (X_t^\pi - \pi_t X_t^\pi)\frac{dS_t^0}{S_t^0} = (rX_t^\pi + (\mu - r)\pi_t X_t^\pi)dt + \sigma\pi_t X_t^\pi dW_t .$$

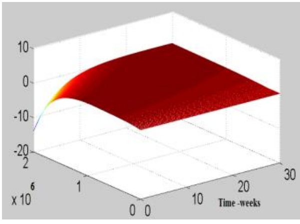
The objective of decision maker- is to choose the control (the partπ_t invested in the risky process) so as to maximize the expected DM's utility at moment T (final wealth):

$$V(T,X_T) = \sup_{\pi\in[0,1]} E[U(X_T,\pi_T)].$$

The symbol E denotes mathematical expectation. The optimal control is determined step by step from the Hamilton-Jacobi-Bellman (HJB) partial differential equation in agreement with the dynamical programming principle []:

$$\frac{\partial B}{\partial t}(t,X) + \sup_{\pi\in[0,1]}[(rX + (\mu - r)\pi X)\frac{\partial B}{\partial X}(t,X) + \frac{1}{2}\sigma^2\pi^2 x^2 \frac{\partial^2 B}{\partial^2 X}(t,X)] = 0.$$

The coefficients in the stochastic differential equation are continuous, the objective function U(.,.) is continuous and that the optimal control is continuous by parts. These conditions determine that there is a smooth by parts solution of the HJB partial differential equation [12. 13]. According to the evaluation procedures of Touzi, Gabassow and Kirilova [5, 8, 13] and using B-spline approximation for determination of the Hamilton-Jacobi-Bellman (HJB) function we found a polynomial approximations of B(t, X) and based on this approximation is determined the control manifold π(t,X) [9]. They are shown in Figures 2,3,4 and 5.



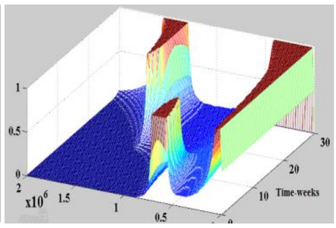
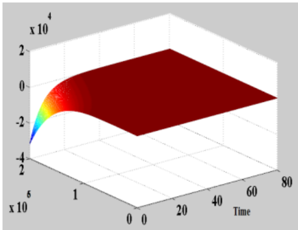


***Fig.2.** **HJB** function,(**T=30)Fig.**3. Optimal control manifold, (**T**=30)*



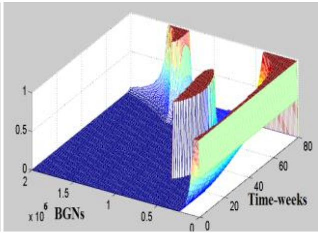
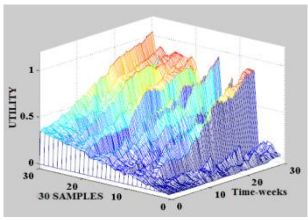


***Fig.4.** **HJB** function,(**T=80)** **Fig.**5. Optimal control manifold, (**T**=80)*

The stochastic process is started in 30 different initial points; from 1000 BGN's to 30000 BGN's. The optimal control solutions could be seen in Figure 6 [9]. In Figure 7 is shown an optimal flow projected over the objective function measured in utilils (1 utilils ≈ 10^6 BGN's). The black seesaw line under the objective utility function in Figure 5 is a sample of stochastic optimal control flow.



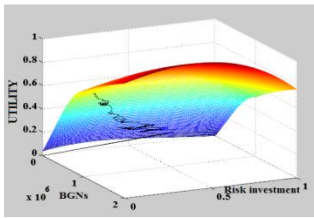
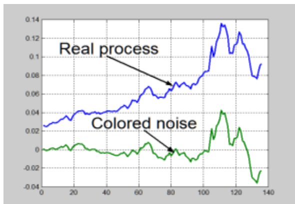


Fig.6.**Samples of control,(**T=30) Fig.**7. An **o**ptimal control flow, (**T=30)

The Wiener process W is an abstraction, sometimes far away from the reality because the white noise assumption is too strong. In the restwill be investigated an optimal portfolio control allocation in the case of a financial process with colored noise. Data from a real process are used: „*GNP in 1982 Dollars, discount rate on 91-day treasury bills, yield on long term treasury bonds, 1954Q1-1987Q4; source: Business Conditions Digest*". The noise of the real financial process (Figure 8) is far away from the white noise as could be seen by the correlation function in Figure 9.



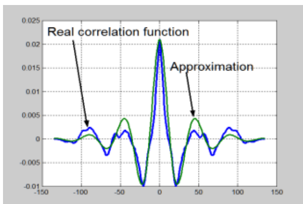


***Fig.8.**Experimental data**Fig.**9.Correlation functions*

Linear control theory discusses the colored noise modeling and filtering in and has practical significance [9]. The noise of the financial process is approximated by colored noise as is shown in Figure 9 by the approximation of the correlation function. This assumption permits modifications of the Black-Scholes model. The stochastic differential equation is extended to a three dimensional differential equation [9]:

$$dX_t^{\pi} = (rX_t^{\pi} + (\mu - r)\pi X_t^{\pi})dt + \pi X_t^{\pi} N_1$$
$$dN_1 = N_2 dt + 0.0028 dW_t$$
$$dN_2 = -(0.135 N_1 + 0.07 N_2)dt + 0.0047 dW_t$$

It is obvious that the colored noise N appears autonomously in the second and the third row. The second and the third rows described a linear system. The wealth X_t appears only with its first derivative in the Hamilton-Jacobi-Bellman (HJB) partial differential equation.

$$\frac{\partial B}{\partial t}(t,X,N_1,N_2) + \sup_{\pi\in[0,1]} [(rX + (\mu - r)\pi X) + \pi X N_1] \frac{\partial B}{\partial X}(t,X,N_1,N_2)$$
$$+ N_2 \frac{\partial B}{\partial N_1}(t,X,N_1,N_2) - (0.135 \frac{\partial B}{\partial N_1}(t,X,N_1,N_2) + 0.07 \frac{\partial B}{\partial N_2}(t,X,N_1,N_2)) +$$
$$+ \frac{1}{2}(0.0028)^2 \frac{\partial^2 B}{\partial^2 N_1}(t,X,N_1,N_2) + \frac{1}{2}(0.0047)^2 \frac{\partial^2 B}{\partial^2 N_2}(t,X,N_1,N_2) = 0.$$

These observations permit a decomposition of the HJB partial differential equation to a partial differential equation of the first degree with variables X_t and (t) and to an autonomous HJB partial differential equation with variables N_1 and N_2. We will look for a solution of the HJB partial differential equation of the form $B_1(t, X)B_2(t, N_1, N_2)$. The function $B_2(t, N_1, N_2,)$ is a positive smooth function, solution of the following partial differential equation:

$$\frac{\partial B_2}{\partial t}(t,N_1,N_2) + N_2 \frac{\partial B_2}{\partial N_1}(t,N_1,N_2) - [0.135 \frac{\partial B_2}{\partial N_1}(t,X,N_1,N_2) + 0.07 \frac{\partial B_2}{\partial N_2}(t,N_1,N_2)] +$$
$$+ \frac{1}{2}(0.0028)^2 \frac{\partial^2 B_2}{\partial^2 N_1}(t,N_1,N_2) + \frac{1}{2}(0.0047)^2 \frac{\partial^2 B_2}{\partial^2 N_2}(t,N_1,N_2) = 0.$$

The function $B_1(T, X)$ is chosen to be equal to the DM's utility function $U(X_T, \pi)$ in the final moment T. This function is solution of the partial differential equation:

$$\frac{\partial B_1}{\partial t}(t,X,N_1,N_2) + \sup_{\pi\in[0,1]}[(rX + (\mu - r)\pi X) + \pi X N_1] \frac{\partial B_1}{\partial X}(t,X,N_1,N_2) = 0$$

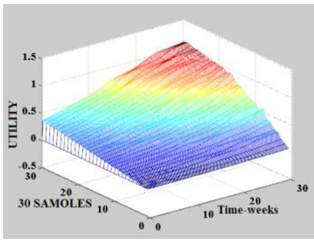
The decomposition permits determination of the partial derivative on X of the Belman's function $B(t, X, N_1, N_2)$ as follows:

$$\frac{\partial B}{\partial X}(t,X,N_1,N_2) = \frac{\partial U}{\partial X}(t,X) e^{\int_t^T (r + (\mu - r)\pi_S + \pi_S E(N_1(s)))ds} B_2(t, N_1, N_2)$$

In the formula π_t, $t\in[0,T]$ is the optimal control policy and $E(N_1(t))$ is the mathematical expectation of the colored noise at moment t. We remind that the color noise is generated by a linear system with Gauss white noise as input. Now is clear that the two partial derivatives have the same sign and the formula of the optimal control law becomes [9]:

$$\sup_{\pi\in[0,1]}[(rX_t + (\mu - r)\pi_t X_t) + \pi_t X_t N_{1_t}] sign(\frac{\partial U}{\partial X}(t,X))$$

The stochastic process with colored noise is started in 30 different initial points; from 1000 BGN's to 30000 BGN's. and is evaluated for two periods (30 and 80 weeks).The solutions in the condition T=30 are shown in Figure 10.



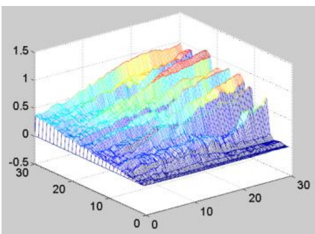
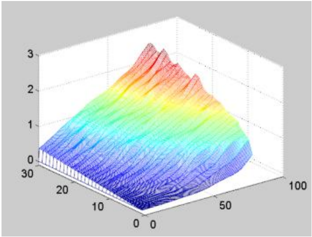


*Fig.10.*Optomal contr. (Colored noise) *Fig.11.*Opt. contr. (White noise)

In the next Figure 11 are shown the same solutions but with the classical control law in the case of Wiener process. We underline that in Figure 10 and Figure 11 are shown two different optimal solutions - wealth evaluated in Utilities measures U(X) in the case of colored noise and in the case of white noise respectively. In figures 12 and 13 is shown the same comparison but in the condition T=80:



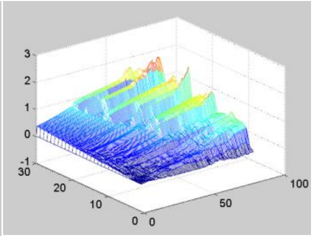


*Fig.12.*Optimal contr. (Colored noise) *Fig.13.*Opt. contr. (White noise)

It is well seen that the solution that renders an account of the colored noise gives much better results. The approach described in this case study is in fact is a value driven design and control [1, 2, 7].

3. Conclusions

The proposed modeling in the case of colored noise permits a much easy solution in comparison with the classical stochastic control solution. It is seen that the evaluation of the HJB function could be omitted and the control law depends only from the derivative of the objective utility function.

In the paper is shown that in complex systems the incomplete qualitative decision maker's information could be compensated by measurement and utility evaluation of qualitative human preferences. This is achieved through a stochastic machine learning procedure that based on the gambling approach. In this way, it is realized a relationship between value-driven modeling and machine learning, which is rarely under research.

The analytical presentations of the expert's preferences as value or utility function allow the inclusion of the decision maker mathematically in the model "Human-process". The suggested approach can be regarded as a realization of the *prescriptive decision making*. The utility function is an abstraction presented in the area of the normative approach, the axiomatic systems of Von Neumann. The utility functions are measured in the interval scale on the base of the DM's preferences over lotteries and are polynomial approximations of Von Neumann's utility functions.

The described methodology and procedures allow the design of individually oriented portfolio allocation models and optimal control in accordance with the decision-makers expectations and preferences.

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