# CONSTRUCTION AND CLASSIFICATION OF OPTIMAL (v,5,3,1) OPTICAL ORTHOGONAL CODES

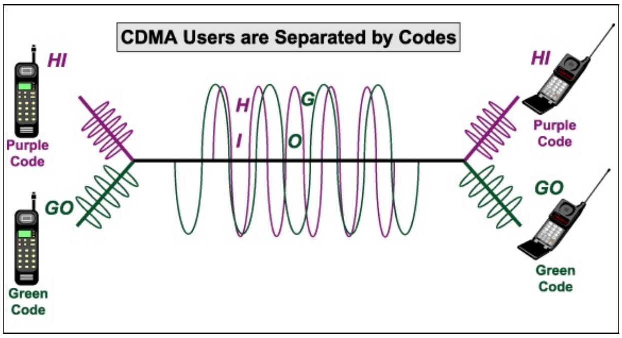
Prof. Dr. Sci Tsonka Baicheva, Prof. Phd Svetlana Topalova[1]
Institute of Mathematics and Informatics – BAS, Bulgaria[1]
tsonka,svetlana@math.bas.bg

***Abstract:*** *We classify up to multiplier equivalence maximal (v, 5, 3, 1) optical orthogonal codes (OOCs) with lengths up to 100.*

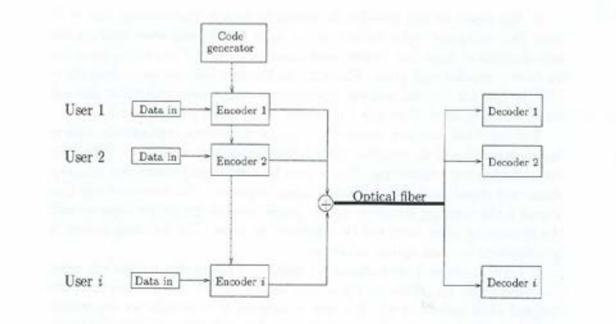
**Keywords**: OPTICAL CODE-DIVISION MULTIPLE ACCESS SYSTEM, OPTICAL ORTHOGONAL CODE

## *1. Introduction*

One of the basic concepts in data communication is the idea of allowing several transmitters to send information simultaneously over a single communication channel. This is called multiple access. There are several techniques to provide multiple access and one of them is code division multiple access (CDMA). Although used in various radio communications systems, the most widely known application of CDMA is for cellphones.




A binary signature sequence (codeword) is assigned to each user in a CDMA system. The data to be sent is mapped onto the codewords and the different users codewords are mixed together and sent over the channel. At the receiver end a decoder, which is individual for each user, compares the incoming sequence with stored copies of the codewords to be able to extract the information bits.




Optical code-division multiple access (OCDMA) systems attract much attention as they have several benefits such as asynchronous transmission, being flexible in network design, accommodation of burst traffic, etc. A main problem connected with the use of OCDMA systems is the search for powerful code structures that allow a large number of users to communicate simultaneously with a low error probability. Among the most famous codes considered to date are optical orthogonal codes (OOCs). They also have applications in mobile radio, frequency-hopping spread-spectrum communications, radar, sonar signal design, constructing protocol - sequence sets for the M-active-out-of T users collision channel without feedback, etc.

Since the introductory paper by Chung, Salehi and Wei [7] the optical orthogonal codes existence problem has been intensively studied. Many constructions of optimal optical orthogonal codes are known, see for instance [4,5,6,8,9,11]. In particular, OOCs of weight 5 have been considered in [1,2,3,4,9,10,11]. Most classes of optimal (v,5,1,1) OOCs that are known were obtained by the powerful difference family apparatus either as ordinary (v,k,1) difference families (corresponding to the perfect (v,k,1,1) OOCs) or relative (v,k,n,1) difference families with $n \le k^2$-k.

Constructions and classification of (v,5,2,1) OOCs of small lengths are presented in [2] and [4]. In this paper we classify (v,5,3,1) OOCs with lengths up to 100.

## *2. Basic definitions*

For the basic concepts and notations concerning the classified combinatorial objects we follow [4] and [9]. We denote by $Z_v$ the ring of integers modulo v and by $\oplus$ addition in it.

**Definition 1**. A $(v,k,\lambda_a,\lambda_c)$ OOC is a collection $C=\{c_1,\dots,c_s\}$ of k-subsets (codewords) of $Z_v$, such that any two distinct translates of a codeword share at most $\lambda_a$, elements, and any two translates of two distinct codewords share at most $\lambda_c$ elements:

$$|c_i \cap (c_i \oplus t)| \le \lambda_a, \ 1 \le i \le s,\ 1 \le t \le v-1 \qquad (1)$$

$$|c_i \cap (c_j \oplus t)| \le \lambda_c, \ 1 \le I < j \le s,\ 1 \le t \le v-1 \qquad (2)$$

Codewords of an OOC can also be represented as {0,1} sequences of length v having ones at positions $c_i$.

Condition (1) is called the auto-correlation property and (2) the cross-correlation property.

**Definition 2.** The size of C is the number s of its codewords.

Consider a codeword $C=\{c_1,\dots,c_s\}$. Denote by $\Delta'C$ the multiset of the values of the differences $c_i - c_j$, $i \neq j$, i,j=1, 2, … , k and by $\Delta C$ its corresponding set. The type of C is the number of elements of $\Delta C$, i.e. the number of different values of its differences. The auto-correlation property means that at most $\lambda_a$ differences are the same. For $\lambda_c$ =1 the cross-correlation property means that $\Delta C_1 \cap \Delta C_2=\varnothing$ for two codewords $C_1$ and $C_2$.

**Definition 3.** A $(v,k,\lambda_a,\lambda_c)$ OOC is perfect if $|\cup^s_{i=1} \Delta C_i|$= v-1, that is if all nonzero differences are covered.

**Example.** Codewords of a perfect (37, 5, 3, 1) OOC

$C_1$ = {1110000100000000000000000000000010000} or

{0, 1, 2, 7, 32}

$C_2$ = {1001000000010000000100100000000000000} or

{0, 3, 11, 19, 22}

$C_3$ = {1000100000000100010000000001000000000} or

{0, 4, 13, 17, 27}

$\Delta C_1$ = {1,2,5,6,7,12,25,30,31,32,35,36}

$\Delta C_2$ = {3,8,11,15,16,18,19,21,22,26,29,34}

$\Delta C_3$ = {4,9,10,13,14,17,20,23,24,27,28,33}

Among the OOCs with given parameters those ones which have more codewords are more interesting from application point of view and research efforts are directed there.

**Definition 4**. A $(v,k,\lambda_a,\lambda_c)$ OOC is optimal if it has maximum size.

Since we want to classify all OOCs with given parameters, we need to define an equivalence relation on them.

**Definition 5**. Two $(v,k,\lambda_a,\lambda_c)$ OOCs C and C' are isomorphic if there exists a permutation of $Z_v$, which maps the collection of translates of each codeword of C to the collection of translates of a codeword of C'.

**Definition 6**. Two $(v,k,\lambda_a,\lambda_c)$ OOCs are multiplier equivalent if they can be obtained from one another by an automorphism of $Z_v$ and replacement of codewords by some of their translates.

There can exist OOCs which are isomorphic, but multiplier inequivalent.

## *3. Classification method*

We classify up to multiplier equivalence the (v, 5, 3, 1) OOCs by a modification of the algorithm used in [2]. The algorithm performs backtrack search on the set of all possible codewords, i.e. all 5-sets which meet the auto-correlation requirement. In the present modification we check which types of codewords are available in this set and exclude from it the codewords of types that are impossible for a code with the predefined size.

At each stage of the back-track search we add a codeword to the current partial solution. In order to make the classification feasible we speed up the algorithm by performing a minimality test and a type test to the partial solutions.

*Minimality test*: we check if the current partial solution can be mapped to a lexicographically smaller one by the automorphisms of $Z_v$. If it can, an equivalent partial solution has already been considered, and we look for the next possibility for the current codeword.

*Type test*: Suppose that r codewords of the code have been already found. Let T be the type of the r-th codeword, and let d be the number of distinct differences covered by the r codewords. We only look for codes with a definite number s of codewords. The type of the remaining codewords (of the array we choose them from) is at least as big as that of the r-th chosen one. That is why

$$d+(s-r)T \leq v-1.$$

If this does not hold, the next possibility for the (r-1)-st codeword is considered.

## *4. Classification results*

The classification of maximal (v, 5, 3, 1) OOCs with $29 \leq v \leq 100$ is presented in Table 1. We start with v=29 because s < 2 for all smaller lengths. For each v we give the size $s_{opt}$ of the optimal OOCs, the number of all multiplier inequivalent optimal OOCs, and the number of the perfect ones among them.

All computer results are obtained by our own C++ programs. Files with the OOCs we construct can be downloaded from http://www.moi.math.bas.bg/~tsonka. All codes are available online to everybody who is interested and further investigations of their properties are possible.

## *5. Conclusion*

The classified codes can be of use both directly in relevant applications, and as parts of constructions of new infinite families.

Table 1: Multiplier inequivalent maximal (v, 5, 3, 1) OOCs

| v | $s_{opt}$ | OOCs | perfect | v | $s_{opt}$ | OOCs | perfect |
|---|---|---|---|---|---|---|---|
| 29 | 2 | 2 | 0 | 65 | 6 | 544 | 399 |
| 30 | 2 | 6 | 0 | 66 | 5 | 132707 | 286 |
| 31 | 2 | 9 | 0 | 67 | 6 | 105 | 54 |
| 32 | 2 | 28 | 0 | 68 | 6 | 4 | 4 |
| 33 | 2 | 63 | 0 | 69 | 6 | 1008 | 330 |
| 34 | 2 | 153 | 0 | 70 | 6 | 101614 | 28159 |
| 35 | 2 | 257 | 0 | 71 | 6 | 14572 | 2228 |
| 36 | 2 | 852 | 0 | 72 | 6 | 197039 | 4274 |
| 37 | 3 | 1 | 1 | 73 | 6 | 48986 | 3109 |
| 38 | 2 | 1816 | 0 | 74 | 6 | 155414 | 10287 |
| 39 | 3 | 3 | 0 | 75 | 7 | 2 | 2 |
| 40 | 3 | 9 | 0 | 76 | 6 | 779392 | 18383 |
| 41 | 3 | 13 | 0 | 77 | 7 | 13445 | 0 |
| 42 | 3 | 91 | 0 | 78 | 7 | 4 | 4 |
| 43 | 3 | 138 | 1 | 79 | 7 | 1962 | 58 |
| 44 | 3 | 449 | 0 | 80 | 7 | 38484 | 1570 |
| 45 | 3 | 1360 | 0 | 81 | 7 | 220990 | 15449 |
| 46 | 3 | 2138 | 0 | 82 | 7 | 54166 | 4584 |
| 47 | 3 | 3171 | 0 | 83 | 7 | 254095 | 21646 |
| 48 | 4 | 30 | 0 | 84 | 8 | 1831920 | 1109328 |
| 49 | 4 | 1 | 0 | 85 | 7 | 2927999 | 88910 |
| 50 | 4 | 16 | 0 | 86 | 7 | 4657069 | 125855 |
| 51 | 4 | 79 | 4 | 87 | 8 | 540 | 274 |
| 52 | 4 | 90 | 0 | 88 | 8 | 21070 | 2590 |
| 53 | 4 | 651 | 0 | 89 | 8 | 46713 | 8076 |
| 54 | 4 | 3581 | 0 | 90 | 8 | 1382960 | 146554 |
| 55 | 5 | 21 | 0 | 91 | 8 | 40991084 | 9158887 |
| 56 | 4 | 19017 | 4 | 92 | 8 | 960686 | 234936 |
| 57 | 4 | 31009 | 23 | 93 | 8 | 3435231 | 289834 |
| 58 | 5 | 4 | 0 | 94 | 8 | 7820174 | 1131812 |
| 59 | 5 | 31 | 0 | 95 | 9 | 14580 | 8 |
| 60 | 5 | 443 | 12 | 96 | 8 | $\geq$ 83000000 | |
| 61 | 5 | 613 | 61 | 97 | 9 | 6023 | 2397 |
| 62 | 5 | 1749 | 8 | 98 | 9 | 1920 | 1792 |
| 63 | 5 | 20689 | 92 | 99 | 10 | 343488 | 343488 |
| 64 | 5 | 23820 | 62 | 100 | 9 | 154808 | 42088 |

## *6. References*

[1] T. Baicheva, S. Topalova, Classification results for (v, k, 1) cyclic difference families with small parameters, *Mathematics of Distances and Applications*, M. Deza, M. Petitjean, K. Markov eds., in International book series: Information Science and Computing, book 25, 2012, 24-30.

[2] T. Baicheva, S. Topalova, Optimal (v,5,2,1) optical orthogonal codes of small v, *Applicable Algebra in Engineering, Communication and Computing*, 24 (3-4), 2013, 165-177.

[3] M. Buratti, A. Pasotti, Further progress on difference families with block size 4 or 5, *Designs, Codes and Cryptograthy*, 56, 2010, 1-20.

[4] M. Buratti, A. Pasotti, D. Wu, On optimal (v,5,2,1) optical orthogonal codes, *Designs, Codes and Cryptograthy*, 68(1–3), 2013, 349-371.

[5] W. Chu and S. W. Golomb, Optimal (n, 4, 2)-OOC of small order, *Discrete Math.* 279, 2004, 163-172.

[6] W. Chu and S. W. Golomb, A new recursive construction for optical orthogonal codes, *IEEE Trans. Inform. Theory*, 49 (11), 2003, 3072-3076.

[7] F. R. K. Chung, J. A. Salehi and V. K. Wei, Optical orthogonal codes: Design, analysis, and applications, *IEEE Trans. Inform. Theory*, 35, 1989, 595-604.

[8] H. Chunk and P. V. Kumar, Optical orthogonal codes - new bounds and an optimal construction, *IEEE Trans. Inform. Theory*, 90 (4), 1990, 866-873.

[9] Ch. Colbourn, and J. Dinitz, Eds, *Handbook of Combinatorial Designs*, 2nd edition (Discrete mathematics and its applications, ser. ed. K. Rosen), CRC Press, Boca Raton, FL., 2007.

[10] S. Martirosyan and A. J. Han Vinck, A construction for optical orthogonal codes with correlation 1, *IEICE Trans. Fundamentals*, E85-A (1), 2002, 269-272.

[11] K. Momihara, *Existence and construction of difference families and their applications to combinatorial codes in multiple-access communications*, Ph. D. Thesis, Graduate School of Information Science, Nagoya University, Japan, 2009.