# MATHEMATICAL MODEL FOR INCREASING THE EFFICIENCY OF PASSENGER RAILWAY TRANSPORT IN BULGARIA

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***Abstract:*** *A mathematical model has been derived to assess the potential of the rail passenger transport to increase his capacity and efficiency. This potential has been evaluates in comparison with the competition of the bus transportation. A specific transportation route has been chosen from Sofia to Varna and the potential for increase of the rail transport has been evaluated. The mathematical model uses optimization problem, related to the evaluation of a maximal flow in a transportation network.*

**Keywords:** MAX FLOW PROBLEM, OPTIMIZATION, NETWORK DESIGN

## 1. Introduction

This research derives and mathematical formulation of a problem for evaluation of the potential of the rail transport to increase its capacity and performance. The model is a=base =d on comparisons between he capacities of bus and rail passenger transportation for a redefined transportation destination. The mathematical model applies theoretical formulation and evaluation of maximum flow design in a network topology. Such a model targets the solution of very practical problem for optimization of the passenger rail transport and its schedule in the National Transportation scheme of Bulgaria. Thus the rail transport will play main role in National long distances travels. The improvements in the rail schedules is based on comparisons between the current transportation flows, performed by bus and rail transport and the opportunity to track the components of the max flows in the network by rail transport.

## 2. Theoretical background to control flows in networks

This research derive a mathematical model to evaluate the potential for passenger transportation, performed both by rail and bus transport. The model defines a specific optimization problem to evaluate the capacities for transport, performed by buses and rails in a network. The results from these capacity evaluations can be used to improve the economic performance of the passenger rail transport; to make quantitative assessments of the competition berween rai and bus transportation; to identify measures for intensification of the rail transport in compliance with European directives (Directive 2010/40 / EU of European Parliament and of the Council of 7 July 2010 on the framework for the deployment of Intelligent transport Systems in the field of road transport and for interfaces with other models of transport) to reduce pollution from emissions and improving logistics services through intensification of rail transport. The derived model applies hierarchical optimization with particular definition of bi-level optimization problems. [1, 2]

The derived model comprises the following evaluation steps:
- Identification of transport flows, simultaneously performed both by rail and bus transport for the predefine destination;
- Quantification of the capacity of transport with buses and trains for the selected area of PTC;
- Definition of graph topology presenting the transportation flows between source and destination nodes ;
- Definition of an optimization problem for evaluation of the maximum flow in the defined transportation network.

This problem is a linear optimization one with large number of unknowns and constraints. The analytical definition of max flow problem is in the form:

$$\begin{aligned}
&\max X(f_i^k) \\
&\sum_i f_i^k - \sum_j f_j^k = 0 \\
&\sum_i f_i^s = -X \qquad (1) \\
&\sum_j f_j^d = X \\
&\sum_i f_i^k \le V_{i,k}
\end{aligned}$$

where

$\sum_i f_i^k$ denotes the sum of all inflows in the node *k*,

$\sum_j f_j^k$ denotes the sum of all outgoing flows from the node *k*,

*s*, *d* are notations for the source and destination nodes,

*X* is the unknown value of the maximal flow, which can be transported from node *s* to node *d* trough the transportation network,

$V_{i,k}$ is the capacity for transportation for link *i,k*.

The engineering meaning of the solution of this problem gives the values of the components of the maximal flows and the link in the network, where this flow is passed.

## 3. Practical application of the derived mahematical model

The current practice in Bulgaria defines so called There is a Republican Transport Scheme (RTS), which is approved by the Ministry of Transport and Information Technology. RTS network is supported both by rail and buses transportation means. The rail transport is exploited by a state enterprise, named BDZ- PT= The bus transportation is performed by a set of private companies. Each company has to apply and receives after approval a license document to exploit exactly defined transportation link. Particularly the State Executive Agency for Automobile Administration approve the applications for transport licenses. .

For the application of the derived mathematical the chosen destination by RTS: Sofia-Varna.

The definition of the max flow optimization problem, the values of the capacities of the transportation links $V_{i,k}$ have to be estimated. These capacities are defined according to considerations for improvement of the transport service.

From customer point of view, the quality of the transportation service is strongly related with the duration of the transport service. Hence if the ravel time is short, it can be assumed that the transportation capacity through the corresponding link is high or the duration of the transport service is reciprotially related with the capacity of the transportation link. Following this engineering assumption the relation between is transport capacity and travel time is analytically defined as

V = 1/ travel time. (2)

The reason for the usage of this relation results from the lack of real data for the intensities of passenger transportation per different destinations of National transportation scheme RTS. Because for given destination the transportation can be performed by different means and with a set of transport schedule, the total transport capacity of network link is evaluated as a sum of the individual capacities of each transportation schedule.

Example: The travel time from Sofia to Levski is 3 hours and 23 minutes or 203 minutes. The transportation capacity for this link is 1/203, or 0.0049 relative units. If the train schedule is 3 times per day the capacity for this transport link in the network will be 3 x 0.0049 = 0.014 relative capacity units.

The links, served by bus transportation are taken into consideration also.

## *5. Definition of integrated network for transport*

The transportation network for the destinations Sofia – Varna is generated by all transport services, which are supported both by train and buses. The network topology is generated by all nodes, where bus transports intersect with the rail transport. The network topology insists to be identified the set of all these intersection points between the bus lines with the rail route for Sofia – Varna destination. After generation of the network, the capacities of all links have to be evaluated. These capacities are calculate according to the travel time and the number of schedules per dayq performed by rail and buses. Finally the total capacity per link is evaluated, according to relation (2).

Having he set of capacities $V_{i,k}$ and the topology of the network, the optimization max flow problem (1) can be defined. The solution of the problem (1) will give the value $X$ of the maximal flow, which can be transported between Sofia and Varna, according to the current schedules of buses and trains.

The solution of problem (1) gives also the routes, where the components of the maximal flow are passed in the graph. Thus the solution of (1) will give picture if the rail transport is used on its maximal capacity and which links of the rail can support additional transportation services due to available capacities for transportation.

The solution of problem (1) will identify also the links of the rails, which are on its upper capacity bounds. This means that they cannot increase its efficiency or if increase is needed, new train schedule has to be design or the travel time per these links haves to be decrease.

This paper presents the current case, which is actual for transportation from Sofia to Varna by uses and trains for 2016 schedules. Having data for the train schedules for trains from www.bdz.bg and the schedules for buses from http://www.rta.government.bg/index.php?page=scategories&scategory=otrts the transportation network is presented on fig 1. The notations on the network correspond to geographically places and towns, which are defined as:

| | |
|---|---|
| 1. Sofia | 9.Popovo |
| 2. virtual point | 10.virtual point |
| 3. Mezdra | 11.Targovishte |
| 4. Pleven | 12.virtual point |
| 5. Levski | 13. Shumen |
| 6. Gorna Oryahovitsa | 14.virtual point |
| 7. Starzhitsa | 15. Veliko Tarnovo |
| 8. virtual | 16. Antonovo |
| | 17. Varna. |

The virtual points are introduced to separate the transportation link, generated by bus and/or rail transport. This is needed for the personal assessment of rail and bus transportation.

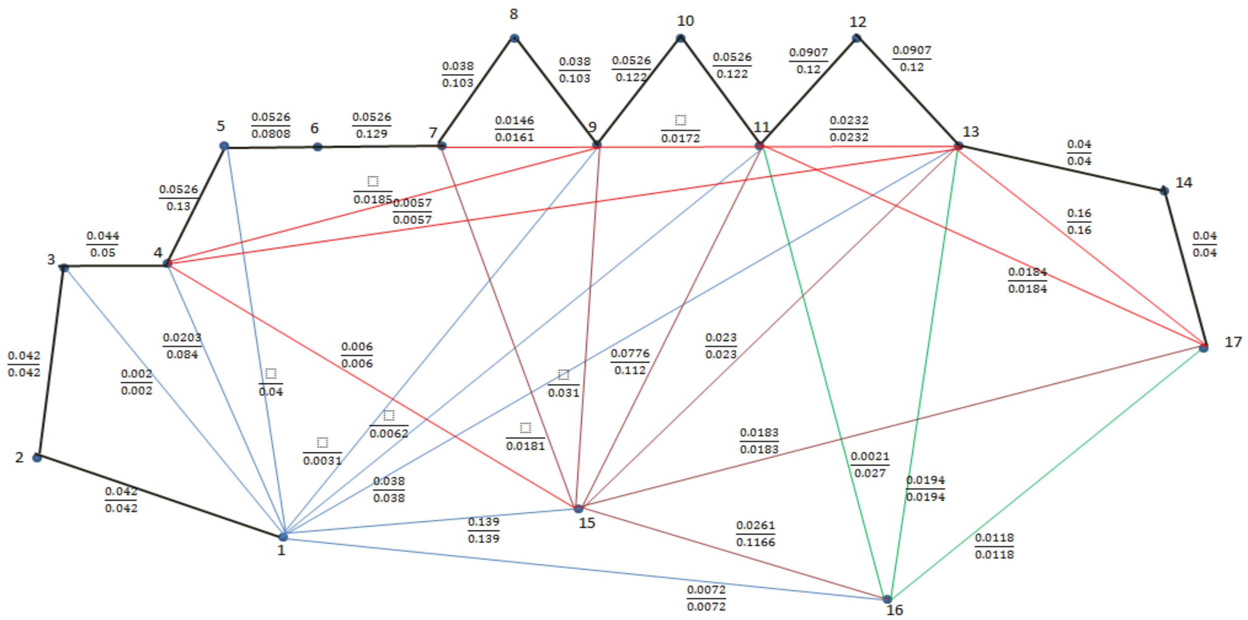

Fig.1 The transportation network Sofia Varna

The notations on the network links are given in form of nominators and denominators. The denominator is the value of the transport capacity on the corresponding link, which is supported by bus or train transport. The nominator of the notation gives the value of the part of the maximal flow, which is passed on this link. If the nominator is less than the denominator, this means that such a link has capacity to undertake additional transportation services.

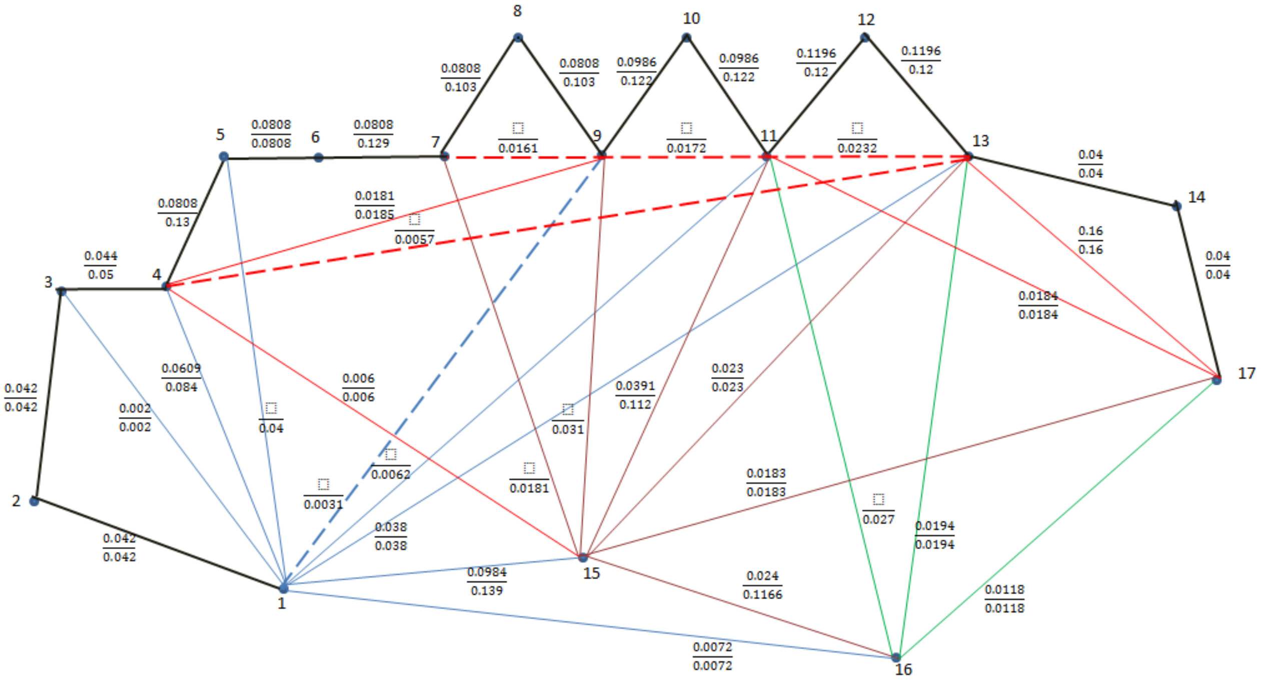


Fig.2 Allocation of the components of max flow per link in the network

Fig.1q which presents graphically the solution of problem(1) identifies that it exists potential for increasing the exploitation of the rail transport by decreasing the transportation, performed by buses on appropriate links. On fig.2 are given potential solutions for increase of the rail transport. The potential solution is graphically presented by dotted lines. If these lines are removed, the value of the maximal flow is preserved, but the rail transport is increase close to its capacity values. For example by removing bus lines, serving s=directions 4-13 an№ДФНП-98/04.05.16, signature 210273 d 7-13, the rail transport increases from 0.038 till 0.808 for link 7-9. This a beneficial decision for the increase of the rail transport services.

## *Conclusions*

The paper derives a mathematical model for accessing the potential for exploitation of transportation services, performed simultaneously by trains and buses. The idea of this assessment is to increase the rail transport by means of definition right transport schedule and decrease the bus exploitation. Such policy will benefit National policy for improvement of the rail transport and passenger transportation on National level. The Model is based on definition and solution of appropriate optimization problem, stated as maximal flow problem. The solution of this problem identifies weak points in the exploitation of the rail transport and gives potential solutions for increase of it efficiency.

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