DESIGN AND OPTIMISATION OF 2DOF VEHICLE SUSPENSION SYSTEM

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Abstract: In this paper is presented the optimal design of quarter car vehicle suspensions. During the optimisation the three design criteria which have been used are; vertical vehicle body acceleration, dynamic tire load and suspension working space. For implementing a optimization a comparison of two optimisation algorithms: Sequential Quadratic Program (SQP) and Genetic Algorithms (GAs). will be chosen five design parameters: sprung and un-sprung mass, spring stiffness, damping coefficient and tire stiffness. Through the simulation in Matlab it will be shown that GA is more powerful tool to find the global optimal point, without any restrictive requirements on the gradient and Hessian Matrix, while SQP has local convergence properties. The main focus of this research will be on minimising the vertical vehicle body acceleration subjected to a suspension working space and the dynamic tire load. At the end of this paper, it will be shown the comparison between the simulation carried out with nominal and optimal values of design parameters.

KEY WORDS: OPTIMISATION ALGORITHMS, DESIGN PARAMETERS, SQP, GA.

1. Introduction

Vehicle suspension design includes a number of compromises that have to do with suspension system which should be smooth to provide good levelling and ride comfort. On the other hand must be strong, to fix changes the behaviour of the vehicle and to ensure road holding for varying external conditions. Traditional design practices of vehicle suspensions have been based on trial and error approaches. Now the focus of vehicle suspension design has switched from pure numerical analysis to extensive design synthesis using optimization approaches. There are numerous methods available and even the choice of an efficient optimization algorithm is a non-trivial problem [2]. Genetic algorithms (GAs) have been used in various applications such as function optimization, system identification and control systems. GAs are general-purpose stochastic optimization methods for solving search problems to seek a global optimum. However, GAs is characterized by a large number of function evaluations [1]. The pattern search algorithm (PSA) is typically based on function comparison techniques. Most of these procedures are heuristic in nature and derivative evaluations are not needed. They can be used to solve problems where the objective function is not differentiable and continuous [3].

On the other hand, traditional methods, such as sequential quadratic programming (SQP), are well known to exploit all local information in an efficient way, provided that certain conditions are met and the function to be minimized is 'well-conditioned' in the neighbourhood of a unique optimum. These methods require adequate local information to be known (such as the gradient and Hessian matrix) [2, 3]. If the basic requirements are not satisfied, the reliability of the SQP method is greatly jeopardized [2]. By means of the ride quality analysis in the frequency domain, the vertical vehicle body acceleration (VBA), suspension working space (SWS) and dynamic tire load (DTL) can be obtained [1]. In this design optimization, the main objective is to minimize the VBA acceleration. In the meantime, the SWS and DTL are constrained. If the SWS is too small, the sprung mass will strike the un-sprung mass and this may lead to damage of the vehicle. If the DTL is greater than the static tire load, the vehicle's tires will bounce off the road [4] and this will result in unstable modes of vehicle motion. Therefore, it is necessary to optimize the suspension working space and dynamic tire load.

2. *Vehicle System Modeling*

The model of the simplified quarter-car active suspension system used in this paper with two degree of freedom is shown in Figure 1. The model represents a single wheel of a car in which the wheel is connected to the quarter portion of the car body through a hydro-pneumatic suspension.

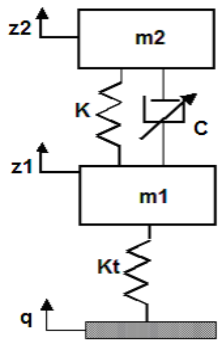

***Fig.1**. Quarter-car model*

where:

$z_2 - z_1$: body displacement; $z_1 - q$: wheel displacement

$\dot{z}_2 - \dot{z}_1$: suspension velocity;

$\dot{z}_2$: velocity of the body; $\dot{z}_1$: velocity of the wheel

m_2 = 637 kg : body mass ; m_1 = 104 kg : wheel mass

K =3200N/m and K_t = 100600 N/m : respective spring constants

C = 3200 Ns/m : damping ratio

The equations of motion:

$$\begin{aligned} m_1\ddot{z}_1 + C(\dot{z}_1 - \dot{z}_2) + K(z_1 - z_2) + K_t(z_1 - q) = 0 \\ m_2\ddot{z}_2 + C(\dot{z}_2 - \dot{z}_1) + K(z_2 - z_1) = 0 \end{aligned} \tag{1}$$

Performing a Fourier transform of equation (1) yields:

$$\begin{aligned} z_2(-\omega^2 m_2 + j\omega C + j\omega + K) = z_1(j\omega C + K) \\ z_1(-\omega^2 m_1 + j\omega C + K + K_t) = z_2(j\omega C + K) + qK_t \end{aligned} \tag{2}$$

The amplitude ratio between the un-sprung mass displacement, z_1, and the road excitation, q is given as follows:

$$\left|\frac{z_1}{q}\right| = \gamma\left[\frac{(1-\lambda^2)^2 + 4\xi^2\lambda^2}{\Delta}\right]^{\frac{1}{2}} \tag{3}$$

The amplitude ratio between the sprung mass displacement, z_2, and the road excitation, q is:

$$\left|\frac{z_2}{q}\right| = \gamma\left[\frac{1+4\xi^2\gamma^2}{\Delta}\right]^{\frac{1}{2}} \tag{4}$$

Therefore, the amplitude ratio between the sprung mass acceleration, $\ddot{z}_2$ and the road excitation, q, can be expressed as:

$$\left|\frac{\ddot{z}_2}{\dot{q}}\right| = \omega\gamma\left[\frac{1+4\xi^2\gamma^2}{\Delta}\right]^{\frac{1}{2}} \tag{5}$$

The suspension working space is the allowable maximum suspension displacement, f_d. The suspension working space in response to the road displacement input is:

$$\left|\frac{f_d}{\dot{q}}\right| = \frac{\gamma}{\omega}\lambda^2\left[\frac{1}{\Delta}\right]^{\frac{1}{2}} \tag{6}$$

The amplitude ratio between the relative dynamic tire load, $\left|\frac{F_d}{G}\right|$, and the road input, q, becomes:

$$\left|\frac{F_d}{G_q}\right| = \frac{\gamma\omega}{g}\left[\frac{\left(\frac{\lambda^2}{1+\mu}-1\right)^2+4\xi^2\lambda^2}{\Delta}\right]^{\frac{1}{2}} \tag{7}$$

3. Stochastic road modeling

Road irregularity or unevenness represents the main disturbing source for either the rider or vehicle structure itself. The road profile elevation is usually expressed in terms of the power spectral density (PSD). The PSD of the road profile elevation is expressed as:

$$G_q(n) = G_q(n_0)(n/n_0)^{-w} \tag{8}$$

For the purposes of design optimization, according to James' principle, the root mean square (RMS) of the sprung mass acceleration $\ddot{z}_2$ can be expressed as:

$$\sigma_{\ddot{z}} = \left\{\pi RV\left[\frac{K_t C}{2m_2^{3/2}K^{1/2}} + \frac{(m_1+m_2)K^2}{2Cm_2^2}\right]\right\}^{1/2} \tag{9}$$

The RMS of the suspension working space f_d is:

$$\sigma_{fd} = \left\{\pi RV\left[\frac{(m_1+m_2)(m_2K)^{1/2}}{2m_2C}\right]\right\}^{1/2} \tag{10}$$

The RMS of the relative dynamic tire load can be calculated as:

$$\sigma_{Fd/G} = \left\{\pi RV\left[\frac{K_t^2 m_1}{2C(m_1+m_2)^2} + \frac{(m_1+m_2)K^2}{2m_2^2 C} - \frac{K_t K m_1}{Cm_2(m_1+m_2)} + \frac{CK}{2m_1m_2}\right]\right\}^{1/2} \tag{11}$$

4. Design and optimisation

In this section, the sprung mass vertical acceleration is minimized, while the design constraints on the suspension working space and dynamic tire load should be satisfied. To implement the design optimization, the two optimization algorithms, i.e, SQP and GA, will be applied, respectively.

4.1 Optimization based on SQP algorithm and GA

The SQP algorithm is a non-linear programming technique that is used for the purpose of minimizing a smooth non-linear function subjected to a set of constraints with upper and lower bounds. The objective function and the constraint functions are assumed to be at least twice continuously differentiable. This algorithm is a gradient-based search method [2, 3]. This algorithm is well-suited for constrained design optimizations.

The reliability for finding the optimum decreases with the increase of number of design variables when using SQP method. In contrast, whether the number of design variables increase the GA can still reliably find the optimum. This can be explained by the fact that GA works on a population of variables in parallel, not on a unique point. GAs are global search methods that are based on the Darwin's principle of natural selection and genetic modification. The GA has higher reliability to find the global optimum with minimum number of computational operations.

The RMS of the acceleration of a sprung mass $\sigma_{\ddot{z}_2}$ is frequently used to evaluate the riding quality of a vehicle. A rider's comfort improves as the acceleration decreases. Ride comfort is chosen to be the design criterion. The suspension working space and dynamic tire load σ_{fd} are selected as the design constraints. The design variables are m_1, m_2, K_t, K and C, respectively. Thus, the design optimization problem can be described as:

Minimise:

$$\sigma_{\ddot{z}_2}(m_1,m_2,K_t,K,C) = \left\{\pi\ R\ V\left[\frac{K_t C}{2m_2^{3/2}K^{1/2}} + \frac{(m_1+m_2)K^2}{2Cm_2^2}\right]\right\}^{1/2} \tag{12}$$

$$\text{Subject to: } \begin{cases} \sigma_{Fd/G}(m_1,m_2,K_t,K,C) \le a = 0.5 \\ \sigma_{fd} = (m_1,m_2,K_t,K,C) \le b = 0.05 \\ 83.2 \le m_1 \le 124.8 \\ 509.6 \le m_2 \le 764.4 \\ 559440 \le K_t \le 839170 \\ 80480 \le K \le 120720 \\ 2560 \le C \le 3840 \end{cases} \tag{13}$$

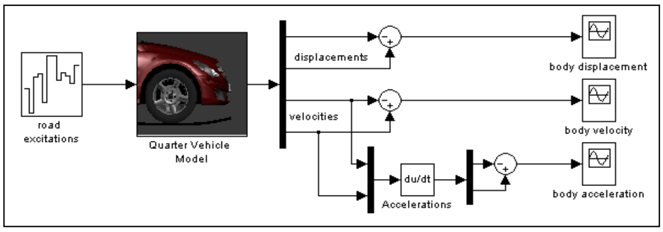

Fig 2. *Simulation scheme of Quarter Vehicle Model*

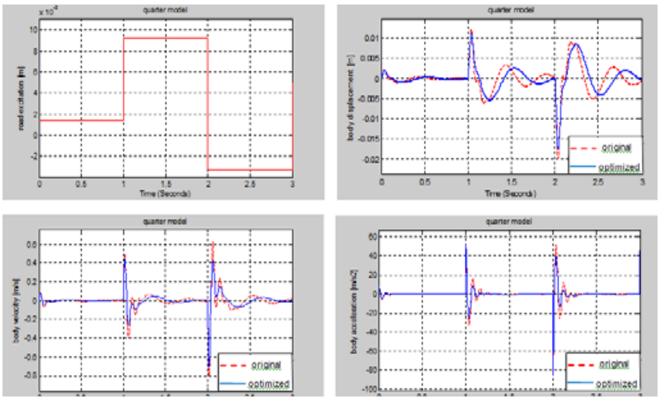

Fig 3. Simulation results for original --- (nominal) and optimized variables

In this sub-section, the optimization results are derived for a vehicle travelling at the speed of 40 m/s on the road with an irregularity coefficient of power spectrum taking the value of $6.5x10^{-6}$ m^3.

Table 1. *Optimal design variables based on the SQP and GA for minimizing the sprung mass vertical acceleration, vehicle speed 40 m/s*

	Original values	SQP method	GA
Initial		[10 10 10 10 10]	[10 10 10 10 10]
m_1 [kg]	104	83.2	83.2
m_2 [kg]	637	764.4	764.4
K [N/m]	699300	80480	80480
K_t [N/m]	100600	559692	559692
C [Ns/m]	3200	3840	3840
$\sigma_{\ddot{z}_2}$	5.6206348677	1.0703660446	1.0703417681
		Optimum found: 47 iteration	Optimum found: 19 generates

5. Conclusions

A comparative study of two optimization algorithms (genetic algorithms, GAs and sequential quadratic programming, SQP), has been conducted through minimizing the vertical sprung mass acceleration subjected to a suspension working space and dynamic tire load. A quarter-vehicle model was used to implement the design optimization of the vehicle suspension systems. The SQP has very strong theoretical and local convergence properties. The numerical results demonstrate these features of the algorithm, since the SQP is located at local optimal points. The GA is more powerful to find global optimal points, without restrictive requirements on the gradient and Hessian matrix.

By optimizing the design parameters compared with the original design, the sprung mass (body) acceleration decreases. The suspension working space and the dynamic tire load satisfy the specified design constraints. Based on the simulation results the optimum found by GAs at 19 generations, while by using the SQP the optimum is found after 47 iterations. As shown from the numerical simulation results in Figure 3 the max amplitude of body displacement using optimized design variables is reduced for 9%, while maximal amplitude of body acceleration is reduced around 22%.

Numerical experiments reveal the fact that to improve vehicle ride quality and satisfy the specified suspension working space and relative dynamic tire load, different vehicle speed and road irregularity have different requirements on the design variables, in particular, the un-sprung mass. In order to solve this problem, application of multi-level optimization approach it is recommended and the resulting solutions will compromise the conflicting requirements on design variables for vehicles travelling at different speed and on roads with different irregularity.

6. References

Likaj R.; Shala A.; Bruqi M.; Qelaj M. (2009). *Optimal Design of Quarter Car Vehicle Suspension System*, Trends in the Development of Machinery and Associated Technology – ISSN 1840-4944; pp. 417-420; Livorno, Italy.

Likaj R.(2005). *Fuzzy Logic Control on Nonlinear Vehicle Suspension System*, Phd thesis, Prishtina, Kosovo,

Likaj, R.; Geca, A.; Pajaziti, A.; Osmanaj S. (2005). *Parametric and sensitivity analysis of four DOFs vehicle model*, 4th DAAAM International Conference on Advanced Technologies for developing Countries, September 21-24, 2005, Slavonski Brod, Croatia

Qelaj M.: Optimisation of 2DOFs VSS, MTH, Pristina, Kosovo 2009.

Zhongzhe Ch., Yuping H., Natarer G.: Design Optimization of suspension with a Quarter-Vehicle Model, CSME/ de la SCGM, Vol.32, No .2, pp 297-312, 2008.

Yuping H: Design Optimization of a Quarter Vehicle suspension system using a Genetic Algorithm and SQP, DSDE, University of Waterloo, 1999.

Mohammadzadeh A. and Haidar S.: Analysis and Design of Vehicle Suspension system using MATLAB and SIMULINK.Grand Valley State University Michigan 2009.