COMPARISON BETWEEN THE PURSUIT GUIDANCE AND THE PROPORTIONAL NAVIGATION GUIDANCE LAWS REGARDING A PREDETERMINED SCENARIO

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Abstract: *In tactical missile design, it is highly crucial to select the proper guidance method in order to meet the scenario requirements. As it is applicable in every aspect of life, trade-off conditions should be measured meticulously to reach the most advantageous and economical outcome. In this study, a particular scenario with a manoeuvring target is taken into consideration. Two of the guidance laws, namely, the pursuit guidance and the proportional navigation guidance are then implemented and compared. Lastly, the effect of the missile acceleration is examined within the proportional navigation guidance to consolidate the study.*

Keywords: MISSILE GUIDANCE, PURSUIT GUIDANCE, PROPORTIONAL NAVIGATION GUIDANCE, TACTICAL MISSILE DESIGN

1. Introduction

The concept of Tactical Missile Design (TMD) is in the special interest of the Defence Industries, therefore, it should come as no surprise that the main objective is to accomplish this task with the most efficient way as possible in order to reduce the cost and development time. This design process requires a tedious analysis of various considerations, such as the scenario that is expected to be met, development and production phases of the missile, and its economical aspect to the customer.

Generally speaking, guidance laws are separated into 3 different categories. These are line-of-sight guidance (LOSG), pursuit guidance (PG), and finally proportional navigation guidance (PNG) [1] . Each of these guidance laws have trade-offs. Electronic systems of the LOS guidance missiles is relatively simple. However, LOS guidance requires commands from a control station, and therefore, susceptible to jamming. Seekers used in the pursuit guidance missiles are quite simple and durable against the noise. But their efficiency against fast moving targets is not as high as the proportional guidance missiles [2].

This study focuses on the two major guidance methods of the tactical missiles, pursuit guidance (PG) and proportional navigation guidance (PNG). In the forthcoming sections, these guidance laws are examined closely within a certain scenario, and their suitable utilizations are presented in the results and discussions section. Also, there is another section in which the impact of the acceleration of the missile is analysed in proportional navigation guidance method.

2. Prerequisites and Definition of the Scenario

Even though there are various specifications that need to be handled in designing a tactical missile, the very first step that should be considered is the threat analysis of the target. Professor Lindsley claims that experience has proven one missile cannot be designated for all types of threats without seriously compromising performance of effectiveness [3]. For this reason, PG and PNG will be performed within the same scenario, and they will be compared. For the rest of the study, subscript "M" will refer to the missile, and subscript "T" will refer to our target.

Missile that is going to be operated must meet certain functions to complete its mission. It should detect the threat, and seek it. This is achieved by the various optical instrumentations that missile includes, which will detect the target, and acquire the angular location of the target relative to a fixed system. The acquisition process provides us the radial distance between the missile and the target, r, and the angle that velocity of the target makes with the line-of-sight of the missile, α_T. Angle ϕ which is the angle between the line-of-sight and the horizontal line and angle α_M which is the angle between the velocity of the missile and the line-of-sight are determined according to the angle α_T and to some pre-established criteria. Angles θ_T and θ_M, which are the angles between the velocity vectors and the horizontal line, can be determined from *Fig. 1* and *Fig. 2*. For our case, the following values are selected:

- $\phi = 1.354\, rad$
- $\alpha_T = 0.750\, rad$
- $\alpha_M = 0.848\, rad$ for PNG and $\alpha_M = 0$ for PG.
- $r = 2000\, meters$
- $V_T = 300\, m/s$
- $V_M = 600\, m/s$
- Navigation ratio $k = 4$ is for PGN and $k = 1$ is for PG.

In these guidance methods, we need only one additional parameter for the sake of the simplicity of calculations and derivations. For convenience $\dot{\theta}_T$, rate of change of the angle between the target and the horizontal line, is set to value 3.2981×10^{-1} rad/s.

3. Additional Scenario for the Effect of Missile Acceleration in PNG

Apart from the first scenario that is defined in the last section, the scenario that is going to be presented here will reinforce our previous understandings. Every value defined in the section 2 is also valid here, except the missile velocity. For the first case, the missile velocity starts with $400\, m/s$ and it will reach to $600\, m/s$ at the end of the propagation. The second case is the exact opposite of the first one, missile velocity decreases from $600\, m/s$ to $400\, m/s$ over the course. After the brief introduction of PG and PNG laws, results and discussions will take place.

4. Pursuit Guidance (PG) Law

In pursuit guidance, missile velocity vector is always directed towards the target, and the rate of turn of the missile is always equal to the rate of turn of the line-of-sight. As a result, missile constantly moves along the line-of-sight from the missile to the target whose path describes a total pursuit [4]. This is often regarded as tail-chase situation. From the *Fig. 1*, missile and target relationship can be visualized with the corresponding angles. Apart from the translating coordinate system, there is assumed to be an external inertial coordinate system. Leaving aside further

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comments and discussions, PG kinematics will be briefly introduced.

It is evident from the basic kinematic relationships, the velocity of the target with respect to the missile is given by [3],

(1) $$\vec{V}_{T/M} = \vec{V}_T - \vec{V}_M$$

Writing absolute velocities of the target and the missile as well as the relative velocity,

(2) $$\vec{V}_T = V_T \cos \alpha_T \bar{e}_r - V_T \sin \alpha_T \bar{e}_\theta$$

(3) $$V_M = V_M \cos \alpha_M \bar{e}_r - V_M \sin \alpha_M \bar{e}_\theta$$

(4) $$\vec{V}_{T/M} = \dot{r}\bar{e}_r + \mathrm{r}\dot{\theta}\bar{e}_\theta$$

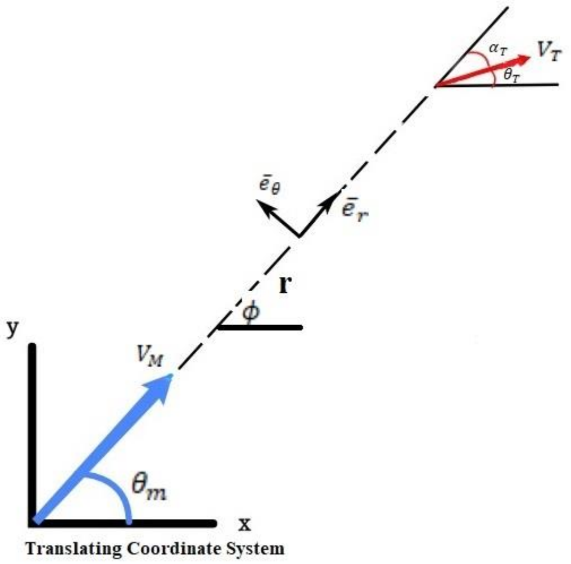

Fig. 1: *Pursuit Guidance Kinematics*

Inserting equations (2), (3) and (4) into the equation (1), we get our fundamental relation and the following terms [3],

(5) $$\dot{r} = V_T \cos \alpha_T - V_M \cos \alpha_M$$

(6) $$\mathrm{r}\dot{\phi} = -V_T \cos \alpha_T + V_M \sin \alpha_M$$

These equations are general and can also be applied to PNG law. It can be readily seen that in PG, ϕ angle is equal to the θ_M and $\alpha_T + \theta_T$ separately. In this case, second term of the right-hand side of the equation (3) vanishes, since there is no α angle for missile. Below relations are also useful ones in the pursuit guidance law,

(7) $$\theta_M = \phi \text{ and } \dot{\theta}_M = \dot{\phi}$$

Also,

(8) $$\theta_M = \alpha_T + \theta_T \text{ and } \dot{\theta}_M = \dot{\alpha}_T + \dot{\theta}_T$$

Since $\dot{\alpha}_M = 0$ for PG, only time derivations of values $\dot{r}$, $\dot{\theta}_M$ and $\dot{\theta}_T$ are required. Using equations (5), (6), (7) and (8), these parameters can be achieved. With these time derivations, it is possible to find each angle at each time step, which will enable us to propagate the guidance method.

5. Proportional Navigation Guidance (PNG) Law

In Proportional Navigation Guidance method, the line-of-sight from the missile to the target does not changes direction as the distance between two objects closes [5]. In other words, if we were to connect all the positional points of the missile and the target at respective times, we would get a parallel set of line-of-sights over the course of action. Therefore, the rate of turn of the missile is proportional to the rate of turn of the line-of-sight from the missile to the target [6]. This proportionality is shown by the following relation,

(9) $$\dot{\theta}_M = \mathrm{k}\dot{\phi}$$

The multiple that is seen in the equation (9) is called navigation ratio, and it is typically between two and six. As distinct from equation (7), we have the following angular relations,

(10) $$\dot{\phi} = \frac{\dot{\theta}_M}{k} \text{ and } \dot{\theta}_M = \left(\frac{-k}{k-1}\right)\dot{\alpha}_M$$

It can be observed that PG can be attained by setting the ratio $k = 1$. Therefore, PNG is designed in such a way that other guidance methods can be achieved by changing this constant. Other than above equations, since equations (5) and (6) were general relations, they are also used in this method. Recalling those equations,

(11) $$\dot{r} = V_T \cos \alpha_T - V_M \cos \alpha_M$$

And,

(12) $$\dot{\phi} = [-V_T \cos \alpha_T + V_M \sin \alpha_M]/\mathrm{r}$$

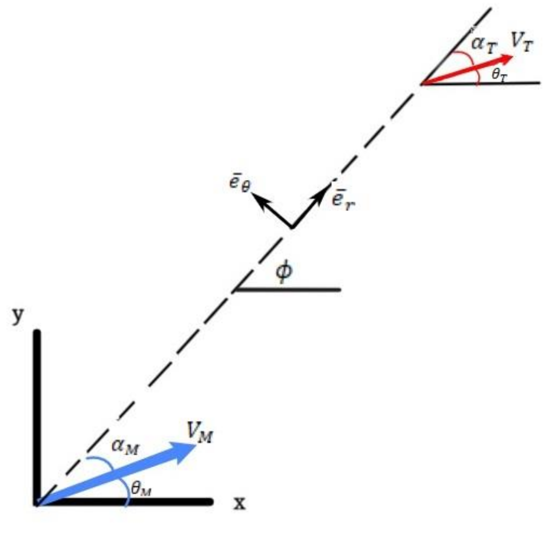

Fig. 2: *Proportional Navigation Guidance Kinematics*

Again, time derivations $\dot{r}$, $\dot{\phi}$, $\dot{\theta}_M$, $\dot{\theta}_T$, $\dot{\alpha}_M$ and $\dot{\alpha}_T$ are required. These parameters can be reached by utilizing equations (9), (10), (11) and (12). In the next section, these findings will be integrated into an algorithm, and that will be briefly introduced.

6. Euler's Method and Algorithm

For manoeuvring targets, the equations should be solved incrementally with time. Euler's method presents an adequate approximation for starters. Assume we have the following initial conditions, $r(0)$, $\alpha_T(0)$, $\alpha_M(0)$, $\phi(0)$, $\theta_T(0)$, and $\theta_M(0)$. Then the algorithm used is as follows [3]:

(13) $$r(t+\Delta t) = r(t) + \dot{r}(t)\Delta t$$

(14) $$\alpha_M(t+\Delta t) = \alpha_M(t) + \dot{\alpha}_M(t)\Delta t$$

(15) $$\phi(t+\Delta t) = \phi(t) + \dot{\phi}(t)\Delta t$$

(16) $$\theta_M(t+\Delta t) = \theta_M(t) + \dot{\theta}_M(t)\Delta t$$

(17) $$\theta_T(t+\Delta t) = \theta_T(t) + \dot{\theta}_T(t)\Delta t$$

Finally, we have the trajectory of the missile and the target as following:

(18) $$X_M(t+\Delta t) = X_M(t) + V_M \cos\theta_M(t)\,\Delta t$$

(19) $$Y_M(t+\Delta t) = Y_M(t) + V_M \sin\theta_M(t)\,\Delta t$$

(20) $$X_T(t+\Delta t) = X_T(t) + V_T \cos\theta_M(t)\,\Delta t$$

(21) $$Y_T(t+\Delta t) = Y_T(t) + V_T \sin\theta_M(t)\,\Delta t$$

Where the initial conditions are $X_M(0) = Y_M(0) = 0$, $X_T(0) = r_0 \cos\phi_0$, and $Y_T(0) = r_0 \sin\phi_0$. Since the algorithm have been provided with the relevant equations and the necessary information, scenario can be analysed.

7. *Results and Discussions*

The algorithm used in this study is implemented in the MatLab with the Euler's method, as mentioned in the preceding section. Entering the pre-determined values from the section 2, and setting the initial time and the time step to 0 and 0.1 seconds respectively will give us the following results and plots:

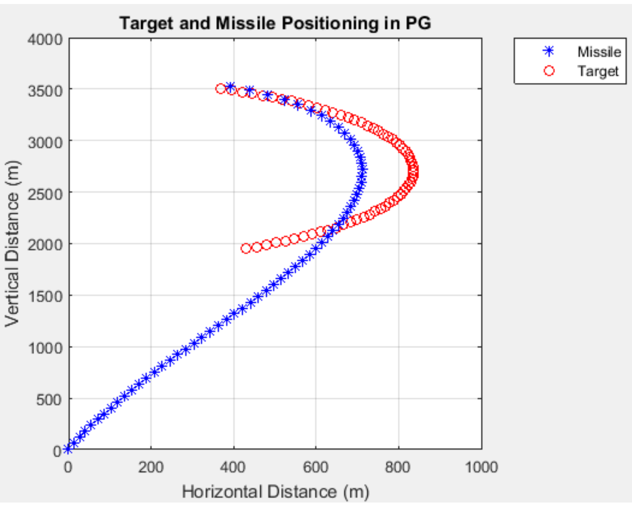

Fig. 3: *Positioning of target and missile in the given scenario (PG).*

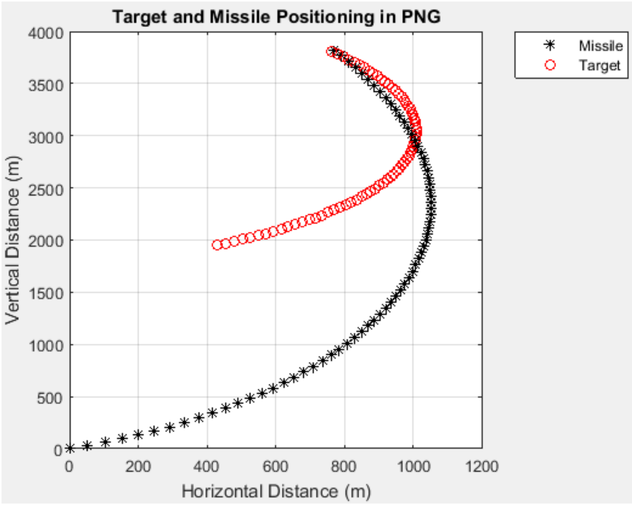

Fig. 4: *Positioning of target and missile in the given scenario (PNG).*

As it can be seen from the figures, even though the target movement is same, missiles have different attitudes in PG and PNG. Pursuit guidance paths are highly curved near the end of the propagation, therefore, missiles need to attain much higher accelerations. However, acceleration may reach to a point which a certain missile may not afford that specific manoeuvre. For example in Fig. 3, the maximum acceleration of the missile is calculated as 148.1 g, and this value may not be supported by the missile structure and electronics. For this reason, the most common application of the pursuit guidance is against slow moving targets. Also, since the target position is in special interest in PG, position determining depends on looking and pointing, therefore, avionics and control is simple relative to PNG [3]. This is called target heading, and it can be considered as inexpensive and light.

On the other hand, in Fig. 4, missile propagates on a less curved path, and does not tail-chase the target. Instead, it anticipates the targets future position and tries to cut its way at all times. Maximum acceleration of the missile is calculated to be 30.03 g which is also in the reasonable range. The lead angle of the missile in this procedure is regulated by the navigation ratio. Moreover, since the line-of-sight changes throughout the course of action, it has to be sensed by the missile avionics. Thus, avionics required in PNG is more complex and expensive than other guidance methods.

The following figures, namely Fig. 5 and Fig. 6, present the results for the scenario that is defined in the section 3. It is seen that there is a slight difference in the terminal phase (last phase) of both cases. It is also found that the maximum magnitude of the acceleration attained in accelerated case is 10.2 g, while the maximum magnitude of the deceleration case is 30.1 g.

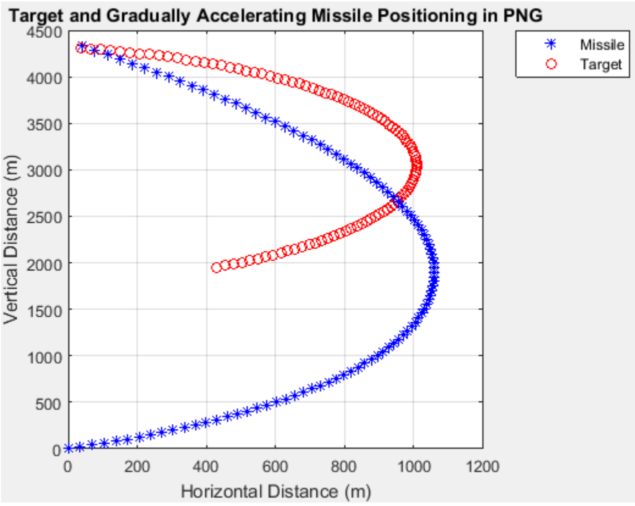

Fig. 5: Accelerated missile scenario in PNG.

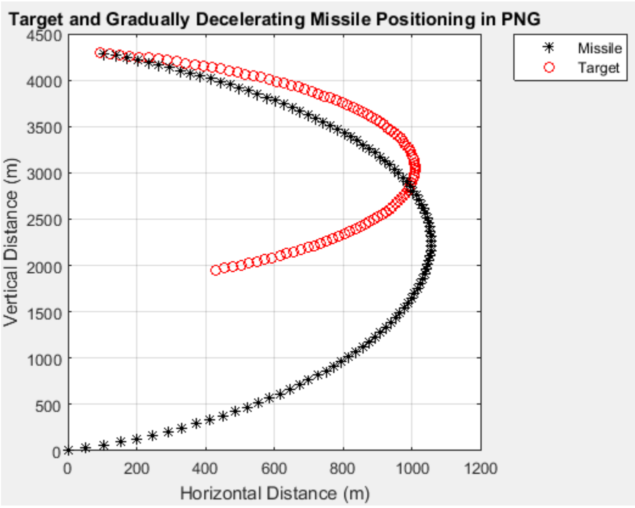

Fig. 6: Decelerated missile scenario in PNG.

As it is obvious from the figures, there is no significant visual distinction between accelerated and decelerated cases. However, this is due to small amounts of acceleration; as the acceleration increases in magnitude, the target is hit much earlier in the accelerated case with respect to the decelerated case. This is also illustrated in the Fig. 7.

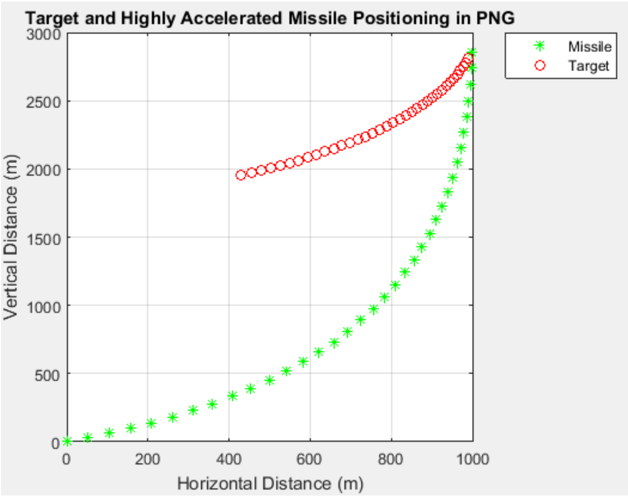

Fig. 7: Highly accelerated missile scenario in PNG.

In the case of the highly accelerated missile, magnitude of its maximum acceleration is 35.1 g. These specifications purely dependent on the scenario and on the mission that is meant to be met. Last, but not least, these attained maximum acceleration values can be deceptive; bear in mind that that this value is the resultant magnitude of the radial and the angular components of acceleration.

8. *Summary and Conclusion*

In this study, PG and PNG laws are compared for the constant speed missile first, then variable speed missile is examined within the PNG method. Besides from comparisons between PG and PNG in the previous section, missile acceleration affects arrival time and miss time. It can be observed that highly accelerated missile reaches the target faster than any of the situation that are mentioned before. For gradually accelerating and gradually decelerating cases, there is no obvious distinction, there is only small change in the missile behaviour.

In the light of above findings, it can be stated that PG performs poorly against accelerated targets, while PNG is more decisive. The situation is not different with target heading accuracy. However, complex avionics of the PNG generates much more noise relative to PG. Even though these methods are examined and compared in terms of superiority in this study, there are way better guidance methods utilizing the newest technologies. However, examination of PG and PNG is vital in terms of understanding the basics and the concept of tactical missile guidance.

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