THEORY OF CALCULATION FOR STRENGTH TRAY PLANTS FOR THE PRODUCTION OF HYDROPONIC GREEN FODDER

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Abstract: Production of hydroponic green fodder is an important task of agriculture, which in developed countries is given enough attention. Currently, the world’s developed a considerable number of various designs installations for the production of hydroponic products, but the main working body of these installations are trays in which the cultivation of hydroponic green fodder, as one of the most simple, but fairly effective devices for seeding, cultivation and the ready products. Due to the fact that each of the trays used for these purposes is a resilient structure which is under the influence of considerable forces and bending moments, to design it requires fairly simple, but fairly effective devices for seeding, cultivation and the ready products. Due to the fact that each of the trays used for these purposes is a resilient structure which is under the influence of considerable forces and bending moments, to design it requires fairly accurate preliminary and final strength calculations. The aim of this study is to develop guidelines on the calculation theory of plant trays strength for the production of hydroponic green fodder. The study used modeling techniques, higher mathematics, mechanics of materials and structures, in particular the theory of elasticity of plates and shells, as well as the methods of calculation and programming on a PC. The study built a mechanical model and the design of the tray scheme, defined analytical expressions to change the maximum and calculated moments in his dangerous sections and constructed diagrams of bending and torque. Further graphs of changes in the safety margin of the tray frame, depending on the area of seed tube parameters and from which it is made. These charts should be used for the calculation and control of the results. A new theory can be used in calculating the strength of similar containers, which are used in the mechanization of agricultural production.

Keywords: HYDROPONICS, PLANT, TRAY, THEORY, STRENGTH, SAFETY FACTOR, MOMENTS, DANGEROUS SECTION.

1. Introduction

Formulation of the problem. One of the main problems of agricultural production, is a year-round maintenance of its major branches - livestock and poultry green fodder. This can be achieved only on the basis of broad general use of modern technology and advanced mechanical equipment (mechanical units) for the production of hydroponic products, which is the hydroponic green forage. The main operating element is a mechanical hydroponic systems tray, which takes place the cultivation of green fodder. From the mechanical strength of the tray depends on the overall performance of the entire hydroponic setup. In this regard, the development of new theoretical foundations of modern calculation of trays for strength as elastic structures of complex configuration, is an actual scientific and technical problem of agricultural mechanization sector.

2. The results of studies and publications.

Features of the tray and, in general hydroponics installations are described in detail in [1-3]. Development of the theory of strength calculations trays should be carried out using the developed classical foundations of the theory of strength of plates and shells, adequately set out in [4, 5]. It is necessary to precisely apply the principles known in the literature for the most common specific designs of trays, which are widely used in installations for the production of hydroponic products produced by the majority of world companies.

A further improvement of both the plants for the production of hydroponic green fodder, as set out in [6-10], as well as their main working body - the tray, in which the process of growing hydroponic forage require the introduction to this theory of the relevant changes, which apparently will be the subject the following studies.

Fig. 1. Structural diagram of the tray for the production of tray hydroponic green fodder

Purpose of the study. To develop the main provisions of the theory of calculation on the strength of the plant trays for the production of hydroponic green fodder.

Research methods. To conduct the study used modeling techniques, mathematics, mechanics of materials and structures, in particular the theory of elasticity, as well as the methods of calculation programming on a PC.

Results of the study. We developed the design of the installation for the production of hydroponic forage provides use her flat swivel trays, section of which is shown in Fig. 1.

As seen from the circuit of Fig. 1, the tray is made in the form of a frame with the corresponding pipe size, which is rigidly attached sheet metal 1,25 mm thick, or metal mesh. Photographic trays sowing area 1,5 ... 3,0 m² can be produced not from the pipes, and parts of 40 × 40 mm.
Fig. 2. Mechanical model and design scheme of the tray: 

a - mechanical model of the tray; b – the design scheme of the tray

For the development of the theory of calculations on the trays strength of this design should be submitted at the beginning a mechanical model, based on which further take design scheme, under the influence of the workload (seeded and sprouting green fodder). Such mechanical model and design scheme shown in Fig. 2 (a, b), which represent the geometric dimensions given plate pivotally resting axle rigidly clamped with two U-shaped framework bearing the same parts are uniformly loaded with hinge supports on the path.

For further calculations will present a general theory, considering in particular the tray with a growth surface 2 m², which is cultivated 100 kg of green fodder. We first define the intensity of the load distribution for the tray area of the vegetation area (2 m²), which will be equal to:

\[ P = \frac{1000}{2 \cdot 0.5} = 500 \text{ N} \cdot \text{m}^{-2} \]  

(1)

where, \( P = 1000 \text{ N} \) - gravity of harvest; \( a \) and \( b \) - the geometric dimensions of the tray, which are respectively, \( a = 2.0 \text{ m} \) \( b = 0.5 \text{ m} \)

Kinematic boundary condition for this case will have the form:

\[ x = \begin{cases} 0, & W = 0, \frac{\partial W}{\partial x} \neq 0, \\ a, & \end{cases} \]

(2)

\[ y = \begin{cases} 0, & W = 0, \frac{\partial W}{\partial y} \neq 0, \\ b, & \end{cases} \]

where: \( W = W(x, y) \) the deflection function.

Static boundary conditions (no points) can be written as follows:

\[ x = \begin{cases} 0, & \frac{\partial^2 W}{\partial x^2} = 0, \frac{\partial^3 W}{\partial x \partial y^2} = 0, \\ a, & \end{cases} \]

\[ y = \begin{cases} 0, & \frac{\partial^2 W}{\partial y^2} = 0, \frac{\partial^3 W}{\partial y \partial x^2} = 0, \\ b, & \end{cases} \]

For the function of the deflections take an infinite number of such type:

\[ W = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}, \]  

(4)

where; \( m = 1, 2, 3, \ldots \)  
\( n = 1, 2, 3, \ldots \) \( A_{mn} \) - Unknown coefficients of an infinite series, (4).

The set thus satisfies static (3), kinematic (2) boundary conditions and the main bending plate equation [4, 5]:

\[ \frac{\partial^4 W}{\partial x^4} + 2 \frac{\partial^4 W}{\partial x \partial y^3} + \frac{\partial^4 W}{\partial y^4} = \frac{q}{D}, \]  

(5)

where - \( D = \frac{E h^3}{12 (1 - \mu^2)} \) - cylindrical rigidity of the plate;

\( E \) - Young's modulus; \( h \) - The thickness of the plate; \( \mu \) - Poisson's ratio.

Expanding the right-hand side of equation (5) in a series, we obtain:

\[ W = \frac{q}{D} \left( \frac{4}{\pi} \right)^2 \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{mn} \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}. \]  

(6)

Differentiating function deflections (4) in the existing argument, and substituting the values of the derivatives in (5) with (6), we obtain expressions for finding the deflection function coefficients:

\[ A_{mn} = \frac{16q}{\pi^6 D m n} \left( \frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^{3/2}. \]  

(7)

Thus obtained is a function of the deflection by rapidly double row.

To set values, \( a = 2.0 \text{ m}; \) \( b = 0.5 \text{ m} \) we find the first five coefficients in this series:

\[ A_{11}' = \frac{1}{1 \left( \frac{1}{2^2} + \frac{1}{0.5^2} \right)^2} = 0.05536; \]  

\[ A_{13}' = \frac{1}{1 \left( \frac{1}{2^2} + \frac{3^2}{0.5^2} \right)^2} = 0.00025; \]  

\[ A_{55}' = \frac{1}{1 \left( \frac{5^2}{2^2} + \frac{1}{0.5^2} \right)^2} = 2.10^{-5}; \]  

\[ A_{31}' = \frac{1}{3 \left( \frac{3^2}{2^2} + \frac{1}{0.5^2} \right)^2} = 0.00853; \]  

\[ A_{51}' = \frac{1}{5 \left( \frac{5^2}{2^2} + \frac{1}{0.5^2} \right)^2} = 0.00190. \]  

(3)

The desired function of troughs will have the following form:

\[ W(x, y) = 0.16qD \left[ 5.536 \frac{\pi x}{2} \frac{\pi y}{0.5} + 0.02537 \frac{3 \pi x}{2} \frac{3 \pi y}{0.5} + 2.10^{-5} \frac{\pi x}{2} \frac{5 \pi y}{0.5} + 0.853 \frac{3 \pi x}{2} \frac{\pi y}{0.5} + 0.19 \frac{5 \pi x}{2} \frac{\pi y}{0.5} \right]. \]  

(8)
Find the corresponding derivatives of the deflection function.

We have:

\[ \frac{\partial^2 W}{\partial x^2} = 0.16q \left( 1.384 \sin \frac{\pi x}{2} \sin \frac{\pi y}{2} + 20.063 \cos \frac{\pi x}{2} \sin \frac{5 \pi y}{2} + \frac{1}{5} \cdot 10^{-5} \cos \frac{\pi x}{2} \sin \frac{5 \pi y}{2} + 5 \cdot 10^{-5} \sin \frac{\pi x}{2} \sin \frac{5 \pi y}{2} \right) \]

\[ \frac{\partial^2 W}{\partial y^2} = 0.16q \left( 0.692 \cos \frac{\pi x}{2} \sin \frac{\pi y}{2} + 0.031 \cos \frac{\pi x}{2} \sin \frac{5 \pi y}{2} + 0.25 \cdot 10^{-5} \cos \frac{\pi x}{2} \sin \frac{5 \pi y}{2} + 0.25 \cdot 10^{-5} \sin \frac{\pi x}{2} \sin \frac{5 \pi y}{2} \right) \]

\[ \frac{\partial^2 W}{\partial y \partial x} = 0.16q \left( 2.768 \cos \frac{\pi x}{2} \sin \frac{\pi y}{2} + 0.0378 \cos \frac{\pi x}{2} \sin \frac{5 \pi y}{2} + 0.5 \cdot 10^{-5} \cos \frac{\pi x}{2} \sin \frac{5 \pi y}{2} + 0.5 \cdot 10^{-5} \sin \frac{\pi x}{2} \sin \frac{5 \pi y}{2} \right) \]

\[ \frac{\partial^2 W}{\partial y^3} = 0.16q \left( 3.84 \sin \frac{\pi x}{2} \cos \frac{\pi y}{2} \cos \frac{\pi y}{2} + 0.212 \sin \frac{\pi x}{2} \sin \frac{5 \pi y}{2} \cos \frac{\pi y}{2} \right) \]

Using the boundary conditions and neglecting high-order harmonic, load intensity distribution obtain the contour of the U-shaped frame, shown in Fig. 2a.

\[ q_a = 0.339 \sin \frac{\pi x}{2} \]

\[ q_b = 0.212 \sin \frac{\pi y}{2} \]

Perform the test results obtained. Static test:

\[ P_{st} = q \cdot ab = q \cdot 2 \cdot 0.5 = q \cdot 1 \text{ N}\cdot m^{-1} \]

\[ P = 2 \left( \int_a^b q_a \, dx + \int_b^c q_b \, dy \right) = \frac{4}{\pi} \left( 2 \cdot 0.339q + 0.5 \cdot 0.212q \right) = 0.9982q \]

\[ \eta_1 = \frac{P - P_{st}}{P_{st}} = 1 - 0.9982 = 100 = 0.178\% . \]

We carry out a check on the bending moment. We have:

\[ M_{b,st} = P_{st} \cdot 0.5b = q \cdot 0.5 \cdot 0.5 = 0.25q \]

\[ M_b = P_{st} \cdot 0.5b = \left( \int_a^b q_a \, dx + \int_b^c q_b \, dy \right) = 2 \cdot 0.339q \cdot 2 + 2 \cdot 0.212q \cdot 0.5 \cdot \frac{5}{\pi} = 0.2495q \]

\[ \eta_2 = \frac{M_{b,st} - M_b}{M_{b,st}} = \frac{0.25q - 0.2495q}{0.25q} \cdot 100 = 0.178\% \]

where \( \eta_1 \) and \( \eta_2 \) - the interest discrepancies.

Fig. 3. Diagrams for calculating the strength of the tray
Fig. 4. Diagrams $M_b$ and $M_{in}$ to calculate the strength of the tray

Since the percentage difference is small enough, the load distribution rate equal to:

$$q_a = 0.339 q \sin \frac{\pi x}{2},$$

$$q_b = 0.212 q \sin \frac{\pi y}{0.5}.$$  \hspace{1cm} (13)

Subsequently, the obtained values $q_a$ and $q_b$ accept for an external load acting on the U-shaped frame of Figure 2, as well. The frame is a flat system, i.e., it is three times statically indeterminate.

To determine the estimated torque in dangerous section construct diagrams of bending and twisting moments acting on the frame of Fig. 3, (a,b,c,d,e,f,g).

The value of the settlement point in the dangerous section will calculate based on this:

$$M_p = \sqrt{M_b^2 + M_{no}^2}.$$  \hspace{1cm} (14)

Substitute values of $M_b$ and $M_{no}$ diagrams of Fig. 3, in the expression (14) we obtain:

$$M_p = \sqrt{0.25q^2 + (0.053q)^2} = 0.26q, \text{ N-m}.$$  

Define the rated voltage. It will be:

$$\sigma_p = \frac{M_p}{W} = \frac{0.26 \cdot 500}{9.28 \cdot 10^{-7}} = 140, \text{ MPa},$$

where $W = 0.928 \cdot 10^{-7} \text{ m}^3 = 9.28 \cdot 10^{-7} \text{ m}^3$ - the moment of resistance of the pipe with an inner diameter of $d_{in} = 89 \text{ mm}$, wall thickness $h = 4$ and height $H = 50 \text{ mm}$ cut mm inner diameter.

We calculate the margin of safety $M_a$. It will be equal to:

$$M_a = \frac{\sigma_T}{\sigma_p} = \frac{240}{140} = 1.71,$$

where, $\sigma_T = 240$ MPa - yield strength steel.

Fig. 5 shows the design scheme and the bending moment diagram for the calculation of the strength – U shaped pipe. The intensity of the load distribution along the length is equal to:

$$q = 2q_a = 2 \cdot 0.339 q \sin \frac{\pi x}{2} = 0.678q \sin \frac{\pi x}{2}.$$  

The values calculated for the $U$ stress - shaped tube is given by:

$$G_p = M_{max} \frac{M_p}{W} = 0.678 \cdot 500 \cdot \frac{1.94 \cdot 10^{-6}}{1.94 \cdot 10^{-9}} = 125, \text{ MPa},$$

where $W = 1.94 \cdot 10^{-6} \text{ m}^3$ - moment of resistance – $U$ shaped pipe with an inner diameter of $d_{in} = 89 \text{ mm}$, wall thickness $h = 4$ and height $H = 50 \text{ mm}$ cut mm inner diameter.

Then the margin is equal to:

$$M_a = \frac{\sigma_T}{\sigma_p} = \frac{240}{125} = 1.82.$$

Since hydroponic installation tray operates in quasi-static mode, the values obtained safety margins for $P$ and – $U$ shaped pipes suggest that this design is quite efficient.

Fig. 5. Scheme of the bending moment diagram for the calculation of $U$-shaped tube strength

Fig. 6. Change the maximum and the calculated points in the dangerous section frames and - $U$ - shaped pipe, depending on the intensity of the load $I$ - $U$ - shaped tube $II$ - frame tray
The calculations on the PC for 18 trays with different geometrical parameters yielded total dependence of the calculated and the maximum points on the intensity of the load $q$, and in a certain way to link the results obtained with the areas considered trays. Fig. 6 shows graphs of the maximum and the calculated hazard moments and frame sections $U$ - shaped pipe, depending on the intensity of the load, and therefore the area of the tray. Knowledge of accounting and maximum values of moments and moments of resistance relevant to determine the stresses in the frame and $U$ - shaped pipe with subsequent calculation of safety margins.

**Fig. 7 and 8 are graphs change safety margin beyond the tray, depending on the area of seed tube and parameters. These schedules should be used for approximate calculations and control of the results, if the tray fits into the range of these parameters.**

3. **Conclusions:**

1. The theory fort calculations on the strength of plant trays for the production of hydroponic green fodder, based on the theory of strength is develop.

2. The analytical expressions to change the maximum and calculated moments in dangerous sections of the tray frame, which made it possible to construct the plots required for their practical application in the design of structures trays of various sizes and capacities is determined.

3. The graphs change the safety margin of the tray frame, depending on the area of seed and tube parameters from which it is made. These schedules should be used for approximate calculations and control of the results, if the tray fits into the range of these parameters.

4. The resulting new theory can be used in calculating the strength of similar containers, which are used in the means of mechanization of agricultural production.

**Bibliography:**


