

# OPTIMAL POLICY FOR USING A RENTED FARMING MACHINE

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**Abstract:** This research analyzes the optimal harvesting policy and the optimal timing for ordering a rented self-propelled blueberry harvester for maximizing the total expected profit. The paper proposes a stochastic model that takes into account two types of uncertainties: 1) the exact date when the crop will be ready for harvesting (fruition) is unknown; 2) the exact quantity of the crop in the field is unknown. Another factor that is taken into account is the decreasing price of the crop during the harvesting season.

**Keywords:** AGRICULTURE MACHINERY, SELF-PROPELLED BLUEBERRY HARVESTER, OPTIMAL HARVESTING POLICY.

## 1. Introduction

The mechanized agriculture allows farmers to attain high yield of crops with few workers who operate sophisticated farming machines. Unfortunately, most of the farming machines are single purpose machines. A farmer that purchases an expensive farming machine uses it only when the crops need the special task that the machine is capable to do. This means that the machine works only several weeks every year/season. Due to the high cost of owning a single purpose farming machine (purchasing and maintenance), small and medium farms rent machines (with their operational teams) for the period of time the machines are needed. The renting company requires from the farmer to determine in advance the exact date he/she will need the machine. At the time when the farmer must set the due date for the arrival of a harvesting machine, he/she faces two types of uncertainties: 1) the exact date when the crop will be ready for harvesting (fruition) is unknown; 2) the exact quantity of the crop in the field is unknown. If the machine would arrive before the crop fruition, the farmer would pay extra rental fee until the work would begin. If the machine would arrive after the crop fruition, the farmer would have the following three possibilities: 1) wait for the machine arrival; 2) use manual harvest until the machine arrival (slow and expensive in comparison to the machine harvest); 3) mix policy (part of time manual harvest and part of time wait for the machine). Another factor that must be taken into account is the price of the crop. At the beginning of the harvest season the selling price is high and then it gradually decreases. This research concentrates in blueberry harvesting.

Blueberry consumption in the U.S. increased threefold from 2000 to 2010, followed by a fourfold increase in acreage plantings in the Southeastern region (Perez et al. [1]). During this period, machine harvesting technology also improved (Rodgers et al. [2]). Fresh market blueberry harvesting in the Southeastern U.S. is generally characterized by high demand for the large workforces that are needed to hand-pick each ripened blueberry; this takes place over a six to eleven weeks ripening window. Labor costs are estimated to be about 60 percent of the average operating costs (Brown et al. [3]; Safley et al. [4]). Rodgers et al. [2] estimate that the price of a mechanical harvester ranges is between \$100,000 to \$200,000, depending on functionality, and that such a machine has a useful lifespan of approximately 20 years.

Assuming that the distribution functions of the two random variables (the fruition date and the quantity of the crop) and the function of selling price during the harvest time are known, the research analyzes the optimal harvesting policy and the optimal timing for ordering a rented self-propelled blueberry harvester for maximizing the total expected profit.

## 2. Notation

Let us introduce the following terms:

$T$  - is the time when the product (blueberry) is ready for harvesting (a positive random variable),  $T \in (0, A)$ ; that is, the earliest possible time for starting the harvest season is defined as zero and the latest possible time for starting the harvest season is defined as  $A$ , where  $A > 0$  is a known deterministic value.

$f_T(x)$  - is the known density function of the random variable  $T$ ,  $0 \leq x \leq A$ ;

$B$  - is a length of the harvest period (i.e., the time interval  $(T, T+B)$ ) and it assumed to have a known positive deterministic value;

$H$  - is the total amount of the product in the field which is measured in thousands of pounds (a positive random variable);

$f_H(x)$  - is the known density function of the random variable  $H$ ,  $x > 0$ ;

$c_1$  - is the booking cost to reserve the use of the machine (a self-propelled harvester) (a one-time payment that guarantees the availability of the machine as well as the option of returning the machine at any point of time),  $c_1 \geq 0$ ;

$c_2$  - is the machine rental cost per hour,  $c_2 > 0$ ; This includes the cost of the operational team of the machine.

$c(t)$  - is the known function of the selling price of the one thousand of pounds of the product (a continuous decreasing function defined on the range  $(0, A+B)$ );

$q_1$  - is the machine capacity per hour (the amount of the product, in thousands of pounds, that can be picked by the machine during one hour,  $q_1 > 0$ );

$q_2$  - is the manual capacity per hour (the amount of the product, in thousands of pounds, that can be picked manually by the existing workers during one hour,  $0 < q_2 < q_1$ );

$c_3$  - is the cost of harvesting one thousand pounds by harvesting machine,  $c_3 > 0$ ;

$c_4$  - is the cost of harvesting one thousand pounds by manually (without the assistance of the harvesting machine),  $c_4 > c_3 > 0$ ;

$t_b$  - is a point in time when the machine arrives at the farm and the rental payments begin to be incurred ( $t_b$  is a deterministic decision variable that should be determined by the farmer,  $0 \leq t_b < A$ );  
 $B(t_b, H, T)$  - is the farmer's profit (benefit) (a random variable).

### 3. An analysis of the optimal harvesting policy

This section presents the farmer's optimal working policy that maximizes his profit, given values of the random variables  $H$ ,  $T$  and the decision variable  $t_b$ . The assumption is that the farmer does not consider the option of not renting a harvesting machine because the available manual harvesting capacity he has is not enough to pick up all the expected amount of the product during the harvesting period. That is, the booking cost  $C_1$  must be taking into account for any harvesting policy. It is also assumed that for any  $0 \leq t \leq A + B$ , the cost function  $c(t) \geq c_4 > c_3$ .

If  $t_b \leq T$ , the optimal policy is to use the machine during the entire harvesting period. In this case the farmer's profit is  $B(t_b, H, T)$

$$B(t_b, H, T) = -c_1 - c_2(T - t_b) + q_1 \int_T^{\min\{T+B, T+H/q_1\}} (c(t) - c_3) dt. \tag{1}$$

If  $t_b > T$ , then the farmer can start with manual harvesting at time  $T$  and he can continue with the manual harvesting for the period  $(T, T + t_1)$ , where  $0 \leq t_1 \leq \min\{t_b - T, H/q_2, B\}$ . After that, the farmer stops manual harvesting and waits for the arrival of the machine. Finally, at time  $t_b$ , the farmer begins with the machine harvesting and he continues with it until all the available yield is harvested (or until the end of harvesting period). In this case, determining the farmer's optimal working policy is to determine an optimal value of  $t_1$  ( $t_1^* = t_1^*(H, T, t_b)$ ) that maximizes the profit. The farmer's profit is

$$B(t_b, H, T, t_1) = -c_1 + q_2 \int_T^{T+t_1} (c(t) - c_4) dt + q_1 \int_{\min\{t_b, T+B\}}^{\min\{T+B, t_b + \frac{H-q_2 t_1}{q_1}\}} (c(t) - c_3) dt, \tag{2}$$

where  $0 \leq t_1 \leq \min\{t_b - T, H/q_2, B\}$ . Thus, for this case,

$$t_1^* = t_1^*(t_b, H, T) = \arg \max_{0 \leq t_1 \leq \min\{t_b - T, H/q_2, B\}} B(t_b, H, T, t_1), \tag{3}$$

and the farmer's profit attained by this optimal policy is

$$B(t_b, H, T) = B(t_b, H, T, t_1^*) = \max_{0 \leq t_1 \leq \min\{t_b - T, H/q_2, B\}} B(t_b, H, T, t_1), \tag{4}$$

where  $B(t_b, H, T, t_1)$  is defined by Equation (2). The following straightforward proposition guarantees that if the selling price function  $c(t)$  is a differentiable function, then for all given values of  $H > 0$ ,  $0 \leq T \leq A$ ,  $B > 0$ ,  $0 \leq t_b < A$ ,  $0 < q_2 < q_1$ ,

$c_4 > c_3 > 0$ , there is a unique feasible value of  $t_1^*$  that satisfies Equation (3).

**Proposition.** Assume that the selling price function  $c(t)$  is a differentiable function. Then for all given values of  $H > 0$ ,  $0 \leq T \leq A$ ,  $B > 0$ ,  $0 \leq t_b < A$ ,  $0 < q_2 < q_1$ ,  $c_4 > c_3 > 0$ , Equation (3) has a unique solution  $t_1^* = t_1^*(t_b, H, T)$ . This solution is defined as follows:

1. If  $t_b \geq T + B$  then  $t_1^* = \min\{H/q_2, B\}$ .
2. If  $T < t_b < T + B$  and

$$QQ = QQ(H, T, t_b, q_1, q_2, B) = \frac{H - q_1(T + B - t_b)}{q_2}, \text{ then:}$$

2.1 If  $QQ > t_b - T$  then  $t_1^* = t_b - T$ .

2.2 If  $QQ < 0$  then:

2.2.1 If  $c(T) - c_4 \leq c\left(t_b + \frac{H}{q_1}\right) - c_3$  then  $t_1^* = 0$ ,

2.2.2 If  $c(T + \min\{t_b - T, H/q_2\}) - c_4 \geq c\left(t_b + \frac{H - q_2 \min\{t_b - T, H/q_2\}}{q_1}\right) - c_3$  then  $t_1^* = \min\{t_b - T, H/q_2\}$ ,

2.2.3 If  $c(T) - c_4 > c\left(t_b + \frac{H}{q_1}\right) - c_3$  and

$$c(T + \min\{t_b - T, H/q_2\}) - c_4 < c\left(t_b + \frac{H - q_2 \min\{t_b - T, H/q_2\}}{q_1}\right) - c_3 \text{ then } t_1^*$$

is the unique feasible solution of the equation

$$c(T + t_1) - c\left(t_b + \frac{H - q_2 t_1}{q_1}\right) = c_4 - c_3 \tag{5}$$

and can be found analytically or numerically, depending on the complexity of the selling price function  $c(t)$ .

2.3 If  $0 \leq QQ \leq t_b - T$  then

$$t_1^* = \begin{cases} QQ & \text{if } q_1 \int_T^{T+B} (c(t) - c_3) dt \geq q_2 \int_{T+QQ}^{T+t_1^{**}} (c(t) - c_4) dt, \\ & \text{where } t_1^{**} = t_b + \frac{H - q_2 QQ}{q_1} \\ t_1^{**} & \text{otherwise} \end{cases}$$

where  $t_1^{**}$  is defined by the following way:

if  $c(T + QQ) - c_4 \leq c\left(t_b + \frac{H - q_2 QQ}{q_1}\right) - c_3$  then  $t_1^{**} = QQ$ ,

if  $c(T + \min\{t_b - T, H/q_2\}) - c_4 \geq c\left(t_b + \frac{H - q_2 \min\{t_b - T, H/q_2\}}{q_1}\right) - c_3$  then  $t_1^{**} = \min\{t_b - T, H/q_2\}$ ,

if  $c(T + QQ) - c_4 > c\left(t_b + \frac{H - q_2 QQ}{q_1}\right) - c_3$  and

$$c(T + \min\{t_b - T, H / q_2\}) - c_4 < c\left(t_b + \frac{H - q_2 \min\{t_b - T, H / q_2\}}{q_1}\right) - c_3, \text{ then } t_1^{**}$$

is the unique feasible solution of Equation (5) and can be found analytically or numerically, depending on the complexity of the selling price  $c(t)$ .

The farmer's profit that incorporates his optimal policy is defined according to Equation (1) for  $t_b \leq T$ , and in according to Equation (2) with  $t_1 = t_1^*$  for  $t_b > T$ , where  $t_1^*$  is stated in the above proposition.

**4. An analysis of the optimal start time for the machine rental**

This section presents a procedure for determining the optimal value of the decision variable  $0 \leq t_b < A$  in order to maximize the farmer's expected profit,  $E(B(t_b, H, T))$ , assuming that the farmer behaves according to his optimal working policy. The aim is to find  $t_b^*$  such that

$$t_b^* = \arg \max_{0 \leq t_b < A} E(B(t_b, H, T)), \tag{6}$$

where  $B(t_b, H, T)$  is defined in Equations (1) and (4). Assuming that the random variables  $T$  and  $H$  are independent, the farmer's expected profit has the form

$$E(B(t_b, H, T)) = \int_0^A \int_0^{t_b} B(t_b, h, \tau) f_T(\tau) f_H(h) d\tau dh \tag{7}$$

$$= \int_0^A \int_0^{t_b} B(t_b, h, \tau) f_T(\tau) f_H(h) d\tau dh + \int_0^A \int_{t_b}^A B(t_b, h, \tau) f_T(\tau) f_H(h) d\tau dh,$$

where  $f_T(x)$  and  $f_H(x)$  are known density functions of the random variables  $T$  and  $H$ , respectively. Straightforwardly by Equations (1)-(4), (7)

$$E(B(t_b, H, T)) = \int_0^A \int_{t_b}^A \left( \begin{aligned} & \left( -c_1 - c_2(\tau - t_b) \right. \\ & \left. \min\{\tau + B, \tau + h/q_1\} \right. \\ & \left. + q_1 \int_{\tau}^{\min\{\tau + B, \tau + h/q_1\}} (c(t) - c_3) dt \right) \\ & \times f_T(\tau) f_H(h) \end{aligned} \right) d\tau dh \tag{8}$$

$$+ \int_0^{t_b} \int_0^A \left( \begin{aligned} & \left( -c_1 + q_2 \int_{\tau}^{\tau + t_1^*} (c(t) - c_4) dt \right. \\ & \left. \min\left\{ \tau + B, t_b + \frac{h - q_2 t_1^*}{q_1} \right\} \right. \\ & \left. + q_1 \int_{\min\{t_b, \tau + B\}}^{\min\left\{ \tau + B, t_b + \frac{h - q_2 t_1^*}{q_1} \right\}} (c(t) - c_3) dt \right) \\ & \times f_T(\tau) f_H(h) \end{aligned} \right) d\tau dh,$$

where  $t_1^*$  is stated by the proposition presented in section 3. Note that in any real life problem, the decision variable  $t_b$ ,  $0 \leq t_b \leq A$ , is practically not continuous and can be considered to have a finite number of values (e.g., the values of  $t_b$  are the integer numbers of hours after the earliest possible time for starting the harvest season). Therefore, the optimal value of  $t_b$  may be attained by examination of all its possible values. That is, the farmer's expected profit  $E(B(t_b, H, T))$  can be calculated (with reasonable accuracy in the context of a considered real problem) for all possible values of  $t_b$ . Then, the value of  $t_b$  that yields the farmer's highest expected profit is the needed solution  $t_b^*$  of Equation (6). The farmer's expected profit  $E(B(t_b, H, T))$  can be approximated by utilizing a numerical integration of the double integrals in equation (8) or it can be approximated by simulation.

**5. Conclusions**

This research proposed a stochastic newsvendor-like model to maximize the farmer's expected profit in the context of an unknown (stochastic) fruition date of blueberry crops and an unknown (stochastic) quantity of the yields in the field. Assuming that the two aforementioned random variables are independent, the results of this research enable the farmer to calculate the optimal due date for the arrival of a rented harvesting machine, and then, when the values of the random variables are revealed (fruition date and quantity), to select the optimal harvesting policy. The presented analysis can be adjusted for a situation where the two random variables, the fruition date and the quantity of the yields in the field are dependent random variables.

**6. References**

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