MATHEMATICAL MODEL OF INTERACTION OF THE FLEXIBLE CLEANING BLADE WITH ROOT CROP HEAD

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Abstract: *Modern technologies of harvesting sugar beet tops provide solid preliminary cutting and collecting its harvest at the increased altitude and subsequent additional root crops heads cleaning from tops residues on a root. This last operation also defines quality of root crops of beet before their digging up. Objective of this research is improvement quality of root crops heads cleaning from tops remains, by development of interaction theory of flexible cleaning blade with a spherical surface of root crop head. In the theoretical study were used methods of modeling, higher mathematics and theoretical mechanics, as well as programming and carrying out numerical calculations on the PC. As a result of the conducted theoretical research the interaction theory of flexible cleaning blade with a root crop head surface in the course of its cleaning from tops remains is developed. On the basis of the received differential equations of blade movement which is pivotally mounted on the drive shaft with a horizontal axis of rotation, new mathematical dependences which justify basic parameters of this technology process are given.*

KEY WORDS: *SUGAR BEET, BEET TOPS, HARVESTING, TOPS RESIDUES, CLEANING FLAIL, EQUIVALENT SCHEME, FORCE, VELOCITY, DIFFERENTIAL EQUATIONS.*

1. Introduction

Cleaning of the root crops heads from the remains of the tops is an important operation of the technological process of harvesting sugar beet. In [1], the theory of the blade cleaner of the beet root heads with a horizontal axis of rotation is considered, and the main analytical dependences describing the impact of the cleaning blade on the head of the root crop are presented.

The next stage of the investigation is the compilation of differential equations of motion of the point of contact of the blade on the head of the root crop in the process of combing the remains of the foliage from it, assuming that this process will occur in the longitudinally vertical plane, that is, when the plane of rotation of the blades is located along the row.

2. Results and discussion

We depict the force interaction of the blade with the root of the root crop during the main process of combing the remains of the leaves, i.e. When the blade moves along the head surface of the root crop (Fig. 1).

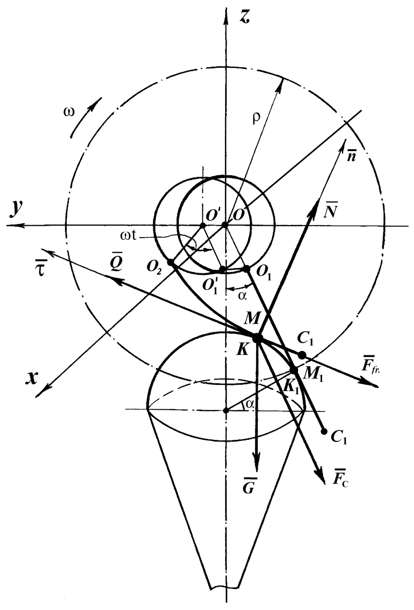

Fig. 1. – Scheme of force interaction of the blade with the root of the root in the process of combing the remains of the foliage

At the contact point K, the following forces will act: $\bar{F}_c$ – the centrifugal force of inertia, which is directed along the radius OK of rotation of the blade about the axis O; $\bar{G}$ – the weight of the blade, which is directed vertically downwards; $\bar{N}$ – normal reaction of the interaction of the blade with the root of the root, is directed along the normal to the head of the root crop, conducted through the given position of the point of contact; $\bar{F}_{fr.}$ – the frictional force that arises when the blade moves along the root of the root $\bar{n}$ is directed in the direction opposite to the direction of the absolute velocity M vector of the point of the blade, which coincides with the point of contact K; $\bar{Q}$ – the force of combing the remains of the leaves from the surface of the head of the root crop, which is directed towards the vector of the absolute velocity of the blade point M.

Let us find the value of these forces. To determine the centrifugal force $\bar{F}_c$ at any point of contact K, it is necessary to consider the kinematics of the motion of the blade O_1C_1 along the head of the root crop after impact contact at a point K_1. Since the impact occurs in a very short period of time, then at the moment of impact, the blade does not move through the head of the root crop. Therefore, the initial position of the blade on the head of the root crop after the impact can be considered as the position of the impact contact K_1.

For a more accurate study of the motion of the blade along the head of the root crop, it is necessary to compile the differential equations of motion of the point M along the head of the root crop, since in this investigation the forces that cause such motion are taken into account.

It should be noted that the main role in forming the combing force $\bar{Q}$ is played by the centrifugal force of inertia $\bar{F}_c$, the traction force $\bar{P}$ and the rotational moment of the blade $M_{ob.}$. It is thanks to the action of these forces that the blade is pressed to the root of the root crop and the deformation of the blade is bent. Indeed, immediately after impact, the centrifugal force of inertia is directed along the blade and tries to straighten the blade along the radius ρ.

If this force was absent, then under the action of the translational motion of the cleaner and the rotational motion of the blade around the axis O, the blade, when in contact with the root of the root, would simply deflect to some angle in the direction opposite to the rotational motion, and without any effort slip through the root of the root without changing its Rectilinear form, since the suspension O_1 is hinged.

After all, under the influence of centrifugal force $\overline{F}_c$, upon reaching the root of the root, the blade remains straightened along the radius ρ, and therefore, as a result of further translational and rotational movements, the blade slides over the root of the root, while experiencing a certain deformation of the bend, which creates a scratching force of the remains of the foliage.

The value of the centrifugal force of inertia $\overline{F}_c$ at the initial point of contact K_1 (point M_1) is:

$$F_{c1} = m\omega^2\rho, \tag{1}$$

where: m – mass of the blade.

Determine the centrifugal force $\overline{F}_c$ of the point M at any point of contact K between the blade and the root of the root. This strength will be equal to:

$$F_c = m\omega^2 \cdot O'K, \tag{2}$$

where: $O'K$ – distance from point K to point O'.

As can be seen from the circuit in Fig. 1, this distance is approximately equal to

$$O'K \approx OK_1 - K_1K + OO', \tag{3}$$

where: $OK_1 = \rho$.

Then, taking into account that $OO' = V_P \cdot t$ and $KK_1 \approx V_P \cdot t$ and (3), we obtain:

$$O'K \approx \rho. \tag{4}$$

Thus, the centrifugal force $\overline{F}_c$ of inertia at each point of contact K approximately remains constant in magnitude and direction and is equal to:

$$F_c \approx m\omega^2\rho. \tag{5}$$

In this case, the mass of the blade m is considered to be concentrated on the working part of the blade. The centrifugal force of inertia that arises from the rotation of the mass of the blade closer to the axis O_1 of suspension causes tension of the blade and is balanced by the reaction in the hinge O_1.

The deformation of the blade bending results from the pressing of the blade at the point of contact K by the forces of inertia $\overline{F}_c$ and the weight of the blade $\overline{G}$ under the action of the traction force $\overline{P}$ of the translational motion of the cleaner and the rotational moment of the blade $M_{ob.}$.

The force of the deformation of the bend will be equal to the force of combing $\overline{Q}$. After all, the force $\overline{P}$ of the translational motion of the cleaner and the rotational moment of the blade $M_{ob.}$ enter into the component of the force $\overline{Q}$, and therefore they are in Fig. 1 are not shown.

The frictional force is known to equal to:

$$F_{fr.} = fN, \tag{6}$$

where: f – coefficient of friction of the blade surface along the surface of the head of the root crop; N – normal reaction at the contact point K of the blade with the root of the root.

Thus, the differential equation of motion of the point of contact K over the root of the root in a vector form will have the following form:

$$m\overline{a} = \overline{F}_c + \overline{G} + \overline{N} + \overline{F}_{fr.} + \overline{Q}, \tag{7}$$

where: $\overline{a}$ – absolute acceleration of the movement of the point of contact K along the head of the root crop; m – mass of the blade.

Since in this case we have a plane system of forces that is located in the plane yOz, the differential equation of motion (7) reduces to a system of two second–order differential equations of the following form:

$$\left.\begin{aligned} m\ddot{y} &= F_{cy} + G_y + N_y + F_{fr.y} + Q_y, \\ m\ddot{z} &= F_{cz} + G_z + N_z + F_{fr.z} + Q_z, \end{aligned}\right\}, \tag{8}$$

where: F_{cy}, G_y, N_y, $F_{fr.y}$, Q_y – projections of the vectors of forces on the axis Oy, respectively F_{cz}, G_z, N_z, $F_{fr.z}$; Q_z – the projections of the vectors of the mentioned forces on the axis, respectively.

Taking into account the value of the projections of the force vectors that enter into the system of differential equations (8) and expressions (5) and (6), the mentioned system acquires the following form:

$$\left.\begin{aligned} m\ddot{y} &= -m\omega^2\rho\sin\alpha + N\cos\left(y,\hat{\overline{N}}\right) - fN\cos\left(\dot{y},\hat{\overline{V}}\right) + Q\cos\left(\dot{y},\hat{\overline{V}}\right), \\ m\ddot{z} &= -m\omega^2\rho\cos\alpha - mg + N\cos\left(z,\hat{\overline{N}}\right) - fN\cos\left(\dot{z},\hat{\overline{V}}\right) + Q\cos\left(\dot{z},\hat{\overline{V}}\right), \end{aligned}\right\}, \tag{9}$$

where: $\cos\left(y,\hat{\overline{N}}\right)$, $\cos\left(z,\hat{\overline{N}}\right)$ are the direction cosines of the force vector $\overline{N}$ to the axes Oy and Oz, respectively; $\cos\left(\dot{y},\hat{\overline{V}}\right)$, $\cos\left(\dot{z},\hat{\overline{V}}\right)$ – the direction cosines of the velocity vector of the contact point on the head of the root crop to the axes and, respectively; $\dot{y}$, $\dot{z}$ – the projection of the velocity vector $\overline{V}$ on the coordinate axis Oy and Oz, respectively.

From the source [2] it is known that these directing cosines are equal to:

$$\begin{aligned} &\cos\left(y,\hat{\overline{N}}\right) = \frac{\partial f}{\partial y}\frac{1}{\Delta f}; \quad \cos\left(z,\hat{\overline{N}}\right) = \frac{\partial f}{\partial z}\frac{1}{\Delta f}; \\ &\cos\left(\dot{y},\hat{\overline{V}}\right) = \frac{\dot{y}}{V}; \qquad \cos\left(\dot{z},\hat{\overline{V}}\right) = \frac{\dot{z}}{V}, \end{aligned} \tag{10}$$

where: $f(y,z) = 0$ is the coupling equation (the surface along which the material point moves); Δf – modulus of the function gradient $f(y,z)$; V – the modulus of the velocity vector of a point.

Since it was initially assumed that the head of the root crop has a spherical shape, the coupling equation is a sphere that has the following equation:

$$f(x, y, z) = x^2 + y^2 + z^2 - R^2 = 0, \tag{11}$$

where: R is the radius of the spherical head of the root crop.

For the plane yOz $x = 0$, and therefore the equation of the sphere (11) goes over into the equation of the circle:

$$f(y, z) = y^2 + z^2 - R^2 = 0. \tag{12}$$

According to the source [2], the modulus of the function gradient and the velocity modulus will be:

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2}, \tag{13}$$

$$V = \sqrt{\dot{y}^2 + \dot{z}^2}. \tag{14}$$

Substituting (10) into (9) and adding the coupling equation (12) to the system of differential equations (9), we obtain the following system of differential equations:

$$\left.\begin{aligned} m\ddot{y} &= -m\omega^2\rho\sin\alpha + \frac{N}{\Delta f}\frac{\partial f}{\partial y} - fN\frac{\dot{y}}{V} + Q\frac{\dot{y}}{V}, \\ m\ddot{z} &= -m\omega^2\rho\cos\alpha - mg + \frac{N}{\Delta f}\frac{\partial f}{\partial z} - fN\frac{\dot{z}}{V} + Q\frac{\dot{z}}{V}, \\ y^2 &+ z^2 - R^2 = 0. \end{aligned}\right\} \tag{15}$$

We calculate the partial derivatives and the gradient of the function that enter the system of equations (15). Will have:

$$\frac{\partial f}{\partial y} = 2y, \quad \frac{\partial f}{\partial z} = 2z. \tag{16}$$

Then, according to (13):

$$\Delta f = \sqrt{\left(2y\right)^2 + \left(2z\right)^2} = 2R . \qquad (17)$$

We substitute the expressions (16), (17) into (15). Then the system of differential equations (15) takes the following form:

$$\left.\begin{aligned} & m\ddot{y} = -m\omega^2\rho\sin\alpha + \frac{y}{R}N - fN\frac{\dot{y}}{V} + Q\frac{\dot{y}}{V}, \\ & m\ddot{z} = -m\omega^2\rho\cos\alpha - mg + \frac{z}{R}N - fN\frac{\dot{z}}{V} + Q\frac{\dot{z}}{V}, \\ & y^2 + z^2 - R^2 = 0. \end{aligned}\right\} \qquad (18)$$

The system of equations (18) is a system of three equations with three unknowns y, z and N. Therefore, it is definite and has a unique solution.

We eliminate the unknown quantities from the obtained system of equations (18) and, thus reducing the given system to one differential equation with one unknown function, $y(t)$. To do this, it is necessary to differentiate twice t according to the constraint equation (12). If we differentiate this equation once, then we get:

$$2y\dot{y} + 2z\dot{z} = 0 . \qquad (19)$$

From where we find:

$$y\dot{y} + z\dot{z} = 0 . \qquad (20)$$

If we differentiate equation (20), then we have:

$$y\ddot{y} + \dot{y}^2 + z\ddot{z} + \dot{z}^2 = 0 , \qquad (21)$$

or

$$\left(y\ddot{y} + z\ddot{z}\right) + \left(\dot{y}^2 + \dot{z}^2\right) = 0 . \qquad (22)$$

Since, $\dot{y}^2 + \dot{z}^2 = V^2$ we get:

$$V^2 = -\left(y\ddot{y} + z\ddot{z}\right). \qquad (23)$$

We multiply the first equation of system (18) by, the second on and add them term by term, we obtain:

$$\begin{aligned} & m\left(y\ddot{y} + z\ddot{z}\right) = -\left(m\omega^2\rho\, y\sin\alpha + m\omega^2\rho\, z\cos\alpha\right) - mgz + \\ & + \frac{N}{R}\left(y^2 + z^2\right) - f\frac{N}{V}\left(y\dot{y} + z\dot{z}\right) + \frac{Q}{V}\left(y\dot{y} + z\dot{z}\right). \end{aligned} \qquad (24)$$

Whence, taking into account expressions (20) and (23), we have:

$$-mV^2 = -m\omega^2\rho\left(y\sin\alpha + z\cos\alpha\right) - mgz + RN . \qquad (25)$$

From expression (25) we find a normal reaction. It equals:

$$N = \frac{1}{R}\left[m\omega^2\rho\left(y\sin\alpha + z\cos\alpha\right) + mgz - mV^2\right]. \qquad (26)$$

We make further transformations. From the expression (20) we obtain:

$$\dot{z} = -\frac{y\dot{y}}{z}, \qquad (27)$$

then

$$\dot{z}^2 = \frac{\left(y\dot{y}\right)^2}{z^2}, \qquad (28)$$

or

$$\dot{z}^2 = \frac{\left(y\dot{y}\right)^2}{R^2 - y^2}. \qquad (29)$$

Thus, for the magnitude of the speed of motion V, we can obtain the following expression:

$$V^2 = \dot{y}^2 + \dot{z}^2 = \dot{y}^2 + \frac{\left(y\dot{y}\right)^2}{R^2 - y^2}. \qquad (30)$$

Substituting expression (26) into the first equation of system (18), we obtain:

$$\begin{aligned} & m\ddot{y} = -m\omega^2\rho\sin\alpha + \left(\frac{y}{R} - f\frac{\dot{y}}{V}\right)\left[m\omega^2\rho\left(y\sin\alpha + z\cos\alpha\right) + \right. \\ & \left. + mgz - mV^2\right]\frac{1}{R} + Q\frac{\dot{y}}{V}. \end{aligned} \qquad (31)$$

Since, $z = \sqrt{R^2 - y^2}$ then, taking into account expression (30), finally:

$$\begin{aligned} & m\ddot{y} = -m\omega^2\rho\sin\alpha + \left(\frac{y}{R} - \frac{f\,\dot{y}\sqrt{R^2 - y^2}}{\sqrt{\left(R^2 - y^2\right)\dot{y}^2 + \left(y\dot{y}\right)^2}}\right)\times \\ & \times\left\{ m\omega^2\rho\left(y\sin\alpha + \sqrt{R^2 - y^2}\cdot\cos\alpha\right) + mg\sqrt{R^2 - y^2} - \right. \\ & \left. - \frac{m\left[\left(R^2 - y^2\right)\dot{y}^2 + \left(y\dot{y}\right)^2\right]}{R^2 - y^2}\right\}\frac{1}{R} + Q\frac{\dot{y}\sqrt{R^2 - y^2}}{\sqrt{\left(R^2 - y^2\right)\dot{y}^2 + \left(y\dot{y}\right)^2}}. \end{aligned} \qquad (32)$$

3. Conclusions

Thus, we have obtained a second–order differential equation (32), in which only one function y is known, i.e. a differential equation is obtained in the so–called normal form, when the higher derivative is expressed in terms of the lower derivatives and the desired function.

The unknown force Q that enters into equation (32) must be found from the deformation conditions of the blade bending.

Therefore, to solve this equation, you must first find the force Q, or express it through known quantities.

Since equation (32) is non–linear, it can be solved only by numerical methods on a personal computer with given initial conditions, which will be the subject of the next study.

4. References

1. Bulgakov V.M., Holovach I.V. Utochnena teoriia ochystky holovok koreneplodiv na koreni hnuchkoiu lopattiu (in Ukrainian language) // Mekhanizatsiia s.-h. vyrobnytstva: Zb. nauk. pr. Nats. ahrar. un-tu.– 2002. – Vol. XIII. – p. 11-42.
2. Vasilenko P.M. Vvedenie v zemledelcheskuju mekhaniku (in Russian language). – Kiev: Selkhozobrazovanie, 1996. – 252 p.
3. Vasilenko P.M. Vasilenko V.P. Metodika postroenija raschotnykh modelej funkcionirovanija mekhanicheskih system (mashin i mashinnykh agregatov) . Uchebnoe posobie (in Russian language). – Kiev: USKhA, 1980. – 137 p.
4. Pogorelyj L.V. Sveklouborochnie mashiny (konstruirovanie i raschot) // L.V. Pogorelyj, N.V. Tatjanko, V.V. Brej i dr.; pod obshch. red. L.V. Pogorelogo (in Russian language). – Kiev: Tekhnika, 1983. – 168 p.
5. Pogorelyj L.V., Tatjanko N.V. Sveklouborochnye mashiny: istorija, konstrukcija, teorija, prognoz (in Russian language). – Kiev: Feniks:, 2004. – 232 p.
6. Bulgakov V.M. Sveklouborochnye mashiny. Monografija (in Ukrainian language). – Kiev: Agrarna nauka, 2011. – 351 p.
7. Bulgakov V.M. Teorija sveklouborochnykh mashin. Monografija (in Ukrainian language). – Kiev: Izdatelskij centr Nacionalnogo agrarnogo universiteta, 2005. – 245 p.
8. Butenin N.V., Lunc Ja.L., Merkin D.R. Kurs teoreticheskoj mekhaniky. Tom II. Dynamika (in Russian language). – Moskva: Nauka, 1985. – 496 p.
9. Bulgakov V., Adamchuk V., Nozdrovicky L., Boris M., Ihnatiev Ye. Properties of the sugar beet tops during the harvest // Proceeding of 6th International Conference on Trends in Agricultural Engineering 2016. – 7-9 September 2016. – Prague, Czech Republic. p.p. 102-108.
10. Bulgakov V., Adamchuk V., Ihnatiev Y. Teoriia ochyshchennia holovok koreneplodiv buriakiv elastychnoiu ochysnoiu lopattiu // Visnyk ahrarnoi nauky (in Ukrainian language). – 2016. – №11. – p. 53-59.
11. Bulgakov V., Adamchuk V., Nozdrovicky L., Boris M., Ihnatiev Y. Properties of the sugar beet tops during the harvest // Proceeding of 6th International Conference on Trends in Agricultural Engineering 2016. – 7-9 September 2016. – Prague, Czech Republic. p.p. 102-108.