SOIL CHARACTERIZATION BY THE CAPILLARY EFFECT

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Abstract: Granular materials are a collection of distinct macroscopic particles. Consequently soils, which are used for agricultural purposes are a part of them. In comparison with other simulation models, creating a numerical simulation with the discrete element method (DEM) for different soils prove to be difficult. The reason for this is that there are an enormous number of particles. Therefore it is not possible to reproduce every one of them. On this account, parcels are used in the numerical simulation. These are larger and also have different physical properties. However, also for the modelling of parcels characteristic variables are necessary. This variable should be determined by using the capillary effect.

Keywords: GRANULAT MATERIALS, DISCRETE ELEMENT METHOD (DEM), PARCELS, CAPILLARY EFFECT

1. Introduction

Basically, there are some engineering classification systems for soils. A very popular method is the classification by the grain size. This kind of soil classification is widely used in geotechnical engineering and exactly described in numerous standards. Essentially, an equivalent diameter is calculated, which allows a categorization of different soils. This characteristic soil value cannot be used immediately in the numerical simulation. The reason for this is that the Discrete Element Method (DEM) uses parcels whose properties are completely different from reality. Generally, a numerical simulation by DEM is based on an experimental approach, in which the available simulation parameters are varied. This is done until the behavior of the numerically calculated soil is in accordance with reality[3].

Fig. 1 Numerical simulation of soil cultivation with parcels [DEM Simulation by Pöttinger Landtechnik GmbH]

Consequently, such a procedure requires lots of experiments. With the aim of simplifying the determination of suitable simulation parameters, the characterization of soils should be realized by the capillary effect. This effect is defined as the movement of water within the spaces of a porous material due to the forces of adhesion, cohesion and surface tension. Due to the fact that the height and filling speed of water in a capillary depend on the pore system of a soil, this effect is used for the determination of characteristic variables. The characterization itself is carried out by conducting numerous capillary tests for various soils. Furthermore, the in physics used Washburn equation, which is only numerically solvable for soils are used. Using the dimensional analysis, an algorithm is developed for this equation, which enables the calculation of a characteristic soil value.

On the basis of this kind of characterization, in the near future this characteristic soil variables should enable a prediction of the traction requirement, power consumption and wear behavior of agricultural machines during the development process.

2. Determination of characteristic soil variables

2.1 Washburn’s equation

Washburn’s equation describes the movement of a liquid in a capillary tube. In this case, the capillary tube is a variety of soil samples. With the knowledge that the capillary rise of the liquid depends on the soil, this effect is used to determine characteristic variables. Assuming a laminar stationary flow, the volumetric flow through a capillary tube is determined by the law of Hagen-Poiseuille[2],

\[
\frac{dV}{dt} = \frac{r^4 \Delta P \pi}{8 \eta h}
\]

Where \(dV/dt\) is the volumetric flow rate, \(r\) – capillary radius, \(\Delta P\) – pressure difference between the two ends, \(\eta\) – dynamic viscosity of the liquid, \(h\) – height of the capillary tube.

The relationship between the volume of the liquid \(dV\) and the capillary height \(dh\) is given by[2]:

\[
dV = r^2 \pi dh
\]

The pressure difference \(\Delta P\) between the two ends of the capillary tube can be determined by using the capillary \(P_k\) and hydrostatic pressure \(P_h^2\):

\[
P_k = \frac{2 \gamma}{r} \cos \theta
\]

\[
P_h = \rho_{g} g h
\]

\[
\Delta P = P_k - P_h
\]

\[
\Delta P = \frac{2 \gamma}{r} \cos \theta - \rho_{g} g h
\]

Where \(\gamma\) is the liquid surface tension, \(\theta\) – contact angle, \(\rho\) – density of liquid, \(g\) – acceleration due to gravity.

By inserting \(\Delta P\) and \(dV\) in the law of Hagen-Poiseuille (1) a first-order differential equation is obtained. With this differential equation the capillary height can be determined numerically at any time [2].

\[
\frac{dh}{dt} = \frac{r^2}{8 \eta h} \left(\frac{2 \gamma}{r} \cos \theta - \rho_{g} g h\right)
\]

Now, in this case \(r\) represents an equivalent grain radius. This means that the capillary effect of a porous soil is compared with the capillary effect of a geometrically exact capillary. In view of the fact that an exact measurement of the capillary height in a porous soil is hardly possible, a relationship between the capillary height and the absorbed amount of water in the porous soil is established.

\[
m(t) = \varepsilon \ h(t)
\]

The factor \(\varepsilon\) represents a scaling parameter, which has the unit kg/m and can be calculated on the basis of the measured water
absorption and capillary height. Now the first order differential equation can be written in the following way.

\[ \frac{dm}{dt} = \frac{r^2 \varepsilon^2}{8 \eta \gamma} \left( \frac{2}{r} \cos \theta - \rho \gamma_0 \frac{m}{\varepsilon} \right) \]

With the aim of obtaining dimensionless characteristic variables for any soil, a dimensional analysis is carried out for the differential equation. The capillary effect accordingly depends on following physical parameters.

**Table 1:** Physical Properties of the used liquid

<table>
<thead>
<tr>
<th>Influencing variables</th>
<th>( r )</th>
<th>( g )</th>
<th>( \rho )</th>
<th>( \gamma )</th>
<th>( \varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical unit</td>
<td>m</td>
<td>m/s²</td>
<td>m/s</td>
<td>kg/m³</td>
<td>kg/m²</td>
</tr>
</tbody>
</table>

At first a dimensionless state variable \( M \) is introduced:

\[ M = \frac{m \gamma}{\varepsilon \gamma_0} \]

Furthermore, also a dimensionless time variable \( \tau \) is needed:

\[ \tau = \frac{t \gamma}{\eta \varepsilon} \]

After forming the dimensionless derivative

\[ \dot{m} = M \frac{\gamma^2}{\eta \varepsilon} \]

the differential equation (9) can be written as follows:

\[ \dot{M} = \pi_1 \left( \frac{\pi_2 \gamma}{M} - \pi_3 \right) \]

To obtain a simplified form of the dimensionless differential equation (13), the following characteristic variables are introduced:

\[ \pi_1 = \frac{\varepsilon g}{\gamma} \]
\[ \pi_2 = \frac{g \varepsilon \cos \theta}{4 \gamma} \]
\[ \pi_3 = \frac{g r^2 \rho}{8 \gamma \gamma_0} \]

Finally, following form is obtained for the differential equation:

\[ \dot{M} = \pi_1 \left( \frac{\pi_2 \gamma}{M} - \pi_3 \right) \]

Considering the dimensionless differential equation it becomes apparent that the two characteristic variables \( \pi_2 \) and \( \pi_3 \) cannot be determined because the equivalent grain radius \( r \), the contact angle \( \theta \) and the scale factor \( \varepsilon \) is not possible.

### 3. Practical realization

#### 3.1 Experimental setup

For investigating the capillary effect, two different soil samples are considered. One soil is from the central region of Upper Austria and the other one from the eastern part of the Czech Republic. Furthermore, both soils are sieved with a 3 mm screen. The duration of the experiment is on week. During this period, the increasing weight of both soils is measured by using a force sensor. In addition, at the end of this period the capillary height \( h \) of the soil is measured.

**Table 2:** Physical Properties of both soils

<table>
<thead>
<tr>
<th>Designation</th>
<th>Soil Austria</th>
<th>Soil Czech Republic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filling height at the beginning ( h_0 ) [m]</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>Initial mass ( m_0 ) [kg]</td>
<td>9.00</td>
<td>7.20</td>
</tr>
</tbody>
</table>

In order not to falsify the result of the measurement, the water level in the container is kept constant. This happened by an automated water supply. The liquid which is used for the experiment is water with following physical properties.

**Table 3:** Physical Properties of the used liquid

<table>
<thead>
<tr>
<th>Designation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface traction ( \gamma ) [N/m]</td>
<td>72.75 \times 10^3</td>
</tr>
<tr>
<td>Dynamic viscosity ( \eta ) [Ns/m²]</td>
<td>1.002 \times 10^3</td>
</tr>
<tr>
<td>Density ( \rho ) [kg/m³]</td>
<td>1000</td>
</tr>
</tbody>
</table>

The big disadvantage of this expression is that a recalculation of the equivalent grain radius \( r \), the contact angle \( \theta \) and the scale factor \( \varepsilon \) is not possible.

The experiment itself, which is shown in the following figure, is started simultaneously for both soils.
3.2 Experimental results

As already mentioned the absorbed amount of water in the porous soils is continuously measured. The following figure illustrates this.

![Absorbed amount of water](image)

**Fig. 3** Absorbed amount of water

At the beginning both dried soils have the same filling height $h_0$, but a different initial mass $m_0$ and therefore also a different dry density. The soils sample from Austria absorbs an amount of water of 0.8 kg. Compared to the Czech soil sample, the soil from Austria absorbs more than twice as much. Interestingly, the reached capillary height of the Austrian soil is 0.08 m in comparison to the capillary height of the Czech soil. The measurement results after one week are shown in Table 4. This impact can be attributed to a strong dependence of the capillary effect on the pore system of the soils.

**Table 4: Measured characteristics of both soils**

<table>
<thead>
<tr>
<th>Designation</th>
<th>Soil Austria</th>
<th>Soil Czech Republic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absorbed water</td>
<td>0.80</td>
<td>0.33</td>
</tr>
<tr>
<td>capillary height</td>
<td>0.38</td>
<td>0.30</td>
</tr>
</tbody>
</table>

4. Determination of characteristic soil variables with Mathematica

As already mentioned the characteristic variables $\pi_1$, $\pi_2$, and $\pi_3$ cannot be determined due to the unknown contact angle $\theta$, the equivalent grain radius $r$ or the scale factor $\varepsilon$. Thus, solving the differential equation numerically is not possible. Therefore in Mathematica an algorithm is programmed which enables an empirically approximation of the differential equation to the measurement results. First of all, each individual measuring point has to be dimensionless. This is done by plug in into the two equations 10 and 11. The occurring grain radius in both equations is replaced by equation 24. Then the three characteristic variables $\pi_1$, $\pi_2$ and $\pi_3$ are varied with the sliders as long as the results of the differential equation coincide with the measurement. The following figure illustrates this procedure for the soil sample from the Czech Republic.

![Numerical solution of M(t)](image)

**Fig. 4** Numerical solution of $M(t)$

This results in the following characteristic variables for the two soil samples.

**Table 5: Characteristic variables of both soils**

<table>
<thead>
<tr>
<th>Designation</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>$\pi_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil Austria</td>
<td>38.5</td>
<td>5.14</td>
<td>$1.22286 \times 10^{-7}$</td>
</tr>
<tr>
<td>Soil Czech Republic</td>
<td>27.5</td>
<td>1.34</td>
<td>$3.961 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

Furthermore, it is obvious that the water absorption tends to a ground-dependent maximum value. This value for each soil is obtained as follows:

\[
0 = M = \pi_1 \left( \frac{\pi_2}{M} - \pi_3 \right)
\]

\[
M_{\text{Max}} = \frac{\pi_2}{\pi_3}
\]

This results in the following values for the maximum water absorption of both soil samples. However, $M$ is still a dimensionless Value. In order to obtain a unit-related value, equation 10 must be transformed.

\[
m_{\text{Max}} = \frac{M_{\text{Max}} \gamma r}{g}
\]

The equivalent grain radius $r$ can be expressed after transforming the equation 16.

\[
r = \sqrt{\frac{8 \gamma \pi_3}{\rho g}}
\]

**Table 6: Maximum of water absorption**

<table>
<thead>
<tr>
<th>Designation</th>
<th>Soil Austria</th>
<th>Soil Czech Republic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum water absorption</td>
<td>42032612.07</td>
<td>33829840.95</td>
</tr>
<tr>
<td>$m_{\text{Max}}$ [kg]</td>
<td>0.84</td>
<td>0.38</td>
</tr>
</tbody>
</table>
The term \( \frac{\pi^2}{x_5} \) ultimately allows the determination of the maximum amount of water that can be absorbed by a soil sample.

\[
M_{\text{Max}} = \frac{\pi^2}{x_5} = \frac{g \varepsilon \cos \theta}{2 \gamma r^2 \rho} = \frac{2 \varepsilon \cos \theta}{\gamma r^2 \rho}
\]

Thus, the following applies for the maximum water absorption:

\[
m_{\text{Max}} = \frac{2 \varepsilon \cos \theta \gamma}{r g}
\]

A closer look at this term reveals that it the maximum amount of water that can be absorbed only depends on following physical quantities:

- Contact angle \( \theta \)
- Equivalent grain radius \( r \)
- Scale Factor \( \varepsilon \)
- Density of liquid \( \rho \)
- Liquid surface tension \( \gamma \)

The greater the radius, the smaller the maximum amount of water that can be absorbed. An even smaller contact angle causes the opposite. On the other hand, a large scaling factor leads to a larger amount of water, which can be taken up by the soil sample. Subsequently the scaling parameter \( \varepsilon \) is defined as suction capacity of a soil sample.

Although the two investigated soil samples are close to each other in terms of particle size distribution due to the sieving operation, they can be clearly distinguished from one another by the presented method.

5. Conclusion

The calibration of essential model parameters takes the central role in a numerical simulation by DEM. For this reason, suitable tests should be carried out on real soil samples and compared with corresponding simulations. A measurement of the interesting variables such as traction requirement or wear behavior is usually expensive. Therefore, a simpler method has to be used to allow certain soil samples to be assigned. This is possible by using the capillary effect. This allows the determination of characteristic variables, which allow a clear distinguish of both soil samples.

6. References
