1. Introduction

Modern energy-saving Technologies, that are widely implemented currently in agriculture include the use of combined units that can not only reduce the agronomic timing of field work, but also reduce water loss by reducing the inter-operational periods of time, to reduce the impact of sealing machine units on the ground, to save fuel-smazochnye materials and so on.

2. Background and means for solving the problem

We have developed and successfully tested in the field work of combined machine. It consisting of tractor and combined seed drill, to which is connected applicator fertilizer and seeder. Operation of such dynamic systems require high quality rectilinear motion during the manufacturing process and, consequently, the stability of its movement.

The study of complex combined agricultural machine units is possible on the basis of the constructed mathematical models for this, including mathematical models of plane-parallel motion. A method for constructing such machine units are well represented in the works [1-4]. In this case, the main type of movement of agricultural machine (trailed, mounted and self-propelled) is their plane-parallel motion, because this type of motion is determined by the quality of performing the specified processes. Study of combined agricultural machine units and work aggregates are determined to many of the published scientific papers in press [5-8, 10]. Numerous studies have found, that the agronomic and operational and technical performance of the combined tractor units depends largely on the nature of their plane-parallel motion. Therefore, the study of plane-parallel motion of various agricultural machine units as needed when evaluating existing and the design of innovative combinations of such units [4].

The solution of the problem

To study the complex plane-parallel motion of combined machine units will be needed to build their mathematical models, create differential equations of plane-parallel motion that will eventually allow them to find a rational design and kinematic parameters of the sustainability of movement of machine, and, consequently, the quality of the process.

Therefore, we first develop a power scheme trailer combined sowing unit, which consists of aggregated tractor 1, to which is attached the fertilizer distributing drill 2, which produces a stripe of mineral fertilizers, behind which, with the help of the hitch 3 attached grain seeder 4 (Fig. 1).

Fig. 1 – Power scheme trailer combined unit:
1 – tractor, 2 – fertilizer distributing seeder, 3 – hitch, 4 – grain seeder

\[
\begin{align*}
&\sum_{i=1}^{4} m_i\ddot{x}_i + \sum_{i=2}^{4} m_i\ddot{y}_i = \sum_{i=1}^{4} F_{ix}, \\
&\sum_{i=1}^{4} m_i\ddot{y}_i + \sum_{i=2}^{4} m_i\ddot{x}_i = \sum_{i=1}^{4} F_{iy}, \\
&I_1\ddot{\beta}_1 + (l_1 - a_1)\sum_{i=2}^{4} m_i\left(\ddot{x}_i \sin \beta_i - \ddot{y}_i \cos \beta_i\right) = \\
&= M_1 - M_{cl1} + (l_1 - a_1) \times \\
&\times \left[\sin \beta_1 \sum_{i=2}^{4} F_{ix} - \cos \beta_1 \sum_{i=2}^{4} F_{iy}\right], \quad (1) \\
&I_2\ddot{\beta}_2 + m_1 a_1 \left(\ddot{x}_1 \sin \beta_1 - \ddot{y}_1 \cos \beta_1\right) + \\
&+ l_1 \sum_{i=1}^{4} m_i \left(\ddot{x}_i \sin \beta_i - \ddot{y}_i \cos \beta_i\right) = \\
&= M_2 - M_{cl2} + l_1 \left[\sin \beta_1 \sum_{i=1}^{4} F_{ix} - \\
&- \cos \beta_1 \sum_{i=1}^{4} F_{iy}\right], \quad (i = 2, 4).
\end{align*}
\]

In this case, the first three equations in (1) describes the linear motion of the center of mass of the tractor the first three equation, along the axis Ox and Oy and \( \dot{x}_i \) and \( \dot{y}_i \) and angular motion \( \beta_i \) around its center of mass. The fourth equation of system (1) written in general terms, describes the units combined unit turns around the property of their centers of mass. Thus, the index \( i \) varies from 2 to
4 and define the actual twists: Mineral spreaders – \( \beta_1 \), hitch – \( \beta_1 \), and grain drill – \( \beta_2 \).

For practical use of the system of differential equations (1) to determine further force factors, that it contains.

In this case, for analytical expressions for the part of the system (1) power factor using the power circuit (Fig. 1) and the main curves obtained when considering a plane-parallel movement of other tractor units [6-10].

First we define \( \sum_{i=1}^{4} F_{ax} \) and \( \sum_{i=1}^{4} F_{ay} \), that are part of the system of equations (1) Compiled using the power circuit (Fig. 1) can be written such vector equation:

\[
\vec{F}_i = F_{ax,i} + F_{ay,i} + \vec{F}_{ax,i} + \vec{F}_{ay,i},
\]

\[
\vec{F}_i = F_{ax,i} + F_{ay,i} + \vec{R}_x + \vec{R}_y, \tag{2}
\]

When designing these vector equation (2) on the axis \( Ox \) and \( Oy \) obtain the values of their projections on the first axis:

\[
F_{ax,i} = F_{ax,i} \sin \beta_i - F_{ay,i} \cos \beta_i, \tag{3}
\]

\[
F_{ay,i} = F_{ax,i} \cos \beta_i + F_{ay,i} \sin \beta_i, \tag{4}
\]

Further, we also obtain the projection forces behind the expression (2) on the axis \( Oy \):

\[
F_{ax,i} = -F_{ax,i} \cos \beta_i - F_{ay,i} \sin \beta_i, \tag{5}
\]

\[
F_{ay,i} = -F_{ax,i} \sin \beta_i + F_{ay,i} \cos \beta_i, \tag{6}
\]

Similarly, we obtain:

\[
\sum_{i=1}^{4} F_{ax,i} = \sum_{i=1}^{4} F_{ax,i} \sin \beta_i - \sum_{i=1}^{4} F_{ay,i} \cos \beta_i, \tag{7}
\]

and

\[
\sum_{i=1}^{4} F_{ay,i} = \sum_{i=1}^{4} F_{ax,i} \cos \beta_i + \sum_{i=1}^{4} F_{ay,i} \sin \beta_i, \tag{8}
\]

as a:

\[
\sum_{i=1}^{4} F_{ax,i} = -F_{ax,i} \cos \beta_i - F_{ay,i} \sin \beta_i, \tag{9}
\]

and

\[
\sum_{i=1}^{4} F_{ay,i} = F_{ax,i} \sin \beta_i - F_{ay,i} \cos \beta_i, \tag{10}
\]

Determine the values of resistance forces \( \vec{F}_{ax,i} \), \( i = 2, 4 \) when wheels roll without sliding. To this end, we describe the motion of the two wheels of the \( i \)-th level of machine-tractor unit.

\[
\dot{\beta} = \frac{\varphi^i}{\alpha}, \tag{i = 2, 4}.
\]

Using the differential equations of plane-parallel motion of the wheels and the force diagram (Fig. 2), we can write the following system of differential equations:

\[
m_a \ddot{x}_i = \sum_{i=1}^{4} F_{ax,i},
\]

\[
m_a \ddot{y}_i = \sum_{i=1}^{4} F_{ay,i}, \tag{11}
\]

\[
I_{ax,i} \dot{\varphi}_i = \sum_{i=1}^{4} M_{ax,i}, \tag{12}
\]

Where \( m_a \) – wheel weight \( i \)-th unit, \( I_{ax,i} \) – moment of inertia of the \( i \)-th wheel assembly about the axis of rotation, \( \sum_{i=1}^{4} F_{ax,i} \) – the vector sum of all external forces that act on the wheel \( i \)-th unit. Determine the acceleration of the center of mass of the left wheel \( \ddot{x}_i \). At first, we form such vector equation:

\[
\ddot{x}_i = \ddot{x}_i + \ddot{y}_i + \vec{F}_{ax,i} + \vec{F}_{ay,i},
\]

\[
\ddot{x}_i = \ddot{x}_i + \ddot{y}_i + \vec{R}_x + \vec{R}_y, \tag{13}
\]

Where \( \ddot{x}_i \) – acceleration of the center of mass of the \( i \)-th level machine aggregate; \( \alpha_i \) – angular acceleration of the \( i \)-th wheel in the rotational motion around the center of mass of the \( i \)-th level of the unit; \( \alpha_{ax,i} \) – rotational acceleration of the center of mass of the\( \ddot{x}_i \) of the \( i \)-th level of the unit around the center of mass of this link; \( \varphi^i \) – the angular velocity of rotation of the \( i \)-th wheel around the center of mass of the \( i \)-th level of the unit; \( \alpha_{ax,i} \) – centripetal acceleration of the center of mass of the \( i \)-th level of the unit around the center of mass of this link; \( c_i \) – distance from the center of mass of the \( i \)-th level of the unit to the center of mass of the left wheel of this link.

\[
\ddot{x}_i = \frac{\varphi^i}{\alpha}, \tag{i = 2, 4}.
\]

The condition of the wheels rolling without sliding is as follows:

\[
\dot{x}_i = \dot{y}_i \frac{\varphi^i}{\alpha}.
\]

Location by double differentiation we find:
\[ \tilde{\theta}_i = \frac{\tilde{\phi}_i}{r_{wi}} \]  

Where \( r_{wi} \) – wheel radius of the \( i \)-th level of the unit.

To the left wheel the third equation of (11) takes the following form:

\[ I_{iL} \tilde{\phi}_{iL} = F_{\alpha iL} \tilde{r}_{wi} + r_{wi} \cdot d_{zi} - \tilde{d}_{zi} \cdot \tilde{d}_{zi} \]  

Taking into account (15), we obtain:

\[ F_{\alpha iL}' = I_{iL} \tilde{r}_{wi} \]  

Substituting (13) into (17) yields:

\[ F_{\alpha iL}' = I_{iL} \left[ \tilde{x}_i \cos \beta_i + \tilde{y}_i \sin \beta_i + \tilde{\beta}_i (l_i - a_i) - \tilde{d}_{zi} \right]. \]  

Similarly, we obtain:

\[ F_{\alpha iR}' = I_{iL} \left[ \tilde{x}_i \cos \beta_i + \tilde{y}_i \sin \beta_i + \tilde{\beta}_i (l_i - a_i) + \tilde{d}_{zi} \right]. \]  

Then:

\[ F_{\alpha iL}' = F_{\alpha iL} + F_{\alpha iL}' = I_{iL} \left[ \tilde{x}_i \cos \beta_i + \tilde{y}_i \sin \beta_i + \tilde{\beta}_i (l_i - a_i) + \tilde{d}_{zi} \right] \]  

We compute \( M_{zi} \), moment of resistance of rotation of \( i \)-level unit \( i \)-th as the sum of the moments of resistance forces: the resistance of the left and right wheels relative to the center of mass of the \( i \)-th link:

\[ M_{zi} = M_{zi}^L(F_{\alpha iL}' + F_{\alpha iL}^L) - M_{zi}^R(F_{\alpha iL}' + F_{\alpha iL}^R) = I_{iL} \left[ \tilde{x}_i \cos \beta_i + \tilde{y}_i \sin \beta_i + \tilde{\beta}_i (l_i - a_i) + \tilde{d}_{zi} \right] \]  

Rear drive wheels of the tractor depending (20) and (21), with the proviso that \( d_{zi} = d_{zi} = d \) will look like this:

\[ F_{\alpha i} = I_{iL} \left[ \tilde{x}_i \cos \beta_i + \tilde{y}_i \sin \beta_i + \tilde{\beta}_i (l_i - a_i) \right] - 2M_{zi} \]  

\[ M_{zi} = 2I_{iL} \tilde{\beta}_i \left( \frac{d_{zi}}{r_{wi}} \right)^2. \]  

Where: \( M_{zi} = \frac{M_i \eta}{2} \); \( M_i \) – torque that develops engine tractor; \( \eta \) – factor which takes into account the type of tractor transmission.

Define \( F_{\alpha iL}' \) and \( M_{zi} \) for the front steering wheels of the tractor (Fig. 3).

From the kinematics of plane-parallel rigid body motion acceleration define the center of mass right and left wheels according to the following expressions:

![Fig. 3 – Diagram of the forces that are applied to the steered wheels the machine set](image-url)
The moment of resistance of rotation of the wheels of the tractor:

\[ M_{i,1} = M'_{i,1} + M''_{i,1} = \]

\[ 2I_{i,1}k_{i} \left( \frac{d_{i,1}^{2}}{\alpha} \right) + 2I_{i,1}(d_{i,1} + d_{i,\alpha, \cos \alpha}) \times \]

\[ \times \left[ (d_{i,1} + d_{i,\alpha, \cos \alpha}) \cos \alpha + a_{i,1} \sin \alpha \right] + \beta_{i}d_{i,1} \sin \alpha \cos \alpha \right) \times \left( \frac{\alpha}{\beta_{i}} \right) \].

Where \( M_{i,1} \) – moment of resistance of rotation of the wheels of the tractor.

If some wheels will roll unit with sliding, for them the force of resistance (friction) are the maximum and determined by the following expression:

\[ F_{i,1} = N_{i,1}f_{i,1}, \quad (i = 1,4), \]

Where \( N_{i,1} \) – pressure force of the i-th wheel on the ground; \( f_{i,1} \) – the coefficient of friction between the wheel and the ground.

The strength of the wheels on the ground pressure will be equal to:

\[ N_{i} = \frac{m_{i}d_{i,1}}{k_{i}}, \quad (i = 1,4) \]

\[ N'_{i} = \frac{m_{i}(1 - a_{i})}{k_{i}}. \]

Where \( N'_{i} \) – the pressure force of the front wheels on the ground.

Define the lateral forces that act on the tractor axles. According to [6]:

\[ F'_{i,1} = k_{i}a_{i} \varphi_{i,1}, \quad (i = 1,4) \]

\[ F''_{i,1} = k'_{i,1} \varphi_{i,1}. \]

Where \( k_{i} \) – cornering power coefficient of resistance, which is typically determined experimentally; \( \varphi_{i} \) – slip angle bridges.

The force \( F'_{1,1} = 0 \), because it is included in the determination of force \( F_{1,1} \). Numerical analysis of this task should be carried out using a PC.

## 3. Results and discussion

Thus, we find all analytical expressions for the force factors that are composed system of differential equations (1), which describes the motion of a plane-parallel combined sowing unit consisting of aggregated tractor and hooked up behind him fertilizer distributing drills that band-pass method makes fertilizers and suspended on her grain drill.

In this case, the obtained analytical dependence expressed in terms of generalized coordinates, and, consequently, the system of differential equations of motion (1) after substituting the expressions obtained it will be closed, which gives every reason for its numerical solution on the PC.

According to the results of the numerical solutions of these have the possibility to build a combined motion path points sowing unit depending on its design and the kinematic parameters, and thus to determine their values, which provide a large rectilinear trajectories sowing unit etc.

## 4. Conclusion

Compiled earlier system of differential equations of plane-parallel motion combined sowing unit is fully prepared to address, is numerical modeling that will enable analytically determine the rational design and kinematic parameters that ensure its sustainable movement, and thus the quality of the implementation process.

## 5. Literature


2. Vasilenko P.M. Equations of motion of the mobile machine units. The collection of works on agricultural mechanics t. II. – Moscow: Selkhozgiz, 1952. – P. 76-84.


