ECONOMIC SCHEDULING MODEL FOR MECHANICAL-MANUAL MANUFACTURING OF SEASONAL PRODUCT UNDER UNCERTAIN SUPPLY TIME AND QUANTITY OF THE RAW MATERIALS

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Abstract: *The research proposes a newsvendor-like model that enables determining the optimal timing for renting machinery with its operators before the arrival of the raw material, and derives an optimal manufacturing strategy. The research includes a case study with a comprehensive analysis for a blueberry harvester that demonstrates the applicability of the proposed model. The rigorous theoretical analysis of the optimal harvesting strategy is validated by numerical experiments. The presented results may help researchers and agriculture practitioners in planning mechanical-manual integrated manufacturing.*

Keywords: SEASONAL PRODUCTS, MECHANICAL-MANUAL INTEGRATED MANUFACTURING, SCHEDULING UNDER UNCERTAINTY, BLUEBERRY HARVESTER.

1. *Introduction*

Seasonal products have regular fluctuations in their quantities and market prices during the seasons and the timing of the selling. For these products, and especially where uncertainty is involved, decision-makers must set effective plan for manufacturing. Manufacturing of a seasonal product can be defined as a project that has a beginning and end points and a specific goal that should be accomplished. This research explores a mechanical-manual integrated manufacturing planning of seasonal product where the delivery date and the quantity of the raw material are random variables. The goal is to maximize the profit that can be obtained during the season period from the available quantity of a seasonal commodity. An economic schedule of such manufacturing should take into account the fact that the selling price of seasonal products tend to decrease over their season period.

This research develops the model presented in Gurevich et al. [1]. The model assumes that the decision-maker is waiting for the supply of raw materials required for manufacturing the seasonal products but the delivery date and the supplied quantity are random variables. Efficient manufacturing of the seasonal product requires rented machinery which must be ordered with its operator in advance. The planner's problem is to determine the due date for the machinery arrival. Early arrival of the machinery, before the actual delivery day of the raw material, means redundant expenses. On the other hand, late arrival of the machinery may require temporary manufacturing by a constrained manual labor capacity which slower the manufacturing and prevents the achievement of high selling prices which tend to be reduced over the season period. In such case, the following strategies are available:

1. Avoiding manual manufacturing and beginning it mechanically when the machinery arrives.
2. Beginning the manufacturing as soon as possible and do it manually until the machinery arrival.
3. Implementing manual manufacturing partially during the window between the raw material supply and the machinery arrival.

The proposed model is a newsvendor-like model that enables determining the optimal timing for the machinery arrival and the optimal strategy for operating the machinery after the delivery of the raw materials. Newsvendor-like models are widely used to determine optimal order of quantities (e.g., inventory) and/or optimal timing (e.g., Keren and Pliskin, [2]) in stochastic environment.

Grown fruits are seasonal commodities with ripening dates and crop yields that are random variables. The supply and demand of the harvested fruits through the season period determines their market price giving advantage to early appearance in the market. Therefore, in order to illustrate the applicability of the proposed model, a case study with a rented self-propelled blueberry harvester was considered. The issue of optimization of harvest operations scheduling, both with and without harvesting machines, was explored in the scientific literature. Ferrer et al. [3] presented a practical tool for optimally scheduling wine grape harvesting operations, taking into account both operational costs and grape quality. Tan and Çömden [4] proposed a planning methodology for a firm whose objective is to match the random supply of annual premium fruits and vegetables from a number of contracted farms, with the random demand from retailers during the planning period. The case study in this research demonstrates how to derive the optimal harvesting strategy and starting time for rental of a self-propelled blueberry harvester, in order to maximize a farmer's profit.

2. *Notations*

The following terms are used herein:

T - is the time when the blueberry is ready for harvesting (a positive random variable), $T \in (0, A)$; that is, the earliest possible time for starting the harvest season is defined as zero and the latest possible time for starting the harvest season is defined as A, where $A > 0$ is a known deterministic value (measured in hours);

B - is the length of the harvest period (i.e., the length of the time interval $(T, T+B)$, measured in hours), and it has a known positive deterministic value;

H - is the total amount of the blueberry in the field, which is measured in thousands of pounds (a positive random variable);

$f_{T,H}(x, y)$ - is the known joint density function of the random variables T and H, $0 \le x \le A,\ y > 0$;

c_1 - is the booking cost to reserve the use of the machine (a self-propelled harvester). (This is a one-time payment that guarantees the availability of the machine as well as the option of returning the machine at any point of time), $c_1 \ge 0$;

c_2 - is the machine rental cost per hour, $c_2 > 0$;

$c(t, H)$ - is the known function of the selling price of one thousand pounds of the product. For a given value of H, $c(t, H)$ is a continuous decreasing function of t defined in the range

$(0, A+B)$, i.e., $t \in (0, A+B)$;

q_1 - is the machine harvesting capacity per hour (the amount of the product, in thousands of pounds, that can be picked by the machine during one hour, $q_1 > 0$);

q_2 - is the manual harvesting capacity per hour (the amount of the product, in thousands of pounds, that can be picked manually by the existing workers during one hour, $0 < q_2 < q_1$);

c_3 - is the prime cost of one thousand pounds of machine-harvested product, $c_3 > 0$;

c_4 - is the prime cost of one thousand pounds of manually-harvested product, without the assistance of the machine, $c_4 > c_3 > 0$;

t_b - is a point in time when the machine arrives at the farm and the rental payments begin to be incurred (t_b is a deterministic decision variable that should be determined by the farmer, $0 \le t_b < A$);

t_1 - is a point in time when the farmer stops the manual harvesting process and waits for the arrival of the machine (t_1 is a deterministic decision variable that should be determined by the farmer according to the actual values of random variables T, H, and the decision variable t_b, $0 \le t_1 \le min\{t_b - T, H/q_2, B\}$);

$B(t_b, H, T, t_1)$ - is the farmer's profit (benefit) (a random variable);

t_1^* - is an optimal value of t_1, given the values of random variables T, H, and the decision variable t_b, which maximizes the farmer's profit $B(t_b, H, T, t_1)$, $t_1^* = t_1^*(H, T, t_b)$;

$B(t_b, H, T)$ - is the farmer's profit (benefit), attained by implementation of an optimal harvesting strategy (optimal choice of the decision variable t_1) (a random variable, $B(t_b, H, T) = B(t_b, H, T, t_1 = t_1^*)$);

t_b^* - is an optimal value of the decision variable t_b, which maximizes the farmer's expected profit, $E(B(t_b, H, T))$.

3. *An optimal harvesting strategy*

This section outlines the farmer's optimal working strategy presented in Gurevich et al. [1] to maximize profit, assuming that the values of the random variables H, T and the decision variable t_b are known. The farmer does not consider the option of not renting the machine because his available manual harvesting capacity is not enough to harvest the expected amount of product during the harvesting period. That is, the booking cost c_1 must be taken into account for any strategy. Another assumption is that for any $0 \le t \le A+B$ and for any value of H, the selling price $c(t,H) \ge c_4 > c_3$.

Obviously, if $t_b \le T$, the optimal strategy is to use the machine during the entire harvesting period. In this case the farmer's profit is

$$B(t_b, H, T) = -c_1 - c_2(T - t_b) + q_1 \int_T^{min\{T+B, T+H/q_1\}} \left(c(t,H) - c_3\right) dt. \quad (1)$$

If $t_b > T$, then the farmer can start with manual harvesting at time T and can continue using manual harvesting for a period $(T, T+t_1)$, where $0 \le t_1 \le min\{t_b - T, H/q_2, B\}$. After that, the farmer stops manual harvesting and waits for the arrival of the machine. Finally, at time t_b, the farmer begins machine harvesting and continues with it until all the available yield is harvested (or until the end of harvesting period). In this case, determining the farmer's optimal working strategy means to determine an optimal value of t_1 that maximizes the profit. The farmer's profit is

$$B(t_b, H, T, t_1) = -c_1 + q_2 \int_T^{T+t_1} \left(c(t,H) - c_4\right) dt + q_1 \int_{min\{t_b, T+B\}}^{min\left\{T+B,\, t_b + \frac{H - q_2 t_1}{q_1}\right\}} \left(c(t,H) - c_3\right) dt, \quad (2)$$

where $0 \le t_1 \le min\{t_b - T, H/q_2, B\}$. Thus, for this case,

$$t_1^* = t_1^*(t_b, H, T) = arg \max_{0 \le t_1 \le min\{t_b - T, H/q_2, B\}} B(t_b, H, T, t_1), \quad (3)$$

and the farmer's profit attained by this optimal strategy is

$$B(t_b, H, T) = B(t_b, H, T, t_1 = t_1^*) = \max_{0 \le t_1 \le min\{t_b - T, H/q_2, B\}} B(t_b, H, T, t_1), \quad (4)$$

where $B(t_b, H, T, t_1)$ is defined by Equation (2). The following proposition, presented by Gurevich et al. [1], guarantees for any given value of $H > 0$ that if the selling price function $c(t,H)$ is a differentiable function of t, then for all values of $H > 0$, $0 \le T \le A$, $B > 0$, $0 \le t_b < A$, $0 < q_2 < q_1$, $c_4 > c_3 > 0$, there is a unique feasible value of t_1^* that satisfies Equation (3).

Proposition. Assume that the value of $H > 0$ is given and the selling price function $c(t,H)$ is a differentiable function of t. Then for all values of $0 \le T \le A$, $B > 0$, $0 \le t_b < A$, $0 < q_2 < q_1$, $c_4 > c_3 > 0$, Equation (3) has a unique solution $t_1^* = t_1^*(t_b, H, T)$. Furthermore, this solution is defined as follows:

1. If $t_b \ge T + B$ then $t_1^* = min\{H/q_2, B\}$.

2. If $T < t_b < T + B$ and

$QQ = QQ(H, T, t_b, q_1, q_2, B) = \dfrac{H - q_1(T + B - t_b)}{q_2}$, then:

2.1 If $QQ > t_b - T$ then $t_1^* = t_b - T$.

2.2 If $QQ < 0$ then:

2.2.1 If $c(T,H) - c_4 \le c\left(t_b + \dfrac{H}{q_1}, H\right) - c_3$ then $t_1^* = 0$,

2.2.2 If $c\left(T + min\{t_b - T, H/q_2\}, H\right) - c_4 \ge c\left(t_b + \dfrac{H - q_2\, min\{t_b - T, H/q_2\}}{q_1}, H\right) - c_3$ then

$t_1^* = min\{t_b - T, H/q_2\}$,

2.2.3 If $c(T,H)-c_4 > c\left(t_b+\dfrac{H}{q_1},H\right)-c_3$ and

$$c(T+min\{t_b-T,H/q_2\},H)-c_4 < c\left(t_b+\frac{H-q_2\,min\{t_b-T,H/q_2\}}{q_1},H\right)-c_3$$

then t_1^* is the unique feasible solution of the equation

$$c(T+t_1,H)-c\left(t_b+\frac{H-q_2t_1}{q_1},H\right)=c_4-c_3 \qquad (5)$$

and can be found analytically or numerically, depending on the complexity of the selling price function $c(t,H)$.

2.3 If $0\le QQ\le t_b-T$ then

$$t_1^*=\begin{cases} QQ & \text{if} \quad q_1\int\limits_{t_b+\frac{H-q_2t_1^{**}}{q_1}}^{T+B}(c(t,H)-c_3)dt \ge q_2\int\limits_{T+QQ}^{T+t_1^{**}}(c(t,H)-c_4)dt \\ t_1^{**} & \text{otherwise}, \end{cases}$$

where t_1^{**} is defined by the following way:

if $c(T+QQ,H)-c_4 \le c\left(t_b+\dfrac{H-q_2QQ}{q_1},H\right)-c_3$ then $t_1^{**}=QQ$,

If $c(T+min\{t_b-T,H/q_2\},H)-c_4 \ge c\left(t_b+\dfrac{H-q_2\,min\{t_b-T,H/q_2\}}{q_1},H\right)-c_3$ then

$t_1^{**}=min\{t_b-T,H/q_2\}$,

if $c(T+QQ,H)-c_4 > c\left(t_b+\dfrac{H-q_2QQ}{q_1},H\right)-c_3$ and

$c(T+min\{t_b-T,H/q_2\},H)-c_4 < c\left(t_b+\dfrac{H-q_2\,min\{t_b-T,H/q_2\}}{q_1},H\right)-c_3$ then t_1^{**} is the unique feasible solution of Equation (5) and can be found analytically or numerically, depending on the complexity of the selling price $c(t,H)$.

The farmer's profit that incorporates his optimal strategy is defined in accordance with Equation (1) for $t_b\le T$, and in accordance with Equation (2) with $t_1=t_1^*$ for $t_b>T$, where t_1^* is stated in the above proposition.

4. *Determining the optimal starting time point of the machine rent*

This section develops the procedure presented in Gurevich et al. [1] for determining the optimal value of the decision variable $0\le t_b<A$ in order to maximize the farmer's expected profit, $E(B(t_b,H,T))$, assuming that the farmer behaves according to his optimal working strategy, as presented in section 3. The aim is to find t_b^* such that

$$t_b^* = arg \max_{0\le t_b<A} E(B(t_b,H,T)), \qquad (6)$$

where $B(t_b,H,T)$ is defined in Equations (1) and (4). The farmer's expected profit has the form

$$E(B(t_b,H,T))=\int_0^\infty\int_0^A B(t_b,h,\tau)\,f_{T,H}(\tau,h)\,d\tau dh \qquad (7)$$
$$=\int_0^\infty\int_0^{t_b} B(t_b,h,\tau)\,f_{T,H}(\tau,h)\,d\tau dh + \int_0^\infty\int_{t_b}^A B(t_b,h,\tau)\,f_{T,H}(\tau,h)\,d\tau dh,$$

where $f_{T,H}(\tau,h)$ is a known joint density function of the random variables T and H. Straightforwardly, by Equations (1)-(4), (7)

$$E(B(t_b,H,T)) = \int_0^\infty\int_{t_b}^A\left(\begin{array}{l}-c_1-c_2(\tau-t_b)\\ +q_1\int\limits_{\tau}^{min\{\tau+B,\tau+h/q_1\}}(c(t,h)-c_3)dt\end{array}\right)f_{T,H}(\tau,h)\,d\tau dh \qquad (8)$$
$$+\int_0^\infty\int_0^{t_b}\left(\begin{array}{l}-c_1+q_2\int\limits_{\tau}^{\tau+t_1^*}(c(t,h)-c_4)dt\\ +q_1\int\limits_{min\{t_b,\tau+B\}}^{min\left\{\tau+B,t_b+\frac{h-q_2t_1^*}{q_1}\right\}}(c(t,h)-c_3)dt\end{array}\right)f_{T,H}(\tau,h)\,d\tau dh,$$

where t_1^* is stated by the proposition presented in section 3. Note that in any real life problem the decision variable t_b, $0\le t_b\le A$, is practically not continuous and can be considered to have a finite number of values (e.g., the values of t_b are the integer numbers of hours after the earliest possible time for starting the harvest season). Therefore, the optimal value of t_b may be attained by examination of all its possible values. That is, the farmer's expected profit $E(B(t_b,H,T))$ should be calculated for all possible values of t_b. Then, the value of t_b that yields the farmer's highest expected profit is the needed solution t_b^* of Equation (6). The farmer's expected profit $E(B(t_b,H,T))$ can be approximated by utilizing a numerical integration of the double integrals in equation (8) or it can be approximated by simulation.

5. *Case Study*

The case study deals with some farm in the countryside that specializes in blueberry growing. The assumption for this case study is that for any given value of H, the selling price function $c(t,H)$ is a linear function, defined on the time interval $(0,A+B)$, and that it linearly decreases from a known selling price of a_H to a lower value of b_H. (The values of a_H and b_H are given in US dollars for one thousand pounds of the crop.) Thus,

$$c(t,H)=a_H+\frac{b_H-a_H}{A+B}t,\ t\in(0,A+B). \qquad (9)$$

Then, for the assumed selling price function (9), the farmer's profit is directly defined by Equations (1), (2) and the proposition presented in section 3. With respect to the joint distribution of the random variables T and H, the following case was considered: The random variables T and H are dependent. The random variables T has a conditional normal distribution with expectation $\mu_1 = A/2$ and standard deviation $\sigma_1 = A/6$, given $0 \le T \le A$. That is, $T = X \left| 0 \le X \le A \right.$, $X \sim Norm\left(\mu_1 = A/2, \sigma_1^2 = (A/6)^2\right)$, the random variables H is defined as follows

$$H = \begin{cases} Unif(\lambda_1, \lambda_2) & \text{if} \quad T < \mu_1 \\ Unif(\lambda_3, \lambda_4) & \text{if} \quad T \ge \mu_1 \end{cases}, \; \lambda_1 < \lambda_2, \; \lambda_3 < \lambda_4, \; (\lambda_1, \lambda_2) \ne (\lambda_3, \lambda_4).$$

Furthermore, the following values were considered: $A = 600$, $B = 500$, $c_1 = \$200$, $c_2 = \$110$, $q_1 = 0.45$, $q_2 = 0.29$, $c_3 = \$500$, $c_4 = \$950$. That is, $E(T) = A/2 = 300$, $Var(T) = 0.973 \times (A/6)^2 = 9730$, $\sigma(T) = 98.64$. $\lambda_1 = 200$, $\lambda_2 = 300$, $\lambda_3 = 100$, $\lambda_4 = 200$. The selling price is independent of the quantity of the crop at the end points of the selling price function (9), that is $a_H = a$, $b_H = b$, $a = \$2{,}100$, $b = \$1{,}000$. We concentrated on scenarios where $t_b > T$. Table 1, below, presents the farmer's profit $B(t_b, H, T) = B\left(t_b, H, T, t_1 = t_1^*\right)$, which was attained by implementation of the optimal working strategy. The table also presents the farmer's alternative profit $B(t_b, H, T, t_1 = t_c)$, $t_c = min\{t_b - T, H/q_2, B\}$, which was attained by first using manual harvesting as long as possible (without stopping and waiting for the machine, i.e., from time T up to time $T + t_c$), and then using machine harvesting. This was tested for several different values of the time when the product became ready for harvesting (T), as well as the total amount of the product (H).

Table 1. Comparisons of farmer profits for optimal and alternative harvesting strategies.

T	260.00	50.00	60.00	180.00	150.00
H	40.00	110.00	100.00	80.00	60.00
$B(t_b, H, T)$	50,022	129,530	118,702	96,689	73,800
$B(t_b, H, T, t_1 = t_c)$	45,916	117,675	105,805	87,958	61,185
relative advantage of the profit $B(t_b, H, T)$	8.94%	10.07%	12.19%	9.93%	20.62%

In particular, Table 1 illustrates the relative advantage of the farmer's profit $B(t_b, H, T)$ in comparison with the farmer's alternative profit $B(t_b, H, T, t_1 = t_c)$ (i.e., $100\% * \left(B(t_b, H, T) - B(t_b, H, T, t_1 = t_c)\right) / B(t_b, H, T, t_1 = t_c)$). Table 1 clearly shows an essential advantage of using the optimal working strategy. Furthermore, a simulation study was conducted to calculate the farmer's expected profit $E\left(B(t_b, H, T)\right)$, which corresponds to the farmer's optimal harvesting strategy for different values of the decision variable t_b, $0 \le t_b \le A = 600$. Figure 1, below, presents the simulated values of the farmer's expected profit, $E\left(B(t_b, H, T)\right)$, for the considered values of the decision variable t_b.

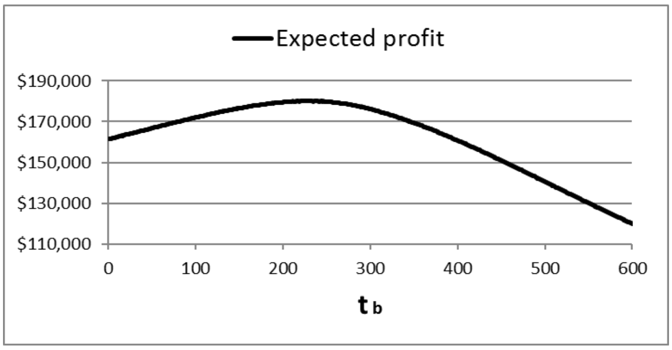

Fig. 1. Simulated expected profit of the farmer versus values of the decision variable t_b.

Figure 1 shows that the simulated expected farmer's profit has a parabola like shape with a respect to the decision variable t_b. The farmer's expected profit continuously increases when t_b increases from zero to 224 (the mean of the time when the product is ready for harvesting, $E(T) = 300$). Thereafter, the farmer's expected profit continuously decreases (when t_b increases from approximately 224 to the latest possible time that the product can be ready for harvesting, $A = 600$). The maximum value of the simulated expected farmer's profit is 180,176.4 US dollars. This value is obtained for $t_b = 224$. For $t_b = 300$ (i.e., for t_b that is defined as the expectation of T), the simulated expected farmer's profit is equal to 176,293.5 US dollars. Thus, in this example, the optimal choice of the decision variable t_b leads to an increase of 2.2 % in the total farmer's profit compared to the "intuitive" choice to set t_b to the expected value of T.

6. *Conclusions*

This research presents a comprehensive analysis for the problem where a decision-maker knows that a specific event will happen in the coming future but the exact timing of the event and its magnitude are random variables. The research includes a case study that demonstrates the applicability of the proposed model. The rigorous theoretical analysis of the optimal harvesting strategy is validated by numerical experiments. The proposed models can be applied to various industry and applications. The results of this research can add the timing dimension for the emergency inventory problem.

7. *References*

1. Gurevich, G., Keren, B., Laslo, Z. (2017). Optimal policy for using a rented farming machine. Mechanization in Agriculture and Conserving of the Resources, 63: 48-50.
2. Keren, B., Pliskin, J.S. (2006). A benchmark solution for the risk-averse newsvendor problem. European Journal of Operational Research, 174(3): 1643-1650.
3. Ferrer, J.C., Mac-Cawley, A., Maturana, S., Toloza, S., Vera, J., (2008). An optimization approach for scheduling wine grape harvest operations, International Journal of Production Economics, 112 (2): 985-999.
4. Tan, B., Çömden, N., (2012). Agricultural planning of annual plants under demand, maturation, harvest, and yield risk, European Journal of Operational Research, 220 (2): 539-549.