# Mathematical model of root head cleaning machine with vertical drive shaft

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***Abstract:*** *The operation of the machine attached to the back of the tractor to clean the heads of root crops from residues in the case of using copying pneumatic wheels causes oscillations of the tops harvester in the vertical plane, which significantly affects the quality of the process. Accordingly, the movement of the horizontal working body will depend on many structural and kinematic factors. Therefore, the development of a mathematical model that will describe the movement of the working body of such a unit is an important task. To solve which the method of construction of calculated mathematical models of functioning of agricultural machines and machine units, on the basis of theoretical mechanics and higher mathematics was used in the work.. Using which the equivalent scheme of movement of the unit is developed and the system of two nonlinear differential equations for detailed research of oscillations of the cleaner of heads of root crops in the longitudinal-vertical plane at movement of its pneumatic copying wheels on roughnesses of a surface of soil is received. The mathematical model of movement of the cleaner with a horizontal cleaning shaft developed on the basis of initial equations of dynamics in the form of Lagrange of the 2nd kind allowed to establish depedencies between constructive and kinematic parameters of the car and its oscillatory characteristics. The found dependences created preconditions for the further mathematical modeling of parameters of the rear-mounted cleaner of heads of root crops, with a horizontal clearing shaft. Using the developed calculation model it is possible to optimize the values of angle* $\beta$ *and coordinates* $y$ *that characterize the oscillations of the machine in the longitudinal-vertical plane.*

***KEY WORDS:*** *HEAD CLEANER, TRACTOR, HITCH, OSCILLATIONS, REACTION, DIFFERENTIAL EQUATIONS*.

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## *1. Introduction*

In the operation process of the machine for cleaning the heads of roots from the bud remnants wich attached to the rear of the tractor, it performs movements in space, which are determined by the translational speed of the tractor, the field surface, placement of copying pneumatic wheels relative to the hitch system.

The use of copying pneumatic wheels causes oscillations of the cleaning machine in the vertical plane, which will have the greatest impact on the quality of the process. Therefore, we consider the motion of the cleaning machine only in the longitudinal-vertical plane, ie we build a mathematical model of the oscillation of the experimental machine when moving along the irregularities of the soil surface.

The use of a sugar beet cleaner of advanced design, which is hung behind the tractor, leads to its free movements in space, which are determined by the translational speed of the tractor, the relief of the field surface, placement of wheels relative to the hitch system, etc.

## *2. Preconditions and means for resolving the problem*

**Аналіз останніх публікацій.** An important issue for the field of mechanization of beet growing is the cleaning of the remnants from sugar beet roots heads after its main cutting at the root by cutting devices of beet harvesters. The importance of this issue is confirmed by a number of scientific papers devoted to the study of designs of root crop cleaners [4-7]. They present the results of theoretical and experimental studies of cleaners of sugar beet root heads of different designs: blade, ring, sector, drum types, as well as a cleaner made in the form of a paraboloid. Concerns is the insufficient study of the question of blades oscillation of machines with horizontal drive shafts on moving their working bodies, which have not yet been reflected in the literature. Therefore, studies that would assess the impact of oscillations perturbed by the movement of the cleaner with a horizontal working shaft on the irregularities of the field surface on the quality of roots heads cleaning from buds remnants are relevant.

For example, using the method described in [9], it is possible to build a calculated mathematical model of this machine, which will allow to study the influence of its design parameters on the movement along rows of sugar beet roots and soil surface irregularities.

Thus, the study of the movement of the root crop cleaner and the impact of its oscillations in the longitudinal-vertical plane on the quality of the process requires more in-depth study and justification of structural and kinematic parameters that will ensure high cleaning efficiency of root heads.

*The aim of the study.* Development of a mathematical model for the theoretical study of the oscillatory motion of the rear-mounted on a wheeled tractor cleaner heads of roots from buds remnants in longitudinal-vertical plane.

*Research methods*. To achieve the goal of the study, methods of construction of computational mathematical models of functioning of agricultural machines and machine units, based on theoretical mechanics and higher mathematics, were used.

## *3. Results and discussion*

Let's make the equivalent scheme of movement of the clearing machine in the longitudinal-vertical plane which rear-mounted to the aggregating tractor (fig. 1).

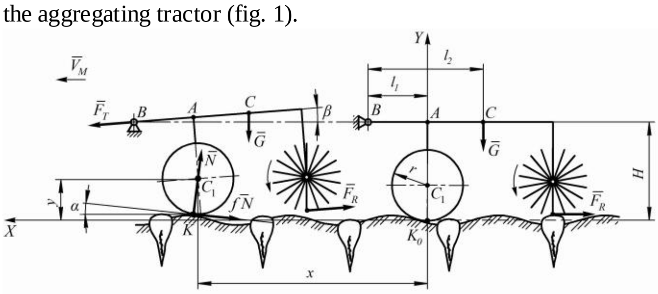

***Fig. 1****. Equivalent scheme of movement of the cleaning machineon the unevenness of soil surface*

In this case, we consider the motion of one of the copy wheels with a radius $r$, given that the second is in the same state as the first. Suppose that in the process of copying wheel, moving in the rows of beet crops on the soil surface of the sinusoidal transverse profile [1], moved from one position to another. In the plane of rotation on the copying wheel are: the force of gravity $\bar{G}$ of the machine and traction force $\bar{P}$. From the side of heads cleaner of root crops there is a resistance force $\bar{F}_R$ equal in magnitude and opposite in direction to the force $\bar{Q}$ of tops crumpling from the root crop head. Under the action of these forces at the point $K_1$ of contact of the copying wheel with the unevenness of the soil surface there are normal $\bar{N}$ and tangential $f\bar{N}$ reactions (where $f$ is the coefficient of rolling resistance) [2]. The directions of action of normal $\bar{N}$ and tangential $f\bar{N}$ reactions are determined by the angle $\alpha$. The angle between the traction force $\bar{P}$ and the direction of movement is denoted by $\beta$, which determines the nature of the vertical oscillations of the cleaning machine in the longitudinal-vertical plane. The center of the suspension of the frame of the machine to the tractor is denoted by $B$, the height of the suspension of the frame – by $H$. Assume that the hinge $B$ does not move in the vertical plane, and copying process occurs only due to the copying wheels of the cleaner. The speed of translational motion of the machine will be considered constant. We attribute this system to fixed Cartesian coordinates $xKy$ [3], assuming that all its points move only in this plane. The center of the coordinate axes is located at the point $K$ of contact of the copy wheel with the ground. Then, as can be seen from Fig. 1, the location of the center of the copy wheel is determined by the coordinates $x$ and $y$. We consider in the first approximation the motion of the point of suspension of the cleaner to the tractor (point $B$) is rectilinear and uniform [8]. The center of gravity of the machine (point $C$) is at a distance $l_2$ from the point of suspension. The distance between the axis of the suspension of the cleaning machine (point $B$) and the axis of attachment of the copying wheels (point $A$) is denoted by $l_1$. The mass of the whole cleaning machine is denoted by $M$, the mass of the copying wheels by $m$, and $m = m_1 + m_2$ ($m_1$ - the mass of the first wheel, $m_2$ - the mass of the second wheel). The force of gravity $\bar{G}$ of the cleaning machine will be considered applied in its center of mass (point $C$). The mass of the copying wheels is concentrated at a point $C_1$.

Pneumatic copying wheels will be presented in the form of elastic damping models, which have a stiffness factor $c$ and a damping factor $\mu$. Since there are two copying wheels, these coefficients will be doubled. We believe that copying the wheels in the General case when moving crumple the upper layer of the soil surface, move along the irregularities of the soil surface of the sinusoidal profile, which varies according to the following law [1]:

$$y = h\left(1+\sin\left(kx-\frac{\pi}{2}\right)\right), \quad (1)$$

where $y$ is the ordinate of the height of the unevenness of the soil surface, m; $h$ - half the height of the unevenness of the soil surface, m; $k$ - frequency of unevenness of the soil surface, m$^{-1}$; $x = V_M \cdot t$ – value of the current coordinate, m; $V_M$ - speed of the cleaner, m·s$^{-1}$.

In the first approximation, we assume that the copying wheel is in contact with the roughness of the field surface at a point, the position of which belongs to the sine (1).

Since the center of mass of pneumatic copying wheels, due to their elastic properties, performs independent oscillating motions (point $C_1$) and the ordinates $y$ of the heights of the unevenness of the soil surface are much smaller than the machine displacement $x$, we can assume that these oscillations can be determined by an independent coordinate $y$.

In addition, the position of the center of mass of the machine (point $C$) in the longitudinal-vertical plane is completely determined by the independent coordinate $\beta$, which is angle of the machine frame to the horizon, thus, the oscillatory system has two degrees of freedom and its motion is completely determined by two independent generalized coordinates $q_1 = \beta$ and $q_2 = y$. These generalized coordinates will be used in compiling the differential equations of motion of the cleaning machine in the longitudinal-vertical plane.

The speed $V$ of movement of the center of mass of the copying wheels in this case will be equal to:

$$V = \sqrt{\dot{x}^2 + \dot{y}^2}. \quad (2)$$

Given that $x = V_M \cdot t$, we obtain the following expression:

$$V = \sqrt{V_M^2 + \dot{y}^2}, \quad (3)$$

where $V_M$ is the speed of translational motion of the machine, m·s$^{-1}$.

To determine the kinetic energy of a given dynamic system, it is also necessary to determine the angular velocity of the copying wheel and express it through known parameters. It is defined by the following expression:

$$\omega = \frac{dS}{dt}\cdot\frac{1}{r}, \quad (4)$$

where $\omega$ is the angular velocity of the copying wheel, rad/s; $S$ - the magnitude of the circular displacement of the copying wheel on the sinusoidal profile of the soil surface, m; $r$ - wheel radius, m

In this case, the differential of the arc of movement of the copying wheel will be equal to:

$$dS = \sqrt{dx^2 + dy^2} = \sqrt{1 + h^2k^2\cos^2\left(kx - \frac{\pi}{2}\right)}dx, \quad (5)$$

or:

$$dS = \sqrt{1 + h^2k^2\cos^2\left(kV_M \cdot t - \frac{\pi}{2}\right)} \cdot V_M dt. \quad (6)$$

Substituting the value of (6) in (4), we obtain the value of the desired angular velocity:

$$\omega = \frac{\sqrt{1 + h^2k^2\cos^2\left(kV_M \cdot t - \frac{\pi}{2}\right)} \cdot V_M}{r}. \quad (7)$$

To compile the differential equations of motion of the considered oscillatory system, we use the initial equations in the form of Lagrange of the second kind of the following type [1]:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = Q_i - \frac{\partial P}{\partial q_i} - \frac{\partial R}{\partial \dot{q}_i}, \quad (8)$$

where $T$ is the kinetic energy of the system; $Q_i$ - generalized force; $P$ - potential energy of the system; $R$ - dissipative function

Define the components included in expression (8). The kinetic energy of the system is equal to:

$$T = T_1 + T_2 + T_3, \quad (9)$$

where $T_1$ is the kinetic energy of the translational motion of the cleaning machine (its center of mass); $T_2$ - kinetic energy of the oscillating motion of the machine frame around the point $B$; $T_3$ - kinetic energy of rotational motion copying the wheels around their axes.

The components of kinetic energy will be equal to:

$$T_1 = \frac{MV^2}{2} = \frac{M}{2}\left(V_M^2 + \dot{y}^2\right), \quad (10)$$

where $M$ is the weight of the cleaning machine, kg; $V$ is the speed of the center of mass of the machine, m·s$^{-1}$.

$$T_2 = \frac{I_B\dot{\beta}^2}{2}, \quad (11)$$

where $I_B$ - the moment of inertia of the frame of the cleaning machine relative to the axis that is perpendicular to the longitudinal-vertical plane and passes through a point $B$, kg·m$^2$; $\dot{\beta}$ - angular speed of rotation of a frame of the car, s$^{-1}$.

$$T_3 = \frac{I_k\omega^2}{2} = \frac{1}{2r^2}I_kV_M^2\left[1 + h^2k^2\sin^2(kx)\right], \quad (12)$$

where $I_k$ is the moment of inertia of the copying wheels relative to their axes of rotation, kg·m$^2$; $\omega$ - angular speed of rotation of the copying wheel, s$^{-1}$.

Then expression (9) will take the following form:

$$T = \frac{MV^2}{2} + \frac{I_B\dot{\beta}^2}{2} + \frac{I_k\omega^2}{2}. \quad (13)$$

The potential energy $P$ of the system is determined by the following expression:

$$P = c\left(l_1\beta - y\right)^2, \quad (14)$$

where $c$ is the stiffness coefficient of the tires of the wheels of the running system, N/m; $l_1$ - distance from the axis of suspension of the machine to the axis of the copying wheels, m

The dissipative function of this dynamic system is as follows:

$$R = \mu\left(l_1\dot{\beta} - \dot{y}\right)^2, \quad (15)$$

where $\mu$ is the damping factor of the copying wheels, N·s/m.

We now find generalized forces $Q$ in generalized coordinates $\beta$ and $y$, which are included in the right-hand side of the Lagrange equation of the second kind (8). To determine the generalized force $Q_y$ on the independent coordinate $y$, we use the expression of the elementary work of forces on a possible displacement $\delta y$:

$$\delta W_y = \sum_{i=1}^{n} Q_y \cdot \delta q_y = -G \cdot \delta y - f\,N \sin\alpha \cdot \delta y + \\ + F_R \sin\beta \cdot \delta y + N\cos\alpha \cdot \delta y - P\sin\beta \cdot \delta y . \quad (16)$$

It should be noted that the normal and tangential reactions of the soil on the copying wheel perform elementary work, respectively, on the deformation of the soil and overcoming friction, ie are active forces.

How do we have that the generalized force $Q_y$ is equal to:

$$Q_y = \frac{\delta W_y}{\delta y} = -G - f\,N\sin\alpha + F_R \sin\beta + N\cos\alpha - P\sin\beta . \quad (17)$$

To determine the generalized force $Q_\beta$ on the independent coordinate $\beta$, we use the expression for the elementary work of forces on a possible displacement $\delta\beta$. We will have:

$$\delta W_\beta = N \cdot BB'' \cdot \delta\beta + fN \cdot BB' \cdot \delta\beta + F_R \cdot H \cdot \delta\beta - G \cdot BC' \cdot \delta\beta \quad (18)$$

where $BB''$, $BB'$, $BC'$ – shoulders of forces $\bar{N}$, $f\bar{N}$, $\bar{G}$ concerning a point $B$ accordingly (fig. 2).

From expression (18) we find the value of the generalized force on the angular coordinate $\beta$, which will be equal to:

$$Q_\beta = \frac{\delta W_\beta}{\delta\beta} = N \cdot BB'' + fN \cdot BB' + F_{on} \cdot H - G \cdot BC' . \quad (19)$$

Thus, the generalized force $Q_\beta$ in this case is the algebraic sum of the moments of all forces acting on a given dynamic system with respect to a point $B$.

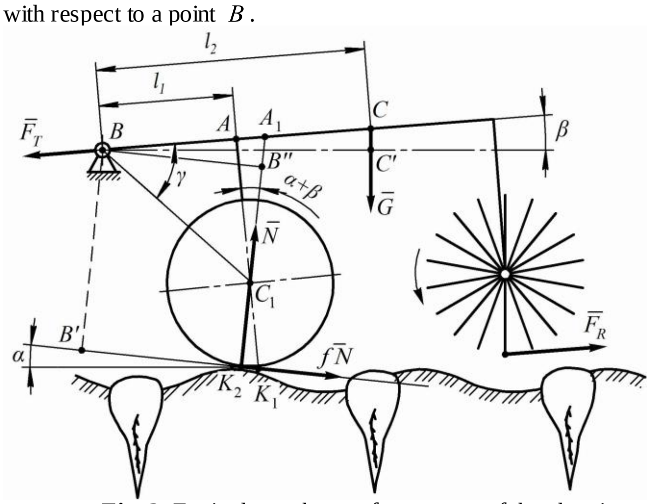


***Fig. 2.*** *Equivalent scheme of movement of the cleaning machine at the time of impact on the unevenness of the soil surface*

Determine the shoulders of the forces included in expression (19), using Fig. 2. As you can see from the figure:

$$BB'' = l_1\cos(\alpha+\beta) + (H-r)\cdot\sin(\alpha+\beta) \quad (20)$$

where $\alpha$ is the angle of tangent inclination to sine (1).

As is known, the tangent of the angle of inclination tangent to the curve $y = f(x)$ at a given point is equal to the derivative of the equation of this curve on the variable $x$ at this point, ie: $tg\alpha = y'_x$. Taking into account expression (1), we obtain:

$$y'_x = hk\cos\left(kx - \frac{\pi}{2}\right). \quad (21)$$

So:

$$\alpha = arctg\left[hk\cos\left(kx - \frac{\pi}{2}\right)\right]. \quad (22)$$

Next, from Fig. 2 we get:

$$BB' = r + \cos\left(90^o + \alpha + \beta - \gamma\right)\sqrt{\left(H-r\right)^2 + l_1^2} . \quad (23)$$

With:

$$\mathrm{tg}\gamma = \frac{AC_1}{AB} = \frac{H-r}{l_1}, \quad (24)$$

then

$$\gamma = \mathrm{arctg}\frac{H-r}{l_1} . \quad (25)$$

Next, from the triangle $BCC'$ we obtain:

$$BC' = l_2\cos\beta . \quad (26)$$

Using the obtained expressions (20), (23), and (26) and substituting them in expression (19) we obtain the value of the generalized force $Q_\beta$ in the coordinate $\beta$:

$$Q_\beta = N\left[l_1\cos(\alpha+\beta) + (H-r)\cdot\sin(\alpha+\beta)\right] + \\ + fN\left[r + \cos\left(90^o + \alpha + \beta - \gamma\right)\sqrt{\left(H-r\right)^2 + l_1^2}\right] + \\ + F_R \cdot H - Gl_2\cos\beta . \quad (27)$$

Having further determined the necessary partial derivatives of the kinetic energy $T$ of a given dynamic system, potential energy $P$, dissipative function $R$ and substituting the obtained values of generalized forces $Q_y$ and $Q_\beta$ the original Lagrange equations of the second kind (8), we finally obtain a system of differential equations of the following form:

$$\left.\begin{aligned} & I_B\ddot{\beta} = N\left[l_1\cos(\alpha+\beta) + (H-r)\cdot\sin(\alpha+\beta)\right] + \\ & + fN\left[r + \cos\left(90^o + \alpha + \beta - \gamma\right)\sqrt{\left(H-r\right)^2 + l_1^2}\right] + F_R \cdot H - \\ & - Gl_2\cos\beta - 2cl_1\left(l_1\beta - y\right) - 2\mu l_1\left(l_1\dot{\beta} - \dot{y}\right), \\ & M\ddot{y} = -G - fN\sin\alpha + F_R\sin\beta + N\cos\alpha - P\sin\beta + \\ & + 2c\left(l_1\beta - y\right) + 2\mu\left(l_1\dot{\beta} - \dot{y}\right). \end{aligned}\right\} \quad (28)$$

Where do we have:

$$\left.\begin{aligned} & \ddot{\beta} = \frac{N\left[l_1\cos(\alpha+\beta) + (H-r)\cdot\sin(\alpha+\beta)\right]}{I_B} + \\ & + \frac{fN\left[r + \cos\left(90^o + \alpha + \beta - \gamma\right)\sqrt{\left(H-r\right)^2 + l_1^2}\right]}{I_B} + \\ & + \frac{F_R \cdot H - Gl_2\cos\beta - 2cl_1\left(l_1\beta - y\right) - 2\mu l_1\left(l_1\dot{\beta} - \dot{y}\right)}{I_B}, \\ & \ddot{y} = \frac{-G - fN\sin\alpha + F_R\sin\beta + N\cos\alpha - P\sin\beta}{M} + \\ & + \frac{2c\left(l_1\beta - y\right) + 2\mu\left(l_1\dot{\beta} - \dot{y}\right)}{M}. \end{aligned}\right\} \quad (29)$$

The obtained system (29) of two differential equations is a calculated mathematical model of the movement of machine for cleaning root heads installed behind the aggregating tractor.

## 4. Conclusions

1. On the basis of the developed equivalent scheme of movement of the unit the is received system of two nonlinear differential equations that allows to investigate in detail oscilations of root crops heads cleaner in the longitudinal-vertical plane at

movement of its pneumatic copying wheels on roughnesses of a soil surface.

2. The calculated mathematical model of movement of the cleaner with a horizontal cleaning shaft is developed on the basis of initial equations of dynamics in the form of Lagrange of the 2nd kind allows to establish dependecies between constructive and kinematic parameters of this machine and its oscillatory characteristics.

3. The purpose of further mathematical numerical modeling of the parameters of the rear-mounted root heads cleaner with horizontal cleaning shaft, using the developed model is to optimize the values of angle $\beta$ and coordinates $y$ that characterize the oscillations of the machine in the longitudinal-vertical plane.

## 5. References


1. Kolychev E.I., Pereltsvaig I.M.On the choice of the calculated case of impact in the study of the smooth running of tractors and agricultural machines. Tractors and agricultural machinery. – 1976. – №3. – pp. 9.–11. (in Russian)

2. Chudakov D.A. Osnovy teorii i rascheta traktor i avtomobilya [Fundamentals of the theory and calculation of a tractor and an automobile]. - 2nd ed., Rev. and add. - M .: Kolos, 1972. – 384 p. (in Russian)

3. Vasilenko P.M. On equations of the dynamics of systems with nonholonomic constraints. - In the book: Agricultural mechanics - a collection of works. - M .: Mashinostroyeniye, 1968. – Vol. II. – pp. 26–34. (in Russian)

4. Khvostov V.A., Reingart E.S. Machines for harvesting root crops and onions (theory, design, calculation). - Moscow: VISKHOM, 1995. - 391 s. (in Russian)

5. Pogorelij L.V., Tatjanko, N.V., Brej V.V. et al. Svekloouborochniej mashiny: (konstruktirovanie i raschot).. Pod obshchej redakciej L.V. Pogorelogo. Kiev, Tekhnika, 1983, pp.168. (In Russian)

6. Adamchuk V., Bulgakov V., Korenko M., Boris A., Boris M. Laboratory and field equipment workingout and the results of experimental studies of pre-harvesting sugar beet field conditions. Mechanization in agriculture. – Sofia, Bulgaria, 2016. – Issue 1. – pp. 3 – 5.

7. Bulgakov V., Adamchuk V., Nozdrovicky L., Ihnatiev Ye.Theory of vibrations of sugar beet leaf harvester front-mounted on universal tractor. Acta Technologica Agriculturae. – 2017. – Issue 4. – pp. 96 – 103.

8. Butenin N.V., Lunc Ja.L., Merkin D.R. Kurs teoreticheskoj mekhaniky. Tom II. Dynamika. – Moskva: Nauka, 1985. – 496 p. (in Russian)

9. Vasilenko P.M. Vvedenie v zemledelcheskuju mekhaniku. – Kiev: Selkhozobrazovanie, 1996. – 252 p. (in Russian)

10. Bulgakov V., Pascuzzi S., Arak M., Santoro F., Anifantis A.S., Ihnatiev Y., Olt J. An experimental investigation of performance levels in a new root crown cleaner. Agronomy Research. 2019. Vol. 17. No 2. p.p. 358–370.

11. Ihnatiev Y. Theoretical substantiation of topping parameters without sugar beet head copying. IV International scientific Congress “Agricultural machinery”. – Varna. – Issue 18 (181). Vol. 1, 22-25 june 2016. – P. 55 – 58.

12. Bosoj E.S. et al. Theory, design and calculation of agricultural machinery. Moscow, Mashinostroenie, 1978, pp. 567. (In Russian)

13. Morozov B.I., Gringauz N.M. Calculation of the movement of a wheeled vehicle on an uneven road. - Mechanization and electrification of socialist agriculture. – 1969. – № 7. – С. 11-14. (In Russian)

14. Reznik N.E. The theory of cutting with a blade and the basics of calculating cutting devices. - Moscow: Mechanical Engineering, 1975. – 311 p. (In Russian)