Kinematic analysis of the slider-crank mechanism of an internal combustion (IC) engine using modern software

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Abstract: The subject of research in this paper is the kinematic analysis of the slider-crank mechanism of an internal combustion (IC) engine. The piston, connecting rod and crankshaft are the most important moving parts of an IC engine. The performance of IC engines largely depends on the design of the mentioned parts. Special attention should be paid to the kinematic analysis of the slider-crank mechanism, when constructing IC engines. Properly performed kinematic analysis is a prerequisite for the design of an efficient IC engine. Today, there are many software packages that make it much easier for engineers to perform fast and efficient kinematic analysis of machine assemblies. This paper presents the use of kinematic analysis software on the example of a slider-crank mechanism, an IC engine.

Keywords: IC ENGINE, PISTON, CRANKSHAFT, CONNECTING ROD, 3D MODELING

1. Introduction

In the general case, an engine is a propulsion machine that converts some kind of energy into mechanical work. Reciprocating internal combustion engines (IC engines) belong to the category of heat engines, which are used as propulsion machines in everyday use to overcome some external resistance in order to perform and perform certain work. The name heat engine refers to a group of devices that use the thermal energy developed by fuel combustion to perform certain work. The name heat engine refers to a group of engines that use heat to perform mechanical work. The IC engine is a heat engine with a four-stroke cycle. The choice of design concept and engine type depends on the following properties:

- low coefficient of friction,
- low thermal expansion,
- high strength, (even at high temperatures),
- low specific gravity (due to inertia),
- high strength, (even at high temperatures),
- good thermal conductivity,
- low thermal expansion,
- low coefficient of friction,
- high wear resistance.

Pistons must meet a number of different requirements, so the material from which the pistons are made must have [11]:

- the mass of the piston must be as small as possible, so that its force of inertia is as small as possible (increases with the square of the angular velocity) at higher angular velocities,
- the stiffness of the piston face must be high, in the area of the shaft bearing as low as possible, and the piston body must be elastic.
- piston grooves must have high strength, so that the links do not damage them.
- the strength must be high even at elevated temperatures, especially in the area of the piston face.
- the piston material must have good thermal conductivity so that the temperature differences on the piston are as small as possible and so that the heat dissipation to the cylinder is as good as possible.

The piston must be designed so that it stretches as much as possible with increasing temperature, so that the gap between the piston and the cylinder is as small as possible even in the cold state, because then the sealing is better and the noise is less [10].

In operation, the piston is exposed to high mechanical and thermal loads and in these conditions it must:

- to separate the combustion space from the crankshaft housing by means of piston rings,
- to transmit gas forces on the connecting rod,
- to transfer the normal (lateral) force generated during the transmission of gas forces on the connecting rod to the cylinder,
- transfer heat from the piston head to the cylinder wall,
- in the case of two-stroke engines, control the gas exchange (switching mechanism)

In order to meet the above requirements, the piston must have the following properties:

Figure 1 shows the piston.

2. Basic parts of the piston mechanism

2.1. Piston

The piston is the most loaded part of the engine. For its proper functioning in the IC engine, it is necessary to correctly determine the shape, dimensions and material of the piston. Therefore, for decades, the construction and production of pistons have been dealt with mainly by specialized manufacturers, who act as partners of engine manufacturers.

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2.2. Piston rings

The basic tasks of piston rings are:
- sealing of the combustion chamber,
- participation in heat dissipation from the piston to the cylinder wall,
- regulation of oil film for lubrication.

These requirements are performed by piston rings:
- adhering with the outer (working) surface to the cylinder wall with a certain pressure,
- impact bearing on the side surfaces of the groove due to axial acceleration under the action of gas forces, friction forces and its own inertial force [10].

2.3. Piston pin with fuses

The basic task of the piston pin is to make the articulated connection of the piston and the connecting rod. The piston pin with the piston is shown in Figure 2.

When the piston pin is in a loose bearing, it must be prevented from moving and damaging the cylinder walls. This is achieved by radial steel rings (Seeger ring or spring wire ring. For easier assembly, the Seeger ring has eyelets and the spring ring has hooked ends) [11].

2.4. Connecting rod

The connecting rod is the element that connects the piston and the crankshaft and converts a straight line into a circular motion. The connecting rod transmits the force from the piston to the crankshaft which converts the force into torque. Due to the great responsibility of the connecting rod in the operation of the engine, high rigidity must be ensured with a minimum weight of the same.

In the operation of the engine on the connecting rod operate:
- pressure forces in the longitudinal direction due to the action of combustion gases on the piston head,
- dynamic stresses due to inertial forces due to changes in velocity,
- bending forces due to piston rod movement.

The connecting rod is shown in Figure 3.

2.5. Crankshaft

The crankshaft transmits torque and is one of the most responsible, complex, stressful and expensive engine parts. The following requirements must be met for the crankshaft to function properly:
- there must be sufficient safety to prevent fatigue fatigue of the entire working space,
- there must be no large amplitudes of torsional, bending and axial oscillations,
- the inertial load must be brought to a reasonable level,
- crankshaft deformations must be kept to a minimum reasonable level.

The tasks of the crankshaft are:
- to convert uneven rectilinear motion into rotation,
- to convert the connecting rod force into torque,
- to transfer most of the torque to the coupling via the flywheel
- to deliver a small part of the torque to the assemblies (distribution mechanism, ignition distributor, fuel and coolant pump, fan, alternator).

The crankshaft must accelerate and decelerate the pistons and connecting rods in each stroke, so large forces of inertia act on the crankshaft [11].

Figure 4 shows the crankshaft.

3. Kinematics of axial piston mechanism

The piston engine mechanism is used to convert the rectilinear oscillatory movement of the piston in the cylinder into the rotational movement of the crankshaft by means of a connecting rod that performs a balancing movement. This kinematic circuit allows the work on the piston, which results from the development of the operating cycle, to be obtained in the form of torque on the engine shaft. A engine mechanism consisting only of a piston, connecting rod and crankshaft is called a simple engine mechanism. With a complex engine mechanism, several elements are involved in the
transmission of movement from the piston to the crankshaft. At a certain stationary velocity mode of operation of the engine, it can be considered that the angular velocity of the crankshaft is constant, since it is maintained within the flywheel within a very narrow range. In this case the knee angle of the crankshaft is given by the expression:

\[ \alpha = \omega t \text{ (rad)} = \frac{180}{\pi} \cdot \frac{mn}{30} \text{ (degree of crankshaft)} \]  

(1)

The angle of the crankshaft is given as a function of time only, so all kinematic quantities, i.e., path, velocity and acceleration can only be expressed as a function of the crankshaft angle, i.e., time \( t \) [3].

3.1. Piston path

![Fig. 5 Schematic representation of the piston mechanism] [6]

Basic structural dimensions from Figure 3.1. are:

- \( r \) - radius of the crankshaft knee,
- \( L \) - connecting rod length,
- \( \lambda = r / L \) - dimensionless constructive characteristic of the curving mechanism,
- \( S = 2 \cdot r \) - piston stroke,
- \( A' \) - top dead center (TDC),
- \( A'' \) - bottom dead center (BDC),
- \( \alpha \) - crankshaft rotation angle,
- \( O \) - crankshaft rotation axis,
- \( B \) - axis of the flying crankshaft arm,
- \( A \) - axis of the piston group shaft,
- \( \omega \) - angular velocity of the crankshaft.

According to Figure 5, the path of the piston \( S_a \) that it crossed from the external dead center is:

\[ S_a = A'O - AC - CO = L + r - L \cos \beta - r \cos \alpha. \]  

(2)

From the triangle ABO, by the sine rule we have:

\[ \frac{L}{\sin \alpha} = \frac{r}{\sin \beta}, \]  

(3)

and from here:

\[ \sin \beta = \frac{r}{L} \sin \alpha = \lambda \sin \alpha. \]  

(4)

Using the known trigonometric relationship:

\[ \sin^2 \beta + \cos^2 \beta = 1, \]  

(5)

we will have:

\[ \cos \beta = \sqrt{1 - \sin^2 \beta} = \sqrt{1 - \lambda^2 \sin^2 \alpha}. \]  

(6)

Developing this expression in Taylor’s order we get:

\[ \cos \beta \approx 1 - \frac{1}{2} \lambda^2 \sin^2 \alpha = 1 - \frac{1}{2} \lambda^2 \sin^2 \alpha - \frac{1}{16} \lambda^4 \sin^4 \alpha - \ldots \]  

(7)

Substituting the expression for \( \cos \beta \) in equation (2) we get after arranging:

\[ S_a \approx r \left( 1 - \cos \alpha + \frac{1}{2} \lambda \sin^2 \alpha + \frac{1}{8} \lambda^2 \sin^4 \alpha - \frac{1}{16} \lambda^4 \sin^6 \alpha + \cdots \right). \]  

(8)

As the values of the kinematic characteristic (\( \lambda = r / L \)) are relatively small, only the first four terms of the previous equation can be retained without major error, so that the approximate expression is obtained for the piston path:

\[ S_a \approx r \left( 1 - \cos \alpha + \frac{1}{2} \lambda \sin^2 \alpha \right). \]  

(9)

Introducing replacement:

\[ \sin^2 \alpha = \frac{1 - \cos 2 \alpha}{2}, \]  

(10)

we get:

\[ S_a \approx r \left[ 1 - \cos \alpha + \frac{1}{4} \lambda (1 - \cos 2 \alpha) \right], \]  

(11)

or:

\[ S_a \approx r \left( 1 - \cos \alpha + \frac{1}{4} (1 - \cos 2 \alpha) \right) = S_{1a} + S_{2a}. \]  

(12)

The last expression allows the construction to be curved \( S_{1a} = r \left( 1 - \frac{1}{2} \cos \alpha \right) \) and \( S_{2a} = r \frac{1}{4} (1 - \cos 2 \alpha) \) and by their graphical addition, the law of change of the piston path as a function of the angle of the crankshaft knee is obtained graphically [3].

The path of the piston is not a simple harmonic function, but a complex harmonic function composed of at least two harmonics, i.e., two simple trigonometric functions. The first part has a significant amplitude \( r \) and a frequency identical to the crankshaft velocity. The second part has a significantly smaller amplitude \( r\lambda / 4 \) (about sixteen times smaller), and twice the frequency of change. Higher harmonics, which have an even higher frequency, can be neglected because they have an extremely small amplitude [1].

The change of the piston path of the first and second row as well as the total path of the piston for, for one full revolution of the crankshaft is shown in Figure 6.
3.2. Piston Velocity

By differentiating the path of the piston by time, the velocity of the piston is obtained:

$$v_p = \frac{dS_0}{dt} = \frac{dS_0}{d\alpha} \frac{d\alpha}{dt} = S_0 \cdot \omega$$  \hspace{1cm} (13)

By differentiating the expression for the piston path and including the found value $S_0$ in the previous expression, the following is obtained:

$$v_p = S_0 \omega = rw \left[ \sin \alpha \left( \frac{\lambda}{2} + \frac{\pi^2}{8} + \frac{15\pi^4}{256} \sin 2\alpha - \left( \frac{3\pi^2}{16} + \frac{31\pi^4}{256} \sin 2\alpha \right) \right]$$  \hspace{1cm} (14)

If only members that do not contain the ratio $\lambda$ at higher degrees are retained, an approximate pattern for piston velocity is obtained:

$$v_p = rw \sin \alpha + r\frac{\pi^2}{2} \sin 2\alpha = v_{0p} + v_{2p}$$  \hspace{1cm} (15)

By constructing the curves $v_{0p} = rw \sin \alpha$ and $v_{2p} = r\frac{\pi^2}{2} \sin 2\alpha$ and summing them, the law of change of piston velocity as a function of the crankshaft angle is obtained graphically [1, 3, 10].

Piston velocity is also a complex harmonic function, with the first harmonic having a significant amplitude and frequency equal to the crankshaft velocity, while the second harmonic has a smaller amplitude (about eight times) and twice the frequency [1].

The change of piston velocity of the first and second row, as well as the total piston velocity for one full turn of the crankshaft is shown in Figure 7.

![Fig. 7 Piston velocity](image)

3.3. Piston Acceleration

Piston acceleration is obtained by differentiating the expression for velocity over time:

$$a_p = \frac{dv_p}{dt} = \frac{dv_p}{d\alpha} \frac{d\alpha}{dt} = v_{0p} \cdot \omega.$$  \hspace{1cm} (16)

By differentiating the expression for the piston velocity or by double differentiating the expression for the piston path and including the found value in the previous expression, the following is obtained:

$$a_p = rw^2 \left[ \cos \alpha \left( \frac{\lambda}{2} + \frac{\pi^2}{8} + \frac{15\pi^4}{256} \cos 2\alpha - \left( \frac{3\pi^2}{16} + \frac{31\pi^4}{256} \cos 2\alpha \right) \right] \right]$$  \hspace{1cm} (17)

Ignoring terms containing $\lambda$ to a power greater than 1, we obtain a simplified expression for piston acceleration in the form:

$$a_p = rw^2 \left( \cos \alpha + \cos 2\alpha \right) = rw^2 \cos \alpha + rw^2 \cos 2\alpha = a_{0p} + a_{2p}.$$  \hspace{1cm} (18)

By constructing the curves $a_{0p} = rw^2 \cos \alpha$ and $a_{2p} = rw^2 \cos 2\alpha$ and their graphical addition, we obtain the law of change of piston acceleration as a function of changing the knee angle of the crankshaft [1, 3, 10].

Piston acceleration is also a complex harmonic function, with the first harmonic having a significant amplitude and crankshaft frequency, while the second harmonic has a smaller amplitude (about four times smaller) and twice the frequency [1].

Change in piston acceleration of the first and second rows, as well as the total piston acceleration for one full crankshaft revolution is given in Figure 8.

![Fig. 8 Piston acceleration](image)

3.4. Connecting Rod Kinematics

The connecting rod performs a complex movement in a plane perpendicular to the longitudinal axis of the crankshaft, with point A (Figure 3.1) corresponding to the axis of the piston shaft, makes a rectilinear oscillatory movement along the axis of the cylinder, while point B, corresponding to the axis of the crankshaft arm, rotates evenly along the trajectory of the knee sleeve axis.

The angle of rotation of the connecting rod in relation to the axis of the cylinder is obtained from relation (3):

$$\beta = \arcsin (\lambda \sin \alpha).$$  \hspace{1cm} (19)

The maximum rotation of the connecting rod is at angles $\alpha = 90^\circ$ and $\alpha = 270^\circ$:

$$\beta_{max} = \arcsin \lambda.$$  \hspace{1cm} (20)

The angular velocity of rotation of the connecting rod is:

$$\omega_{\beta} = \frac{d\beta}{dt} = \frac{d\beta}{d\alpha} \frac{d\alpha}{dt} = \omega \frac{d\alpha}{d\alpha}$$  \hspace{1cm} (21)

By differentiating expressions:

$$\sin \beta = \lambda \sin \alpha,$$  \hspace{1cm} (22)

we get:

$$\cos \beta = \frac{\lambda \cos \alpha}{\cos \beta},$$  \hspace{1cm} (23)

and from here:

$$\frac{d\beta}{d\alpha} = \frac{\lambda \cos \alpha}{\cos \beta}.$$  \hspace{1cm} (24)

Substituting this value into the previous expression for $\omega_{\alpha}$, we have:

$$\omega_{\beta} = \frac{\omega_{\alpha} \cos \alpha}{\cos \beta} = \frac{\omega_{\alpha} \cos \alpha}{\sqrt{1 - \lambda^2 \sin^2 \alpha}}.$$  \hspace{1cm} (25)
The maximum value of the angular velocity of the connecting rod is obtained at $\alpha = 0^\circ$ and $\alpha = 180^\circ$:

$$\omega_{cr,\text{max}} = \pm \omega \lambda.$$  \hfill (26)

By differentiating the angular velocity of the connecting rod over time, we find its angular acceleration:

$$\varepsilon_{cr} = -\frac{d\omega_{cr}}{dt} - \frac{d\omega_{cr}}{d\alpha} \frac{d\alpha}{dt} + \omega^2 \lambda \sin \alpha.$$  \hfill (27)

The maximum angular acceleration of the connecting rod is at $\alpha = 90^\circ$ and $\alpha = 270^\circ$:

$$\varepsilon_{cr,\text{max}} = \pm \frac{\omega^2 \lambda}{\sqrt{1 - \lambda^2}}.$$  \hfill (28)

Figures 9 and 10 show the angular velocity of the connecting rod and the angular acceleration of the connecting rod.

The angular velocity and angular acceleration of the connecting rod can be shown in the common chart as shown in Figure 11.

An approximate expression for the calculation of the connecting rod angular velocity can be written as:

$$\omega_{cr} = \lambda \omega \left( 1 + \frac{1}{8} \lambda^2 \right) \cos \alpha - \frac{1}{8} \lambda^2 \cos 3\alpha.$$  \hfill (29)

An approximate expression for the calculation of the piston angular acceleration is:

$$\varepsilon_{cr} = -\lambda \omega^2 \left( 1 + \frac{1}{8} \lambda^2 \right) \sin \alpha - \frac{3}{8} \lambda^2 \sin 3\alpha.$$  \hfill (30)

4. Kinematic analysis of piston axial mechanism using CATIA software package

4.1. Piston mechanism modeling

In order to perform a kinematic analysis of the piston mechanism, it is necessary to first create a CAD model of the complete mechanism. The creation of the CAD model was done in the CATIA software package. CAD models were created: piston, piston shafts, connecting rod, crankshaft and cylinder with crankshaft bearings. Figure 12 shows CAD models of the mentioned parts of the IC engine.

Figures 9 and 10 show the angular velocity of the connecting rod and the angular acceleration of the connecting rod.

Figure 13 shows the assembly of the mechanism (with the indicated edges of the created parts) for which the kinematic analysis will be performed.

An approximate expression for the calculation of the connecting rod angular velocity can be written as:
4.2. Kinematic analysis of the piston mechanism

In order to perform a kinematic analysis, it is necessary to define the appropriate boundary conditions of the assembly. The piston performs a rectilinear oscillatory movement, the crankshaft a rotational movement, while the piston rod performs a complex movement. It is necessary to define the law of movement of the mechanism, as the angular velocity of the crankshaft. In this case, the crankshaft angular velocity is set at 10 rpm. After defining the boundary conditions, the simulation of movement in a engine with a constant angular velocity of the crankshaft begins. After this, piston kinematics charts were generated. The kinematic analysis of the piston is given through the charts of path traveled, velocity and acceleration shown in Figures 14 to 16.

Fig. 14 Dependence of the piston path on the angle of rotation of the crankshaft for one revolution of the crankshaft

Fig. 15 Dependence of the piston velocity on the angle of rotation of the crankshaft for one revolution of the crankshaft

Fig. 16 Dependence of the piston acceleration on the angle of rotation of the crankshaft for one revolution of the crankshaft

Figures 17 to 19 show charts of the path traveled, the velocity and acceleration of the piston as a function of time for ten full revolutions of the crankshaft.

Fig. 17 Time-dependent Piston path for ten full crankshaft revolutions

Fig. 18 Time-dependent piston velocity for ten full crankshaft revolutions

Fig. 19 Time-dependent piston acceleration for ten full crankshaft revolutions

5. Conclusion

In this paper, the path, velocity and acceleration of the piston are calculated by software, depending on the time and depending on the angle of rotation of the crankshaft. The paper also presents a detailed theoretical kinematic analysis of the piston mechanism. During the kinematic analysis of the piston mechanism, the structural appearance of the parts for which the CAD model was made is not important, only the kinematic characteristic (λ) is important, ie the length of the connecting rod and the radius of the crankshaft. Considering that the same values would be obtained when kinematic calculation in other software such as Excel, which does not require the preparation of CAD models, would significantly reduce the time spent on kinematic analysis. However, the simulation of piston, connecting rod and crankshaft movement offered by CATIA is very interesting. Further research could take place in several directions. First of all, kinematic analysis of the deaxial mechanism should be performed. Since kinematic analysis is practically an introductory analysis for dynamic piston mechanism analysis, it is simply imposed that further research be related to dynamic piston mechanism analysis. One of the further researches could be engine balancing problems, as well as the analysis of crankshaft oscillations.
6. References