Development of mathematical model of plane-parallel movement of trailer harvesting machine

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Abstract. *This paper presents the basic principles of building a mathematical model of trailer harvesting, for which an example of a trailed combine harvester is considered. To develop this mathematical model, all the components are given, starting with the assembly of an equivalent scheme of plane-parallel movement of this harvesting unit, which consists of a wheeled aggregate tractor of classic layout and trailed behind the combine. And behind the combine is attached a cart for the harvest. Differential equations of relative motion of the collecting unit are compiled and solved on the basis of the Lagrange equation of the second kind.*

KEYWORDS*: BRUSHING CALCULATION, TRAILER HARVESTING MACHINE, DIFFERENTIAL EQUATIONS, GENERALIZED COORDINATES, GENERALIZED FORCES.*

1. Introduction

The main indicator of the quality of the technological process of harvesting is the loss. Thus, when harvesting cereals by combing plants at the root, the losses may be remains and not combing. Loss of remains depends on the regulation of the working bodies of the combing device and the kinematic mode of rotation of the combing drums. Losses without combing depend on the straightness and stability of the harvesting machine. If the trajectory of the harvesting machine deviates from straightness, significant grain losses occur due to non-combing. To ensure the stability of the movement of the harvesting machine, it is necessary to study its dynamics.

Fundamentals of stability of motion of a mechanical system are considered by Lyapunov A.M. in [1]. Further development of the theory of stability of motion was obtained in the works of Malkin I.G. [2] and Merkin D.R. [3]. Works by Vasilenko P.M. [4, 5] are devoted to the study of the dynamics and stability of the movement of trailer agricultural machines and units. The most complete questions of the dynamics of trailer units are studied in the works of Gyachev L.V. [6, 7]. Regarding the movement of collecting units, the issues of dynamics and stability of movement are considered in scientific works [8, 9, 10].

The purpose of this study is to compile the differential equations of motion of the harvesting machine in its plane-parallel motion and solve them in general for further use in determining the optimal parameters of this unit.

2. Results and Discussions

When conducting the study, we use a grain-harvesting machine-tractor unit of the combing type. So, consider the movement of the harvesting unit, consisting of a tractor MTZ-80, trailer harvester combing type and two-axle trailer-trolley 2PTS-4.0, which is used to collect the combed heap.

The harvesting unit is a three-link mechanical system, the portable movement of which is translational rectilinear. To simplify the analysis, we assume that the center of mass of the tractor in portable motion moves evenly, i e $V_0 = \text{const}$. The portable movement of the unit occurs together with the plane $X_1O_1Y_1$ Fig. 1 [8]. Under the influence of external factors (inequalities of the field surface), the links of the unit begin to produce relative motion.

The harvesting unit has five degrees of freedom. Thus, its relative motion will be determined by five generalized coordinates.

Consider each of them one by one. The tractor has two degrees of freedom in relative motion, so its position will be determined by two generalized coordinates.

Moving the center of mass of the tractor along the axis O_1X_1 will be determined by the generalized coordinate X_{S1}, rotation around the axis passing through the center of mass of the tractor – a generalized coordinate φ_1. Similarly, the rotation of the harvesting machine relative to the point of the trailer is denoted by the generalized coordinate φ_2.

The 2PTS-4.0 trailer is a two-link kinematic chain with two degrees of freedom. As generalized coordinates we take the angles of rotation φ_3 and φ_4 (Fig. 1).

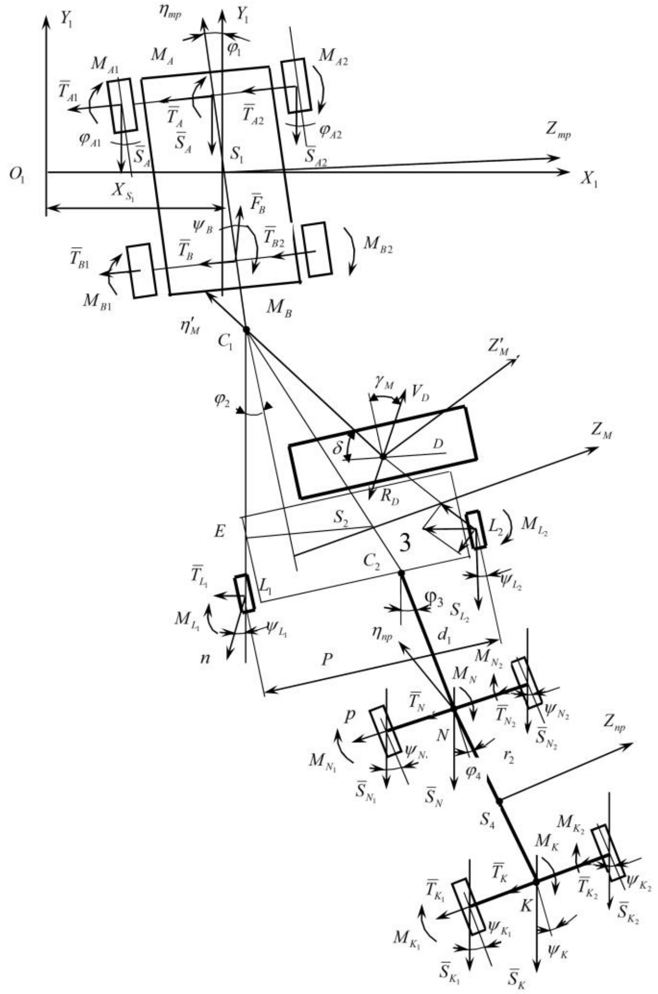

Figure 1. *Estimated scheme of the harvesting unit*

Consider the forces acting on the wheels of the unit.

Two groups of forces act on the wheels of the harvesting unit. The first group of forces acting on the wheels of the harvesting unit is the forces of elasticity of the tires arising from their transverse displacement ($\overline{T}_{A1}, \overline{T}_{A2}, \overline{T}_{B1}, \overline{T}_{B2}, \overline{T}_{L1}, \overline{T}_{L2}, \overline{T}_{N1}, \overline{T}_{N2}$), and the moments of elastic forces of the tires that occur when twisting each tire relative to the axis perpendicular to the surface of the field ($M_{A1}, M_{A2}, M_{B1}, M_{B2}, M_{L1}, M_{L2}, M_{N1}, M_{N2}, M_{K1}, M_{K2}$), (Fig. 1).

In fig. 1 marked: $\overline{T}_{A1}$ and $\overline{T}_{A2}$ – the elastic forces of the tires arising from the transverse displacement of the front wheels of the tractor relative to the bearing surface; $\overline{T}_{B1}$ and $\overline{T}_{B2}$ – the elastic forces of the tires arising from the transverse displacement of the rear wheels of the tractor relative to the bearing surface; $\overline{T}_{L1}$ and $\overline{T}_{L2}$ – forces of elasticity of the tires arising at cross shift of the left and right wheels of the harvesting machine concerning a basic surface; $\overline{T}_{N1}$ and $\overline{T}_{N2}$ – forces of elasticity of the tires arising at cross shift of forward wheels of the trailer 2PTS-4.0 concerning a bearing surface; M_{A1} and M_{A2} – moments of elastic forces of the tires of the front wheels of the tractor, which occur when twisting each tire relative to the axis perpendicular to the field surface; M_{B1} and M_{B2} – moments of forces of elasticity of tires of back wheels of a tractor arising at twisting of each tire concerning an axis of a perpendicular field surface; M_{L1} and M_{L2} – moments of forces of elasticity of tires of wheels of the harvesting machine arising at twisting of each tire concerning an axis of a perpendicular surface of a field; M_{N1} and M_{N2}; M_{K1} and M_{K2} – moments of elastic forces of the tires of the front and rear wheels of the trailer, which occur when twisting each tire relative to the axis perpendicular to the field surface.

The second group of forces acting on the wheels of the harvesting unit is the resistance forces to the unit moving over the field ($\overline{S}_{A1}$, $\overline{S}_{A2}$, $\overline{S}_{L1}$, $\overline{S}_{L2}$, $\overline{S}_{N1}$, $\overline{S}_{N2}$, $\overline{S}_{K1}$, $\overline{S}_{K2}$), (Fig. 1),

where: $\overline{S}_{A1}$ and $\overline{S}_{A2}$ – forces of resistance to rolling of front wheels of a tractor; $\overline{S}_{L1}$ and $\overline{S}_{L2}$ – forces of resistance to rolling of wheels of the harvesting machine; $\overline{S}_{N1}$ and $\overline{S}_{N2}$ – resistance forces to the rolling front wheels of the trailer; $\overline{S}_{K1}$ and $\overline{S}_{K2}$ – resistance forces to the rear wheels of the trailer.

In addition, there is another group of forces acting on the harvesting unit. These are the resistance forces arising during the operation of the computing device. Replace their action with the main vector $\overline{R}_D$, attached at the point D (Fig. 1).

To study the dynamics of a three-link aggregate, the problem of compiling differential equations of relative motion arises.

To do this, we use the original Lagrange equations of the second kind in generalized coordinates. Since the assembly unit has five degrees of freedom, using the recommendations [10] we will make five differential equations [9]:

$$\left.\begin{aligned}
&\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_1}\right)-\frac{\partial T}{\partial \varphi_1}=Q_1;\\
&\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_2}\right)-\frac{\partial T}{\partial \varphi_2}=Q_2;\\
&\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_3}\right)-\frac{\partial T}{\partial \varphi_3}=Q_3;\\
&\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_4}\right)-\frac{\partial T}{\partial \varphi_4}=Q_4;\\
&\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_{S_1}}\right)-\frac{\partial T}{\partial x_{S_1}}=Q_5
\end{aligned}\right\}. \tag{1}$$

where T – kinetic energy of the harvesting unit; Q_1, Q_2, Q_3, Q_4, Q_5 – generalized forces; $\dot{\varphi}_1$, $\dot{\varphi}_2$, $\dot{\varphi}_3$, $\dot{\varphi}_4$, $\dot{x}_{S_1}$ – generalized speeds.

After determining the kinetic energy of the aggregate unit, generalized forces and substituting them in equation (1) we obtain a system of differential equations:

$$\begin{aligned}
&I_{S_1}\ddot{\varphi}_1+\ddot{X}_{S1}\cdot a\left(m_{у.м.}+m_{np}\right)+\ddot{\varphi}_1 a^2\left(m_{у.м.}+m_{np}\right)+\\
&+\ddot{\varphi}_2 abm_{у.м.}+\ddot{\varphi}_2 anm_{np}+\ddot{\varphi}_3 ad_1m_{np}+\ddot{\varphi}_4 ar_2m_{np}=\\
&=T_A\cdot h_1-T_B\cdot h_2-M_A-M_B+F_B\cdot\psi_B\cdot h_2-\\
&-2T_L\cdot a+2S_L\cdot a\cdot\left(\varphi_2-\varphi_1-\psi_L\right)+R\cdot a\cdot\left(\varphi_2-\varphi_1-\gamma_M\right)-\\
&-T_N\cdot a-T_K\cdot a+S_N\cdot a\cdot\left(\varphi_3-\varphi_1-\psi_N\right)+\\
&+S_K\cdot a\cdot\left(\varphi_3+\varphi_4-\varphi_1-\psi_K\right).
\end{aligned}$$

$$\begin{aligned}
&I_{S_2}\ddot{\varphi}_2+\ddot{X}_{S1}\cdot nm_{np}+\ddot{\varphi}_1 abm_{у.м.}+\ddot{\varphi}_1 anm_{np}+\\
&+\ddot{\varphi}_2 b^2m_{у.м.}+\ddot{\varphi}_2 n^2m_{np}+\ddot{\varphi}_3 nd_1m_{np}+\ddot{\varphi}_4 nr_2m_{np}=\\
&=-T_L\cdot l-S_L\cdot l\cdot\psi_L-M_L-T_N\cdot n\cdot\sqrt{1-\frac{r_1^2}{b^2}}-\\
&-T_K\cdot n\cdot\sqrt{1-\frac{r_1^2}{b^2}}-S_N\cdot n\cdot(\varphi_3-\varphi_2-\psi_N)\cdot\sqrt{1-\frac{r_1^2}{b^2}}-\\
&-S_K\cdot n\cdot\left(\varphi_3+\varphi_4-\varphi_2-\psi_K\right)\sqrt{1-\frac{r_1^2}{b^2}}-\\
&-S_L\left(l\cdot\psi_L\sin\left(\arccos\frac{p}{l}\right)+\left(p+l\varphi_2\right)\right)-\\
&-T_L\cdot l\cdot\sqrt{1-\frac{p^2}{l^2}}-R\cdot\gamma_M\cdot c.
\end{aligned} \tag{2}$$

$$\begin{aligned}
&I_{S_3}\ddot{\varphi}_3+\ddot{X}_{S1}\cdot d_1m_{np}+\ddot{\varphi}_1 ad_1m_{np}+\ddot{\varphi}_2 nd_1m_{np}+\ddot{\varphi}_3 d_1m_{np}=\\
&=-T_Nd_1-S_Nd_1\psi_N-T_Kd_1-M_N-S_Kd_1\psi_K.
\end{aligned}$$

$$\begin{aligned}
&I_{S_4}\ddot{\varphi}_4+\ddot{X}_{S1}\cdot r_2m_{np}+\ddot{\varphi}_1 ar_2m_{np}+\ddot{\varphi}_2 nr_2m_{np}=\\
&=-T_Kd_2-S_K\psi_Kd_2-M_K.
\end{aligned}$$

$$\begin{aligned}
&\ddot{X}_{S1}\left(m_{mp}+m_{у.м.}+m_{np}\right)+\ddot{\varphi}_1 a\left(m_{у.м.}+m_{np}\right)+\\
&+\ddot{\varphi}_2 bm_{у.м.}+\ddot{\varphi}_2 nm_{np}+\ddot{\varphi}_4 r_2m_{np}+\\
&+\ddot{\varphi}_3 d_1m_{np}=-T_A-T_B-2T_L-T_N-T_K+S_A\left(\varphi_1-\psi_A\right)+\\
&+2S_L\left(\varphi_2-\psi_L\right)+S_N\left(\varphi_3-\psi_N\right)-\\
&-\left(S_A+S_L+S_N+S_K+R_D\right)\left(\varphi_1-\psi_B\right)-R_D\cdot\gamma_y.
\end{aligned}$$

The resulting differential equations are quite cumbersome because the harvester is a complex dynamic system with five degrees of freedom, on which a fairly large number of forces and moments act. To simplify further analysis, consider the motion of the harvester separately, replacing the bindings by their reactions.

In relative motion, the harvesting machine performs plane-parallel motion with one degree of freedom. The following forces and moments of forces act on the harvesting machine (Fig. 2) [12]: $\overline{T}_{L_1}$ and $\overline{T}_{L_2}$ – elastic forces of the left and right wheels of the harvesting machine; M_{L_1} and M_{L_2} – moments of elastic forces of the tires of the left and right wheels of the harvesting machine; $\overline{S}_{L_1}$ and $\overline{S}_{L_2}$ – the resistance forces of the left and right wheels of the harvesting machine; $\overline{R}'_{C_1}$ – elm response with a tractor; $\overline{R}_{C_2}$ – the reaction of the elm with the trailer-cart to collect the combed heap; $\overline{R}_D$ – the main vector of eye resistance forces.

To compile the differential equation of motion of the harvesting machine, we use the Lagrange equation of the second kind in generalized coordinates [11]. As a generalized coordinate we take the angle φ_2 (Fig. 2):

$$\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{\varphi}_2}\right]-\frac{\partial T}{\partial \varphi_2}=Q_3\,. \tag{3}$$

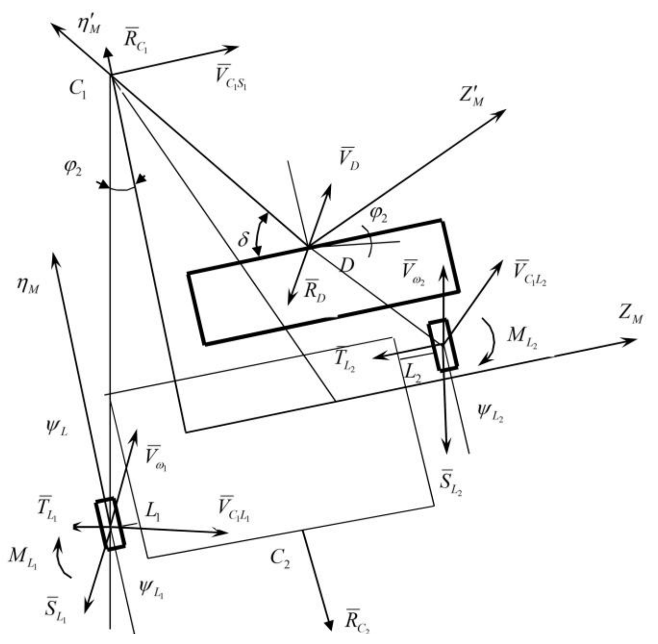

Figure 2. *Diagram of forces and moments of forces applied to the harvesting machine when replacing elms with their reactions*

Finally, we obtain the differential equation of the form [12]:

$$C_0 \cdot \dddot{\varphi}_2 + C_1 \cdot \ddot{\varphi}_2 + C_2 \cdot \dot{\varphi}_2 + C_3 \cdot \varphi_2 = 0; \qquad (4)$$

where:

$$\begin{aligned} &C_0 = I_{C_1}; \\ &C_1 = \frac{R_D \cdot C_R^2}{V_0} + V_0 \cdot k_L \cdot I_{C_1}; \\ &C_2 = R_D \cdot C_R + R_{C_2} \cdot n - \ell \cdot L + R_D \cdot C_R^2 \cdot k_L; \\ &C_3 = -V_0 \cdot L + V_0 \cdot k_L \cdot R_{C_2} \cdot n + R_D \cdot C_R \cdot V_0 \cdot k_L \end{aligned} \qquad (5)$$

Expressions (5) are the coefficients of the differential equation (4).

In the equations of expression (5) the following notations are accepted:

I_{C_1} – the moment of inertia of the harvesting machine relative to the point of its application to the tractor;

C_R – distance from the point C_1 hitching the machine to the tractor to the point D, application of the main vector of forces of resistance to combing;

n – the distance between the point of attachment C_1 of the machine to the tractor and the hitch point C_2 of the carts to the machine;

V_0 – the speed of the tractor that assembles the harvesting machine;

l – distance from the point C_1 of the attachment of the harvesting machine to the tractor to the wheel L_1;

k_L – coefficient of proportionality, which characterizes the elastic properties of the tires of the wheels of the harvesting machine;

L – designation determined from such an expression:

$$\begin{aligned} L = &-C_L \cdot l - C_L \cdot l \cdot \sqrt{1 - \frac{p^2}{l^2}} - 2k_L \cdot f_L - S_L \cdot l \cdot k_L - \\ &- S_L \cdot l \cdot k_L \cdot \sqrt{1 - \frac{p^2}{l^2}}; \end{aligned} \qquad (6)$$

where C_L – tire stiffness coefficient, at displacement; f_L – torsional stiffness of the tire; p – the distance between the wheels of the harvesting machine.

To find the angle φ_2 of deviation of the harvester from the rectilinear trajectory, solve differential equations (4), for which we divide the left and right parts of equation (4) by the coefficient C_0, as known coefficient $C_0 = I_C$ (I_C – moment of inertia of the harvester relative to the point C_1), that is $C_0 \neq 0$:

$$\dddot{\varphi}_2 + \frac{C_1}{C_0} \cdot \ddot{\varphi}_2 + \frac{C_2}{C_0} \cdot \dot{\varphi}_2 + \frac{C_3}{C_0} \cdot \varphi_2 = 0 . \qquad (7)$$

The following should be noted:

$$\frac{C_1}{C_0} = a; \quad \frac{C_2}{C_0} = b; \quad \frac{C_3}{C_0} = c. \qquad (8)$$

Then the differential equation (7) will look like:

$$\dddot{\varphi}_2 + a \cdot \ddot{\varphi}_2 + b \cdot \dot{\varphi}_2 + c \cdot \varphi_2 = 0 . \qquad (9)$$

Compose the characteristic equation:

$$n^3 + a \cdot n^2 + b \cdot n + c = 0 . \qquad (10)$$

As a result, a cubic equation was obtained. To solve it, we use the Cardano method [13], the essence of which is to reduce equation (10) to "incomplete form".

We introduce a substitution:

$$n = k - \frac{a}{3};$$

$$\left(k - \frac{a}{3}\right)^3 + a \cdot \left(k - \frac{a}{3}\right)^2 + b \cdot \left(k - \frac{a}{3}\right) + c = 0 . \qquad (11)$$

Perform algebraic transformations and then obtain the equation:

$$k^3 + k \cdot \left(-\frac{a^2}{3} + b\right) + \left[2 \cdot \left(\frac{a}{3}\right)^3 - \frac{a \cdot b}{3} + c\right] = 0 . \qquad (12)$$

Denote:

$$-\frac{a^2}{3} + b = f; \quad 2 \cdot \left(\frac{a}{3}\right)^3 - \frac{a \cdot b}{3} + c = \eta . \qquad (13)$$

In the final form, an incomplete cubic equation is obtained:

$$k^3 + k \cdot f + \eta = 0 . \qquad (14)$$

The roots of the incomplete cubic equation are equal to [14]:

$$k_1 = A + B; \quad k_2 = -\frac{A+B}{2} + i \cdot \left(\frac{A-B}{2} \cdot \sqrt{3}\right);$$

$$k_3 = -\frac{A+B}{2} - i \cdot \left(\frac{A-B}{2} \cdot \sqrt{3}\right), \qquad (15)$$

where $A = \sqrt[3]{-\frac{\eta}{2} + \sqrt{Q}}$; $B = \sqrt[3]{-\frac{\eta}{2} - \sqrt{Q}}$. (16)

In turn:

$$Q = \left(\frac{f}{3}\right)^3 + \left(\frac{\eta}{2}\right)^2 . \qquad (17)$$

Thus, obtained as a result of solving the cubic equation (14), we have one real root and two complexly conjugate roots. Then the general solution of differential equation (9) will look like [15, 16]:

$$\begin{aligned} \varphi = &C_1 \cdot e \cdot \frac{A+B}{2} \cdot t + e \cdot \left(-\frac{A+B}{2} \cdot t\right) \times \\ &\times \left[C_2 \cdot \cos\left(\frac{A+B}{2} \cdot \sqrt{3}\right) \cdot t + C_3 \cdot \sin\left(\frac{A+B}{2} \cdot \sqrt{3}\right) \cdot t\right], \end{aligned} \qquad (18)$$

where C_1, C_2, C_3 – integration steels, which are determined from the initial conditions.

Thus, the solution of the differential equation of relative motion of this harvesting unit is obtained. Using this solution, it is possible in the future with the help of PC to investigate the plane-parallel motion of the specified unit to determine the optimal construction and kinematic parameters.

4. Conclusion

As a result of the analytical studies, a new mathematical model of the relative motion of the trailed harvesting machine was constructed. To consider the specified motion of the harvesting machine, its equivalent scheme was initially constructed. Using the output Lagrangian equations of type II, the differential equation of relative motion of the given harvester and tractor unit was compiled and solved.

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