Determination of Resonance Parameters for Vibratory Machine Drives with Inertial and Kinematic Excitation

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Abstract. This article presents the general solution to the differential equations of motion for vibratory drives with both inertial and kinematic excitation in the first approximation during the passage through the resonance zone. Expressions for the maximum resonant amplitudes and the corresponding values of the phase and frequency were determined. An analysis of the resulting expressions is provided for the characteristic periods of acceleration and deceleration. The necessity of utilizing coupled dynamic models is underscored by the inherent instability observed in traditional simplified systems when operating beyond the natural frequency.

KEYWORDS: *MECHANICAL VIBRATIONS, VIBRATORY MACHINE, KINEMATIC DRIVE, RESONANCE.*

Introduction

Today, vibration technologies are exerting a wide influence on the intensification of technological processes across global and domestic production sectors. The distinctive characteristics and methods of applying vibration in agricultural production have proven beneficial for enhancing productivity and processing quality in tasks such as separation, transportation, and scattering [1-3]. Furthermore, vibratory working bodies are commonly employed in widely known machinery, including tillage equipment and root-tuber harvesting machines, in addition to other systems where the calculation of oscillations and vibrations is fundamental to operation [4, 5].

Despite the often relatively simple structural schemes of vibratory systems, the practical implementation and creation of robust vibratory machines for agricultural and processing enterprises frequently encounter significant engineering difficulties [6]. These challenges stem from the inherent complexity of the physical processes that occur during the dynamic interaction between the vibrating working parts of the machine and the processed medium or raw material. To design reliable vibratory equipment and ensure its effective operation, engineers must possess a clear conceptualization of the physical mechanism by which vibration acts on the material, alongside a solid grasp of the dynamic principles governing the initiation and maintenance of optimal machine operating parameters [7].

Consequently, the objective substantiation of the theoretical foundations and operational principles of vibratory machines, specifically targeting the enhancement of their technological and energy efficiency, remains an extremely vital engineering task.

A particular focus must be placed on mechanical systems where a reverse influence exists, meaning the movement of the oscillating part exerts a reciprocal effect on the excitation source. In such coupled systems, the active forces determining the system's dynamics depend significantly not only on the instantaneous state of the oscillating component but also on the state of the vibratory drive itself. The behavior of the drive, in turn, must be described by an additional differential equation or a complete system of equations.

The traditional engineering approach to analyzing the interaction between an oscillating system and its excitation source typically simplifies the problem by assuming the electric motor rotor's rotation is uniform, thus deeming its angular velocity independent of the system's movement. Under this assumption, the oscillations can be described simply by a linear differential equation. However, empirical and experimental studies have unequivocally established that the dynamic parameters found beyond the resonance frequency in such simplified systems are unstable. This instability underscores the inherent limitations of uncoupled linear models in predicting real-world behavior, especially when traversing critical frequency zones.

Therefore, the oscillatory process must be investigated jointly with the vibratory drive, explicitly accounting for the reverse influence exerted by the oscillating system back onto the drive. Analyzing the interaction between the oscillating system and the excitation source is critically important when examining transient operating modes, particularly the process of passing through resonance in vibratory machines utilizing inertial excitation. Analytical solutions for these non-linear, coupled systems are generally achieved through methods involving a small parameter or through asymptotic methods, which allow for the accurate mathematical description of non-linear vibratory effects and other characteristic phenomena.

The primary goal of this investigation was the theoretical study of the maximum resonant amplitudes and the corresponding values of the phase and rotation frequency for vibrator drives specifically featuring kinematic excitation, along with a detailed analysis of the expressions derived during the critical transient moments of run up and run out.

Results and Discussion

The dynamic behavior of the coupled mechanical systems under consideration—vibratory drives subject to either inertial or kinematic excitation—necessitates a comprehensive mathematical model that incorporates the feedback loop.

In the general case, the motion of these mechanical systems is defined by a coupled system of non-linear differential equations of the form:

$$\left.\begin{aligned} \ddot{x} + \omega^2 x &= \varepsilon f_1(x, \dot{x}, \ddot{x}, \varphi, \dot{\varphi}, \ddot{\varphi}), \\ \ddot{\varphi} &= \varepsilon f_1(x, \dot{x}, \ddot{x}, \varphi, \dot{\varphi}, \ddot{\varphi}). \end{aligned}\right\} \quad (1)$$

In this system, x represents the linear displacement of the system, and φ is the angular displacement of the drive. The variable ε is introduced as the small non-linear parameter of the system, quantifying the magnitude of the coupling and non-linear terms, while ω denotes the natural oscillation frequency of the primary system.

To address the inherent difficulty in solving these non-linear, coupled differential equations directly, the solution to the system (1) is sought in the zone of the resonant frequency using the well-established asymptotic method (also known as the Method of Averaging or Method of Small Parameter). The general solution structure derived through this methodology takes the following form in the first approximation:

$$\left.\begin{aligned} x &= A\sin\psi + \varepsilon u_1(A, \theta, v, \varphi) + \dots \ , \\ \psi &= \frac{p}{q}\varphi + \theta, \\ \frac{d\varphi}{dt} &= v + \varepsilon v_1(A, \theta, v, \varphi) + \dots \ . \end{aligned}\right\} \quad (2)$$

Here, the terms A, θ, and ν represent the amplitude, phase, and frequency of the oscillations, respectively, considered as slowly varying functions of time. Specifically, ν is defined as the frequency, representing the average value of the angular velocity of the drive shaft rotation over a cycle. The phase relationship ψ introduces the integers p and q, which are mutually prime numbers selected to match the specific order of the resonance being investigated.

The parameters A (amplitude), θ (phase), and ν (frequency) as functions of time are determined by a system of first-order differential equations:

$$\left.\begin{aligned} \frac{dA}{dt} &= \varepsilon a_1(A, \theta, v) + \varepsilon^2 \dots \ , \\ \frac{d\theta}{dt} &= \omega - \frac{p}{q}v + \varepsilon b_1(A, \theta, v) + \varepsilon^2, \\ \frac{dv}{dt} &= \varepsilon c_1(A, \theta, v) + \varepsilon^2. \end{aligned}\right\} \quad (3)$$

The numerical integration of these reduced equations (3), given the specific system parameters and initial conditions, provides a complete picture of the dynamic system's path as it passes through resonance. Crucially, this allows for the precise determination of the maximum transient values of the amplitudes and their corresponding phases and frequencies during non-stationary operation. This approach moves beyond the limitations of steady-state analysis, providing the necessary tools to predict and mitigate potentially destructive transient peaks.

The analysis of inertial vibratory drives, typically featuring rotating unbalanced masses (debalances) to generate excitation, requires a model that captures the physical constraints and forces acting on the coupled system. The motion of an inertial vibratory drive with two symmetrical rotating debalances is described by the general system of differential equations derived from mechanics:

$$\left.\begin{aligned} \ddot{x} + \omega^2 x &= \varepsilon[Q(\dot{\varphi}^2 \sin\varphi - \ddot{\varphi}\cos\varphi) - h\dot{x}], \\ \ddot{\varphi} &= \varepsilon[M_1(\dot{\varphi}) - Q_1(g + \ddot{x})\cos\varphi - L(\dot{x}, \varphi, \dot{\varphi}, \ddot{\varphi})], \end{aligned}\right\} \quad (4)$$

where the parameters (4) are defined as $\omega^2 = c/m$ (natural frequency squared); $\varepsilon Q = rm_0/m$; $\varepsilon h = \alpha/m$; $\varepsilon M_1(\varphi) = M_\varphi/J$; $\varepsilon Q_1 = rm_0/J$. Here, m is the mass of the oscillatory system, J is the moment of inertia of the rotating unbalances, α is the viscosity coefficient, and M_φ is the rotational moment supplied by the motor. The term $\varepsilon L(x, \dot{x}, \ddot{x}, \varphi, \dot{\varphi}, \ddot{\varphi})$ represents the complex friction moment acting in the vibrator bearings, where μ is the reduced friction coefficient.

Through the application of asymptotic methods to system (4), expressions for the maximum amplitudes (A), the corresponding phase (θ), and the rotational speed (ν) of the unbalances during the passage through the main resonance are determined. These critical values are found to be:

$$\left.\begin{aligned} A &= -\frac{2rm_0v^2 \sin\theta}{\alpha(\omega+\nu)}, \\ v &= \sqrt{\frac{\omega^2 + \frac{2m(\omega+\nu)}{J\alpha}\left(1 + \frac{rm_0v(2\omega+\nu)}{mA(\omega+\nu)^2}\cos\theta\right)}{1 + \frac{rm_0\cos\theta}{m\alpha}}} \times \\ &\times \sqrt{M(v) - \mu m_0 v^2 - \frac{\alpha A^2\omega^2(\omega+\nu)}{4v^2}} \\ \cos\theta &= \frac{rm_0v^2}{mA(\omega^2+v^2)}\left[1 - \left(\frac{\alpha A(\omega+\nu)}{2rm_0v^2}\right)^2\right] - \\ &- \frac{2[M'(v) - 2\mu m_0 r^2 v]}{Jrm_0A\omega^2(\omega-v)}\left[M(v) - \mu r^2 m_0 v^2 - \frac{\alpha A^2\omega^2(\omega+\nu)}{4v^2}\right]. \end{aligned}\right\} \quad (5)$$

These analytical expressions provide the quantitative basis for understanding the non-linear relationship between mechanical and electrical system parameters during dynamic transition.

A detailed analysis of the second equation in system (5) reveals a defining characteristic of inertial vibratory systems operating during acceleration (run up). The frequency ν that corresponds to the maximum transient amplitude is found to be greater than the natural oscillation frequency ω ($\nu > \omega$). This phenomenon, often associated with the Sommerfeld effect, where the motor must overcome a high-resistance region near resonance, is crucial for design.

The extent to which the maximum amplitude frequency ν exceeds the natural frequency ω is directly proportional to the engine's power reserve. This available power reserve is explicitly quantified by the term derived from the angular motion equation:

$$M(v) - \mu r^2 m_0 v^2 - \frac{\alpha A^2\omega^2(\omega+\nu)}{4v^2}. \quad (6)$$

In contrast to the run-up phase, the dynamics during the coasting period are distinctly different. During run out, the motor moment is effectively zero ($M(\nu) \equiv 0$). Consequently, the conditions shift such that the maximum amplitudes occur when the drive frequency is less than the natural frequency ($\nu < \omega$). This asymmetry confirms that the drive's nonlinear electrical characteristics govern acceleration behavior, while pure mechanical dissipation governs deceleration behavior, pushing the transient peak amplitude location toward the safer, sub-resonant frequency range during power cut off.

In systems employing kinematic excitation, the vibration is generated by the prescribed motion of a supporting element rather than an internal inertial mass. The motion of the vibrator with kinematic excitation is described by a structurally different set of coupled differential equations:

$$\left.\begin{aligned} m\ddot{x} + \beta x + cx &= c_1 r \sin\varphi, \\ J\ddot{\varphi} &= L(\dot{\varphi}) - H(\dot{\varphi}) + c_1 r(x - r\sin\varphi)\cos\varphi, \end{aligned}\right\} \quad (7)$$

where $c = c_0 + c_1$ represents the stiffness of the oscillatory system, β is the viscous damping coefficient, $L(\varphi)$ is the driving moment, and $H(\varphi)$ is the resistance moment applied to the drive shaft.

Applying asymptotic methods to system (7) yields expressions that define the maximum amplitudes and corresponding phase and excitation frequency values within the main resonance zone:

$$\left.\begin{aligned} A &= \sqrt{\frac{1}{2}\left(\frac{c_1 r}{\beta\omega}\right)\left(1 + \sqrt{1 - 4\gamma^2}\right)}, \\ \sin 2\theta &= -2\gamma, \\ L(v) - H(v) - \frac{J\beta\nu}{2m} &= \frac{1}{2}\beta A^2\omega - \frac{J\beta}{2m}\left(\omega - \frac{\beta\alpha}{2\omega(mA)^2}\right), \end{aligned}\right\} \quad (8)$$

where γ is the dimensionless parameter defined as $\gamma = \frac{\alpha m\omega}{\beta\omega c_1 r^2}$.

An important conclusion derived from this analysis is that the maximum amplitude A achieved during the transient regime (when passing through resonance) is inherently less than the corresponding resonant amplitude A_p achieved in the stationary regime. This relationship is approximated by $A \approx A_\text{p}\left(1 - \frac{\gamma^2}{2}\right)$. This difference between transient and stationary peaks is a critical safety margin in machine design, quantified by the dimensionless parameter γ.

Following the cessation of power, the vibratory drive system must decelerate and pass through resonance in the opposite (reverse) direction. Analyzing this run-out scenario is vital for understanding machine longevity.

Assuming the resistance moment during deceleration takes the linear form $H(\nu) = \alpha_1$, the derived transient parameters become:

$$\left.\begin{aligned} \gamma_1 &= \frac{\alpha_1}{m\omega}\left(\frac{\beta\omega}{c_1 r}\right)^2; \\ A_1 &\approx A_\text{p}\left(1 - \frac{\gamma_1^2}{2}\right); \\ \sin 2\theta_1 &= -2\gamma_1; \\ \nu_1 &= \omega\left(1 - \frac{mA_1^2}{J} + \frac{\beta\alpha_1}{2(m\omega A_1^2)^2}\right)\frac{1}{1 + \frac{2m\alpha_1}{J\beta}}. \end{aligned}\right\} \quad (9)$$

The derived expressions provide a direct, actionable quantitative link between the system's rotational resistance and the transient response. It is demonstrated that with an increase in the absolute value of the resistance coefficient to the drive's rotational motion, α_1, the maximum value of the transient amplitude A_1 decreases. This establishes that mechanical engineers can implement deliberate increases in rotational damping or friction during the shutdown sequence as an effective, measurable strategy to limit potentially destructive transient amplitudes.

Furthermore, the inertial drive during coasting, the rotational frequency ν_1 that corresponds to the maximum transient amplitude in the kinematic drive is found to be less than the natural frequency ω ($\nu_1 <$). This confirms the universally asymmetric behavior of transient dynamics in vibratory systems, regardless of the excitation type, during acceleration and deceleration.

Conclusions

This investigation successfully applied asymptotic methods to solve the complex differential equations governing the motion of coupled vibratory systems. The core objective of providing analytical solutions for transient dynamics during resonance passage has been achieved for drives utilizing both inertial (unbalanced) and kinematic excitation. Specific, highly detailed analytical expressions were found for the maximum resonant amplitudes and the corresponding values of the phase and excitation frequency for both drive types. These quantitative expressions are essential tools, replacing empirical uncertainty with predictable, parameter-based relationships. For inertial drives, the analysis confirmed the direct relationship between the system's capacity to shift the maximum amplitude frequency above resonance (run up) and the motor's power reserve, providing a quantifiable metric for non-linear stability control. Conversely, the analysis showed that deceleration (run out) peaks occur below the natural frequency. For kinematic drives, the derived equations established a clear

methodology for mitigating transient peaks during power cut-off. The analysis demonstrated that engineering an increase in the rotational resistance coefficient (α_1) during run-out directly minimizes the maximum transient amplitude, offering a critical design parameter for protecting these machines during non-stationary operation. Overall, the derived expressions are crucial for accurately analyzing and predicting dynamic processes throughout the operational lifespan of vibratory machine drives, specifically during the acceleration and coasting periods.

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