Modeling the Shaking Process of a Walnut Tree: Determining the Natural Vibration Frequency

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Abstract: *The results of a study of the shaking process of a walnut tree are presented. Shakers used in harvesting machines are specialized equipment that is attached to the tree trunk and generates controlled vibrations, causing the fruit to fall to the ground or into special catching devices. Thus, universal shakers for various types of nuts, including walnuts, almonds and hazelnuts, ensure fast and uniform harvesting without significant damage to the trees. When harvesting nuts by shaking, the dynamic properties of the trees are important. Therefore, it is necessary to calculate the natural frequency of the system's oscillations, and it plays a key role in the occurrence of the resonance effect.Thus, during the shaking process, the natural frequency of the system includes the tree and other elements interacting with it, which makes it possible to determine the optimal operating modes of the vibrators to ensure the required completeness of fruit removal with the least energy expenditure. For this purpose, the Euler-Bernoulli theory is used, according to which, when solving a dynamic equation for specific fastening conditions, the natural frequencies and corresponding vibration modes are determined, at which the beam oscillates without external forces.*

KEY WORDS: *OSCILLATION FREQUENCY, SHAKING PROCESS, WALNUT TREE, PHYSICAL MODEL.*

Introduction

Harvesting nuts is one of the most important stages in horticultural technology, determining not only the yield of the crop, but also the quality of the final product. Traditional harvesting methods such as manual labor or simple mechanical devices, which are generally labor-intensive and inefficient, are especially ineffective in large orchards or plantations of nut crops such as walnuts, hazelnuts and almonds.

Fruit shakers used in harvesting machines are specialized equipment that is attached to the tree trunk and generates controlled vibrations, causing the fruit to fall to the ground or into special catching devices. As noted by Bradu N., Melnic Iu. et al., mechanized harvesting of nuts provides a solution to the problem of efficient removal of fruits from trees, while reducing harvesting costs to 35-45% of the total cost of production, which ultimately contributes to an increase in overall production efficiency [1, 2, 3]. Compared with manual collection, the use of machines with pulse vibration exciters, mechanized harvesting can increase labor productivity several times [3, 4].

The most common mechanical harvesting system in orchards is shaking the trunks and crowns of trees. As noted by Mohsen Farajijalal et al., the branch shaking system enables selective fruit harvesting due to the varying vibration characteristics of different branches, as well as branches in the tree canopy, depending on their age, water content in the trunk, and elasticity [5]. The morphological characteristics of fruit trees, the diversity of fruit structures, growth patterns and agricultural practices in the orchard are the main factors in choosing the appropriate type of mechanized fruit harvesting system. These basic characteristics will help to select optimal vibration parameters such as frequency, amplitude, duration and type of shaking [4, 5].

Fruit shakers used in harvesting machines are specialized equipment that is attached to the tree trunk and generates controlled vibrations, causing the fruit to fall to the ground or into special catching devices. Thus, universal shakers for various types of nuts, including walnuts, almonds and hazelnuts, ensure fast and uniform harvesting without significant damage to the trees.

Prerequisites and means for solving the problem

Like any other tree, the walnut tree is a complex natural structure and its properties vary depending on the species and age. When harvesting trees, where their dynamic properties are important, it is necessary to calculate the natural frequency of the system's oscillations, since the natural frequency is the frequency at which the system oscillates on its own, without any external influence on it, and which plays a key role in the occurrence of the resonance effect. Thus, during the shaking process, the natural frequency of the system includes the tree and other elements interacting with it, which makes it possible to determine the optimal operating modes of the vibrators to ensure the required completeness of fruit removal.

From the theories of vibrations it is known that vibrations that are supported only by elastic force are called free or natural vibrations, and their frequency depends on the physical characteristics of the tree, such as the length of the trunk, the elasticity of the wood, the mass of the crown, etc. [6, 7]. In this case, the diameter of the cross-section A of the trunk within each section is considered constant and the material is homogeneous. This allows to apply Hooke's law, which states that for elastic deformations (small in magnitude), the normal stress σ in the body is directly proportional to the relative deformation ε, and the proportionality coefficient depends on the properties of the materials – Young's modulus E. In this case, the mass of the crown is brought to a common center of mass, and its interaction with the air is excluded [8].

It has been established that the falling of nuts when the tree is shaken is due to a physical phenomenon known as resonance. If we take into account that resonance is a phenomenon in which an oscillatory system reacts with a maximum oscillation amplitude δ to an external influence, the frequency of which coincides with the natural frequency of the system under study, then this occurs because the external force transfers maximum energy to this system.

In the particular case under study, the tree vibrates at a frequency close to its natural frequency, when the oscillation amplitude δ increases sharply. Therefore, to ensure fruit fall with minimal impact, mechanical shakers should be tuned to a resonant frequency, which will ultimately:

1) maximize fruit removal (up to 90–95%);
2) minimize damage to the tree trunk and branches;
3) reduce energy consumption.

Solution of the examined problem

To calculate the natural frequency of oscillations of a walnut trunk, physical and mathematical models are used to determine the natural circular frequencies for medium and large trunks, as well as the parameters of the rigidity of walnut wood, which are expressed through the expression of Hooke's law [8].

$$\sigma = E\varepsilon \tag{1}$$

where: ε is the relative deformation;
E - Young's modulus, Pa.

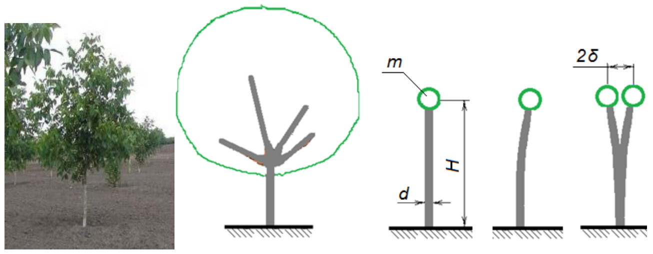
It is also formulated as a function of absolute elongation, load, length, and cross-sectional area.

Based on the selected physical model, electronic dynamometers connected to a computer are used in practice to determine the frequency of free oscillations or natural frequency.

The trunk is usually modeled as a cantilevered cylinder, rigidly fixed at the base and free at the end (top). For this purpose, the Euler-Bernoulli theory is used, according to which, when solving a dynamic equation for specific fastening conditions (for example, clamping), the natural frequencies and corresponding vibration modes are determined, at which the beam oscillates without external forces. It should be noted that the theory is most accurate for thin beams and small deformations. For short and thick beams or large deformations, more complex theories are required that take into account shear deformations and nonlinear effects.

Rezults and discussion

To perform calculations, it is necessary to adopt a calculation scheme, the physical model of which is presented in Fig.1. A real trunk is not perfectly uniform, usually spherical and less commonly conical. Root rigidity may not provide sufficient anchorage, which reduces the oscillation frequency. Branches and foliage on trees during harvesting increase their effective mass and also reduce the oscillation frequency *f* (Fig.1,b).



a) b)

Figure 1. Physical model for determining the frequency of free oscillations: a) walnut tree; b) physical model; *H – height of the tree; d - trunk diameter; m – nominal mass; δ - amplidude of vibration.*

If in the calculation scheme there is a concentrated mass m at the top of the rod (fruits, branches, leaves), then it is necessary to take into account the reduced mass (the usual correction), since with a large mass m the oscillation frequency *f* decreases.

The mathematical model for such a homogeneous cantilever beam of circular cross-section with diameter d expresses the natural angular frequencies ω_n (rad/s) in the following form [6, 7]:

$$\omega_n = \beta_n^2 \sqrt{\frac{EI}{\rho A l^4}} \qquad (3)$$

where: β_n are the coefficients of n modes of natural oscillations;

I - moment of inertia of the section, m^4;

ρ - bulk density of the trunk, kg/m^3;

A - cross-sectional area of the trunk, m^2;

$l=H$ - reduced length of the trunk or height of the tree, m.

In this case, Young's modulus for walnut wood is taken to be within the range of 8.0-14.0 GPa, depending on the variety, humidity and direction of the walnut wood fibers [8].

Since the mass of the tree is distributed along the trunk, according to the Euler-Bernoulli theory, there are n modes of natural oscillations [6, 7]. In formula (3), the coefficients β_n (the first three) are standard numbers:

$$\beta_1 \approx 1.9,\ \beta_2 \approx 4.7,\ \beta_3 \approx 7.9,...$$

When solving a dynamic equation for specific fastening conditions (for example, clamping), the natural frequencies and corresponding vibration modes are determined, at which the beam (trunk) vibrates without external forces.

It should be noted that the theory is most accurate for thin beams and small deformations. For short and thick beams or large deformations, more complex theories must be used that take into account shear deformations and nonlinear effects [8, 9, 10].

Then the natural frequency f_n of the trunk expressed in hertz (Hz) is equal to:

$$f_n = \frac{\omega_n}{2\pi} \qquad (4)$$

In practice, shaking of a walnut tree trunk occurs at the first fundamental frequency (n=1), because under real-world conditions, it is the one that manifests itself most strongly. Then, expression (3) takes the following form:

$$\omega = \omega_1 = \beta_1^2 \sqrt{\frac{EI}{\rho A l^4}} \qquad (5)$$

or simplified

$$\omega = \beta_1^2 \sqrt{\frac{EI}{\rho A l^4}} \qquad (5,a)$$

where $\beta_1 \approx 1.9$.

As a first example, we take an average trunk of a walnut tree with the following parameters:

- Young's modulus E=11.0 GPa;
- trunk diameter d takes values from 0.15 to 0.3 m;
- reduced length of the trunk or height H of the tree takes values from 4.0 to 8.0 m;
- bulk density of the trunk ρ= 650 kg/m^3.

First, the cross-sectional area of the trunk A is determined using the formula:

$$A = \frac{\pi d^2}{4} \qquad (6)$$

The moment of inertia I of a circular cross-section is determined by the formula:

$$I = \frac{\pi d^4}{64} \qquad (7)$$

Substituting the corresponding values of the parameters of the walnut tree trunk into formulas (6) and (7), we obtain the values of the cross-sectional area A of the trunk and the moment of inertia I of the circular cross-section, respectively.

$$\omega = \beta_1^2 \frac{d}{4l^2} \sqrt{\frac{E}{\rho}} \qquad (8)$$

Taking into account formula (4), the natural frequency f_n of the the trunk, expressed in hertz (Hz), is equal to:

$$f = \beta_1^2 \frac{d}{8\pi \cdot l^2} \sqrt{\frac{E}{\rho}} \qquad (9)$$

After calculating the natural angular frequency ω using formula (5,a) and the natural frequency f_n, expressed in hertz, using formula (4), the calculated data are summarized in Table 1.

Table 1. *Natural frequency of a walnut tree trunk*

Trunk parameters		Geometric characteristics of the section		Natural angular frequency of oscillations *ω, rad/s*	Oscillation frequency *f*, Hz
Diameter *d*, m	Height *H*, m	Cross-sectional area *A*, $\cdot 10^{-2}$ m^2	Moment of inertia of a circular cross-section *I*, $\cdot 10^{-5}$ m^4		
2	*2*	*4*	*5*	*6*	*7*
0.15	4.0	1.77	2.48	34.74	5.53
	5.0	1.77	2.48	22.22	**3.54**
	5.5	**1.77**	**2.48**	**18.37**	**2.93**
	6.0	**1.77**	**2.48**	**15.44**	**2.46**
0.20	**5.0**	3.14	7.85	29.68	**4.73**
	5.5	**3.14**	**7.85**	**24.54**	**3.91**
	6.0	**3.14**	**7.85**	**20.64**	**3.29**
	6.5	3.14	7.85	17.57	2.80
0.30	**5.0**	**7.10**	**39.74**	**44.45**	**7.08**
	5.5	**7.10**	**39.74**	**36.73**	**5.85**
	6.0	**7.10**	**39.74**	**30.87**	**4.92**
	6.5	7.10	39.74	26.31	4.19
	7.0	7.10	39.74	22.69	3.61
	8.0	7.10	39.74	17.35	2.76

From the analysis of the data from Table 1 it follows that, with the same diameter of the trunk of a walnut tree, as its height increases, the natural frequency of oscillations of the trunk decreases (Fig.2). However, for the same tree height, the natural frequency of trunk vibrations increases with increasing trunk thickness (Fig.2). For example, with a conventional height of the trunk of a walnut tree H=5 m, the frequency of natural f oscillations of the trunk will be equal to 3.54 Hz for a diameter of d=0.15 m, 4.73 Hz for d=0.20 m, and 7.08 Hz for d=0.30 m.

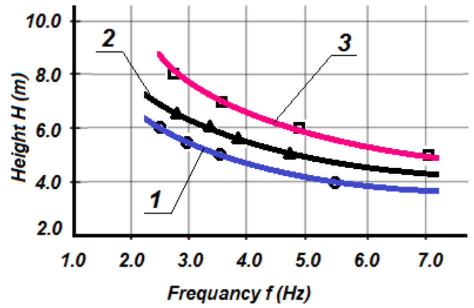


Figure 2. Dependence of natural frequency f of oscillations on the height H of the trunk: *1 - d=0,15 m; 2 - d=0,20 m; 3 - d=0,30 m.*

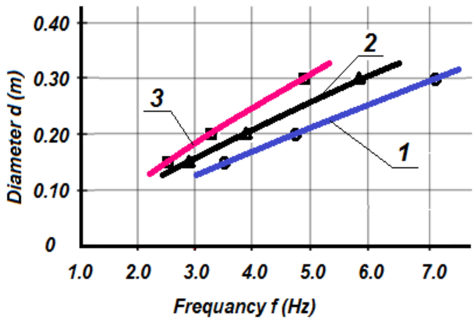


Figure 3. Dependence of natural frequency f of oscillations on the diameter d of the trunk: *1 - H=0,50 m; 2 - H =0,55 m; 3 - H =0,60 m.*

From formula (9) we determine the diameter d of the trunk:

$$d = \frac{8\pi \cdot H^2 f}{\beta_1^2}\sqrt{\frac{\rho}{E}} \qquad (10)$$

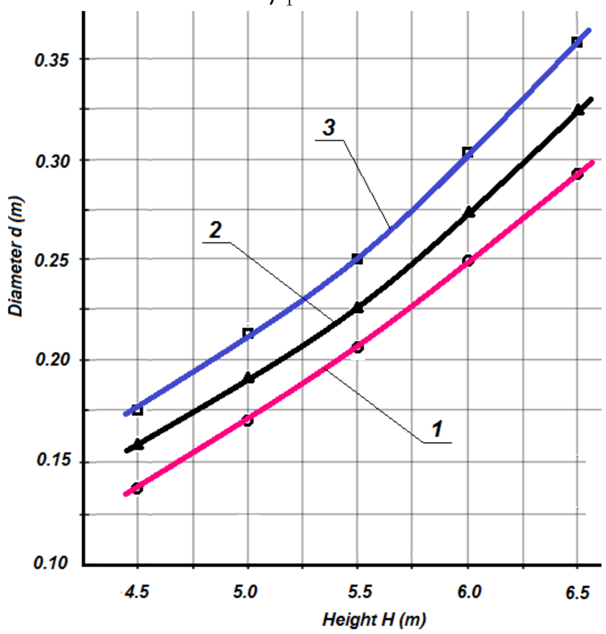


Figure 3. Dependence of the height H of a walnut tree on the diameter d of the trunk at fixed vibration frequencies f: *1 - f=4,0 Hz; 2 - f=4,5 Hz; 3 - f=5,0 Hz.*

Figure 4 shows the dependence of the height H of a walnut tree on the diameter d of the trunk at fixed vibration frequencies f. It means that if, for example, we initially set f=5 Hz, i.e. f=5 oscillations per second, then in order to maintain this parameter, we need to selectively measure the trunk diameter of several trees in the garden and to determine it arithmetic mean value d_m. The graph in Fig.4 can be used as a nomogram to set the tree height H to be the same for all trees in a nut orchard for the next year, taking into account the annual growth rate of branches and determining the working height for adjusting the machine during mechanized pruning of the orchard. For example, for a trunk diameter d_m=0.24 m and an oscillation frequency f=5 Hz, according to the nomogram, the height of the trees should be equal to H_m=5.35 m (Fig. 4).

Conclusions

1. Modeling the process of shaking a walnut tree made it possible to obtain the dependence of the natural frequency of oscillations f of the tree on the trunk diameter d and on the height of the tree in the range of 0.15-0.30 m and 4,0-8,0 m, respectively. The obtained results show:

- for a walnut tree with the same trunk diameter d, as its height increases, the frequency f of oscillations of the trunk decreases;
- for the same tree height H, the natural frequency of trunk oscillations f increases with increasing of thickness of the tree d.

2. The graph in Figure 4 can be used as a nomogram to set the tree height H to be the same for all trees in a nut orchard for the next year, taking into account the annual growth rate of branches and determining the working height for height adjustment during mechanized pruning of the orchard.

3. Tuning the shakers to the resonant frequency of the tree promotes higher-quality collection of nuts with minimal effort required to break the nut off.

References

1. Assoc. Prof. IURIE MELNIC, PhD, PhD student CONSTANTIN ZÎRNESCU, Assist. ANGELA MELNIC, PhD student PAVLIUC VASILE Current status in mechanized harvesting of nut crops. 63 ANNUAL SCIENTIFIC CONFERENCE of Angel Kanchev University of Ruse and Union of Scientists, Bulgaria, 25-26 october 2024. pp.15-22. ISBN 978-954-712-951-1 (Print); e-ISBN 978-954-712-952-8. https://conf.uni-ruse.bg/bg/docs/cp24/bp/bp-1.pdf

2. IURIE MELNIC, ANGELA MELNIC, VALENTINA TKACHUK, CONSTANTIN ZÎRNESCU Current problems of walnuts growing technology. International Scientific Journal “Mechanization in agriculture & Conserving of the resources”, Issue 2, 2023. pp.56-59. ISSN print 2603-3704, ISSN web 2603-3712. https://stumejournals.com/journals/am/2023/2/56

3. ZÎRNESCU, C., PAVLIUC, V. Review of machines and equipment for nut harvesting *(in Russian)*. Technical Scientific Conference of Undergraduate, Master and PhD Students: Chişinău, 27-29 martie 2024. - Chişinău: Tehnica-UTM, 2024. pp.1350-1355. ISBN 978-9975-64-458-7.

4. SANDERS, K. Orange harvesting systems review. Biosyst. Eng. 2005, 90(2), pp.115–125. https://doi.org/10.1016/j.biosystemseng.2004.10.006

5. Mohsen Farajijalal et al. Assessing Crucial Shaking Parameters in the Mechanical Harvesting of Nut Trees: A Review. Horticulturae 2025, 11(4), 392. https://doi.org/10.3390/horticulturae11040392

6. BIDERMAN, V. Theory of mechanical vibrations (rus). M.: Vysshaya shkola press, 1981. – 408 p.

7. MEIROVITCH, L. Fizica. McGraw-Hill: Harley Iditorial Services, 1986. 574 p. ISBN 0-07-041342-8

8. ZACON, E. Mecanică și rezistența materialelor. Chisinau: State Agrarian University of Moldova, 2010. - 145 p. ISBN 978-9975-64-185-2

9. Donell, L. Beams, plates and shells. Translated from English. Moscow: Nauka, 1982. 567 p.

10. BLEVINS, R. D. Formulas for Natural Frequency and Mode Shape. Published by Van Nostrand Reinhold Company 135 West 50 Street, New York, NY 10020, 1979.