

ON ONE APPROACH TO SOLVING THE PROBLEM OF FORMALIZATION AND MODELING OF THE AIRPORT SECURITY THREATS SPACE

Lev N. Elisov¹, Nikolay I. Ovchenkov²

¹Moscow State Technical University of Civil Aviation, Moscow, Russian Federation

²P.G. Demidov Yaroslavl State University, Yaroslavl, Russian Federation

Abstract. *The paper examines the issue of formalization and mathematical modeling of potential dangers (hereinafter referred to as threats). Such threats occur in the external environment in relation to the object of transport infrastructure (hereinafter referred to as airport), while ensuring aviation security. The issue of formalization is manifested in the significant uncertainty of physical presentation of security threats and difficulties in choosing a class of functions that describe threats' spread dynamics.*

The authors suggest using the heuristic approach and the format of the boundary value problem of field theory when moving from a linguistic description of the threat space to its formal representation and description. A mathematical model based on partial differential equations describing threat space is proposed. In addition to this, assumptions and limitations associated with its hypothetical representation are formulated. Modeling and results are presented in the form of threat intensity distribution on some topology of the transport infrastructure object. The results are interpreted as the initial information for modeling the airport security system.

1. Introduction

There has been a definite tendency to consider aviation security issues from the point of view of optimal management of its level with a consistent solution of identification, measurement, evaluation and decision-making problems in recent years. Results analysis obtained in this direction shows the urgent need to move to the issue of formalization of the subject area and mathematical modeling. It should be made in order to optimize the process of managing airport's aviation security level.

The authors consider aviation security as a state associated with certain parameters of the investigated object. The quantitative display of such parameters in the dynamics of their changes under the influence of external and internal factors remains in the boundaries acceptable for the operation of the object. The state of the object under protection can be considered as some of its characteristics, associated with an uneven, heterogeneous and unstable environment that provides protection for this object. In this case, we should talk about two issues of formalization and modeling. On the one hand, it is issue of modeling object security threat space. On the other hand it is issue of modeling some space that provides object security. The authors examine the first one in this paper.

The effective functioning of civil aviation as an industry that provides transport services is primarily related to the concept of security. Security is a state of complex system where the action of external and (or) internal factors does not lead to the system deterioration or to inability of its functioning and development. Aviation security (hereinafter referred to as AS) is one of the most important components of this concept. It is defined as the state of aviation protection from illegal interference into aviation activities.

AS is ensured on the basis of organizational and regulatory methods at the present time. Such methods exclude questions of optimizing decisions since optimization involves measuring and evaluating the parameters under study on the basis of some formalization. A few attempts to use the mathematical apparatus in this subject area are connected with economic models [1] and risks [2–4], i.e. with the solution of local problems. The issue of modeling AS as a state of the transport infrastructure object has hardly been discussed in the sources that the authors know. At the same time, it is possible and even necessary to solve the issue of ensuring security on the basis of mathematical modeling in response to ever-increasing demands on civil aviation associated with the increase and complexity of threats to the security of its facilities. In their works [5], the authors showed this and substantiated the fundamental possibility of formalizing and modeling the space of airport security threats based on the theory of boundary value problems [6] and heuristic procedures.

2. Formulation of the problem

The problem of providing AS of the transport infrastructure object is solved within the framework of the complex interaction of the following main elements (Fig. 1):

- The transport infrastructure object (airport), characterized by the presence of a certain structure, including, among other things, critical (most important) elements, interconnected within a certain topology.
- Security system (hereinafter referred to as SS) of the facilities. It includes a set of measures, methods, procedures, technical means, aviation personnel, regulatory framework, etc.
- Subject of illegal activity (hereinafter referred to as SIA) (violation). It is complex concept that includes all possible attributes for the realization of potential threats and the commission of an act of illegal interference.

Thus, the level of AS of the transport infrastructure object is determined as the result of opposition (counteraction) between SIA and SS. Two concepts are introduced and used for this purpose and in accordance with the recommendations of the International Civil Aviation Organization (ICAO) [7]. It is object vulnerability and object security levels. The object vulnerability assessment is implemented in order to determine the degree of its security against potential threats. The object security levels are set depending on the level of potential and immediate threats and define the requirements for the SS. In both cases, the assessment is of high quality and is implemented by expert methods. There are some papers [8] who offer methods for quantifying vulnerability on a qualimetric or probabilistic basis. Such works have not found practical application. Besides, they are of academic interest.

It is necessary to obtain quantitative information on the threats intensity by the topology of object under protection, taking into account critical elements in order to build the structure and configuration of the object security system that meet the vulnerability requirements and the specified security levels (Fig. 1). In a direct formulation, this problem does not have an exact mathematical solution because of formalization problems. In this case, it is proposed not to receive a decision but to take it. For this purpose, the authors have analyzed the structural model of the SIA (violation) [9]. They propose to introduce conditional quantitative estimates of threats level corresponding to the security levels scale. Such the estimates should be obtained by expert methods. It is necessary to obtain the threats' intensity distribution according to the object topology on the basis of these estimates. Such estimates should be considered as boundary and initial conditions if this problem is solved on the basis of the theory of boundary value problems. The proposed heuristic approach is most likely the only possible one since the problem does not have an exact mathematical solution. The second important problem here is to define a class of functions that can describe the expected

distribution. This is even more difficult, because you can only talk about a hypothetical threat space. It is proposed to consider the set of such functions so that, having received a set of distributions of threat intensities, to build the final distribution using heuristic procedures. The resulting distribution of potential threats contains comprehensive information for the formation of an object protection field, whose intensity according to the object topology is aligned with the quality of the SS functioning in quantitative representation. It makes it possible to formulate requirements for a set of aviation security facilities.

The proposed approach was called heuristic modeling. The concept includes a mathematical model and a heuristic formulation of the problem. In this case all the difficulties of formalization go into the permissible inadequacy of the mathematical model with subsequent interpretation of the statistical results of modeling on the basis of decision-making theory.

It is necessary to clarify the difference between two concepts: impact and danger. Impact is any action aimed at an object in order to influence it, cause changes. Danger is the possibility of the occurrence of circumstances in which matter, field, energy, information or a combination of them can thus affect the complex system. In a follow-up it will lead to deterioration or inability to function and develop it. The impact is related to illegal interference act implementation, and the danger is considered as a potential possibility of committing such an act in the context of AS.

The analysis and prediction of undesirable events is carried out on the basis of statistical data on the realized impacts in modern aviation security systems. Considering the fact that the frequency of such events is very small, the statistical sample is significantly limited, with all the expected problems for the forecast. The approach proposed by the authors is based on the examination of potential threats and obtaining the threat intensity distribution by the topology of the protection object in a quantitative matter. The author's approach seems preferable for optimal control of aviation object security level.

3. Heuristic formalization and classes of functions

The structural and logical model of the SIA [9] includes a representative set of threats of different physical nature, significant uncertainty in the description of characteristic and objective functions. As a result, it is characterized by insurmountable difficulties of mathematical formalization. Heuristic formalization here should be understood as the solution of the inverse problem. It means that it is not necessary to give a mathematical interpretation of the threat. It is necessary to provide to a problem setter the right to choose a certain class of equations (equation) for a certain threat, having agreed in advance with the possible incongruity of the real threat and model. The inadequacy of the model is compensated by a set of statistics based on the results of modeling using the chosen equations. It is important that the solution in this case is not obtained as a result of modeling, but is adopted on the basis of certain heuristic rules.

In their works [6, 10] the authors showed the principal possibility of using differential equations in partial derivatives for solving this problem. Some classes of such equations are presented below [6, 11–12].

$$\frac{\partial}{\partial x} \left[A_1(x, y, z, U, t) \frac{\partial U}{\partial x} \right] + \frac{\partial}{\partial y} \left[A_2(x, y, z, U, t) \frac{\partial U}{\partial y} \right] + \frac{\partial}{\partial z} \left[A_3(x, y, z, U, t) \frac{\partial U}{\partial z} \right] = a \frac{\partial^2 U}{\partial t^2} + b \frac{\partial U}{\partial t} + cU + \alpha, \quad (1)$$

where

$$a = f_1(x, y, z, U, t) \geq 0; \quad c = f_3(x, y, z, U, t) \geq 0; \quad (2)$$

$$b = f_2(x, y, z, U, t) \geq 0; \quad \alpha = f_4(x, y, z) \geq 0.$$

The values of the function U are sought inside a certain domain G bounded by the surface Γ (by the line Γ in two-dimensional case), on which the values of the required function are known, determined by:

$$(\alpha U + \beta \frac{\partial U}{\partial n})_{\Gamma} = F, \quad (3)$$

where α and β – prescribed functions of the point on the boundary Γ ;

$F = \varphi(x, y, z, U, t)$ – some known function in the boundary points;

$\frac{\partial U}{\partial n}$ – the derivative of the required function along the normal to the boundary at the boundary point under consideration.

Depending on the nature of the right-hand side, expression (1) turns into equations of:

elliptic type, when $a = b = 0; c \geq 0; \alpha \geq 0$;

parabolic type, if $a = 0; b > 0; c \geq 0; \alpha \geq 0$;

hyperbolic type, when $a > 0; b \geq 0; c \geq 0; \alpha \geq 0$.

Depending on the values of the functions α and β , the boundary value problem described by equation (1) is called:

Dirichlet problem, when $\alpha = \varphi_1(x, y, z) \neq 0; \beta = 0$.

Neumann problem, if $\alpha = 0; \beta = \varphi_2(x, y, z) \neq 0$,

mixed problem, when $\alpha \neq 0; \beta \neq 0$.

Functions A_1, A_2 и A_3 determine parameters of the space substance. Equation (1) is solved in the homogeneous domain G , when

$$A_1 = A_2 = A_3 = const > 0.$$

We have for an inhomogeneous volume domain:

$$A_1 \neq A_2 \neq A_3 \neq A_i; A_i = \varphi(x, y, z) \geq 0$$

Equation (1) is linear when the coefficients $A_1, A_2, A_3, a, b, c, d, \alpha$ and β depend only on the coordinates. It is nonlinear if at least one of them depends on U .

Equations of elliptic type describe processes where function values do not change with time (problems of steady heat conduction, steady filtration flows, stationary magnetic and electric fields, etc.). Equations of parabolic type reflect many nonstationary processes occurring in space (the exploitation of productive oil and gas layers taking into account their elastic capacity, the dynamics of the processes of heating and cooling bodies, etc.).

The necessity of solving hyperbolic equations arises in the study of transient processes in various problems of potential theory. Wave ($a \neq 0; b = c = 0; \alpha \geq 0$) and telegraph ($a \neq 0; b \neq 0; c \neq 0; \alpha \neq 0$) equations are particular cases of equations of hyperbolic type.

The most common problem is the displacement of the interface of two phases with additional conditions on it with respect to its temperature, heat balance, incoming and outgoing fluxes, latent heat when speaking on the problems of equations of parabolic type. These conditions, as well as the non-linearity of the coefficients for coordinate derivatives, arising from the temperature dependence of the thermal substance conductivity, relate such problems to the category of non-linear boundary-value problems of special complexity.

This problem is described by a system of equations:

$$\frac{\partial}{\partial x} \left[A_1^V(x, y, z, T) \frac{\partial T_V}{\partial x} \right] + \frac{\partial}{\partial y} \left[A_2^V(x, y, z, T) \frac{\partial T_V}{\partial y} \right] + \frac{\partial}{\partial z} \left[A_3^V(x, y, z, T) \frac{\partial T_V}{\partial z} \right] = C_T^V(x, y, z, T) \frac{\partial T_V}{\partial t} \quad (4)$$

$$\frac{\partial}{\partial x} \left[A_1^W(x, y, z, T) \frac{\partial T_W}{\partial x} \right] + \frac{\partial}{\partial y} \left[A_2^W(x, y, z, T) \frac{\partial T_W}{\partial y} \right] + \frac{\partial}{\partial z} \left[A_3^W(x, y, z, T) \frac{\partial T_W}{\partial z} \right] = C_T^W(x, y, z, T) \frac{\partial T_W}{\partial t}$$

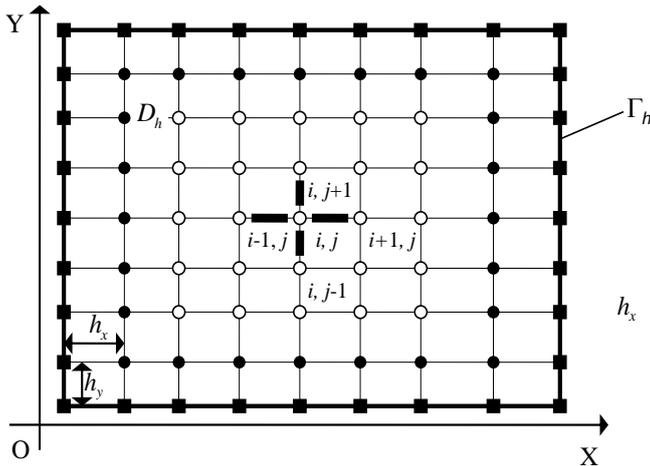
where indexes V and W characterize the belonging of the functions to the liquid or solid phase of the substance respectively. On the space boundaries S_1 and S_2 boundary conditions similar to (3) are defined, in the phase regions V and W , initial conditions $T_V(0) = \varphi_1(x, y, z, 0)$ and $T_W(0) = \varphi_2(x, y, z, 0)$, and on the phase interface $\xi(t)$ – additional conditions:

$$T_V / \xi(t) = T_W / \xi(t) = T_0; \quad (5)$$

$$\theta_V - \theta_W = \theta_c, \quad (6)$$

We consider the nodes that belong to the domain $\bar{D} = D + \Gamma$. Those nodes that are inside the domain D are called *internal* (the inner nodes are indicated by \circ in Fig. 2), and their set is called *grid domain* D_h . The points of intersection of lines (10) with the boundary Γ are called *boundary nodes* (they are indicated by \blacksquare in Fig. 2), and their set is called *grid boundary* Γ_h . In the case of a *non-rectangular domain*, the nodes nearest to the boundary are considered as boundary nodes (the distance from the boundary in the directions of the axes X and Y does not exceed the value of the corresponding step). The nodes closest to the grid boundary are called *boundary nodes* (they are indicated by \bullet in Fig. 2).

Fig. 2. Grid domain example



We consider *grid functions* of grid nodes $u_h(x_i, y_j)$ instead of functions $u(x, y)$ of continuous arguments $x, y \in D$. If a linear differential operator L is given, acting on the function u , then, replacing the derivatives of the Lu with difference relations, we obtain a difference expression $L_h u_h$. It is a linear combination of the values of the grid function u_h on a certain set of grid nodes, called a *template*.

We write the difference approximation of equation (8) for the internal node using the integro-interpolation method [20], a five-point pattern and a constant grid step along the axes X and Y,

$$\left[\frac{\sigma(x_i + h/2, y_j)}{h^2} + \frac{\sigma(x_i - h/2, y_j)}{h^2} + \frac{\sigma(x_i, y_j + h/2)}{h^2} + \frac{\sigma(x_i, y_j - h/2)}{h^2} \right] u_{i,j} + \frac{\sigma(x_i + h/2, y_j)}{h^2} u_{i+1,j} + \frac{\sigma(x_i - h/2, y_j)}{h^2} u_{i-1,j} + \frac{\sigma(x_i, y_j + h/2)}{h^2} u_{i,j+1} + \frac{\sigma(x_i, y_j - h/2)}{h^2} u_{i,j-1} = f(x_i, y_j), (x_i, y_j) \in S_h$$

$$\text{or } a_{i,j} u_{i,j} - a_{i+1,j} u_{i+1,j} - a_{i-1,j} u_{i-1,j} - a_{i,j+1} u_{i,j+1} - a_{i,j-1} u_{i,j-1} = -f_{i,j}, \quad (11)$$

$$\text{Where } a_{i,j} = \frac{\sigma(x_i + h/2, y_j)}{h^2} + \frac{\sigma(x_i - h/2, y_j)}{h^2} + \frac{\sigma(x_i, y_j + h/2)}{h^2} + \frac{\sigma(x_i, y_j - h/2)}{h^2},$$

$$a_{i+1,j} = \frac{\sigma(x_i + h/2, y_j)}{h^2}, \quad a_{i-1,j} = \frac{\sigma(x_i - h/2, y_j)}{h^2},$$

$$a_{i,j+1} = \frac{\sigma(x_i, y_j + h/2)}{h^2}, \quad a_{i,j-1} = \frac{\sigma(x_i, y_j - h/2)}{h^2},$$

$$f_{i,j} = f(x_i, y_j), (x_i, y_j) \in D_h.$$

The above expressions allow us to approximate an

environment with inhomogeneous properties. All the coefficients in the difference formulas will be constant for a domain with homogeneous properties.

The boundary conditions (9) are approximated as follows $u_{ij} = 0, (x_i, y_j) \in D_h$ (12)

The approximation proposed has an error order $O(h^2)$.

We obtain a system of algebraic differential equations with sparse matrices writing the equation (11) for each node where grid function is unknown, numbering nodes sequentially, taking into account boundary conditions and moving all the known members of the right side. Solution is sought only at internal nodes and boundary conditions (12) are taken into account in the right parts of differential equations in the case of the Dirichlet problem.

It is not necessary to apply the general methods of storing sparse matrix when using regular difference template and applying iterative methods for solving systems of differential equations. It is sufficient to store arrays of coefficients of finite-difference approximation for systems of the form (11). We can store two coefficient arrays A_Y, A_X connecting the nodes in the template, respectively, of the axes X and Y (Fig. 3)

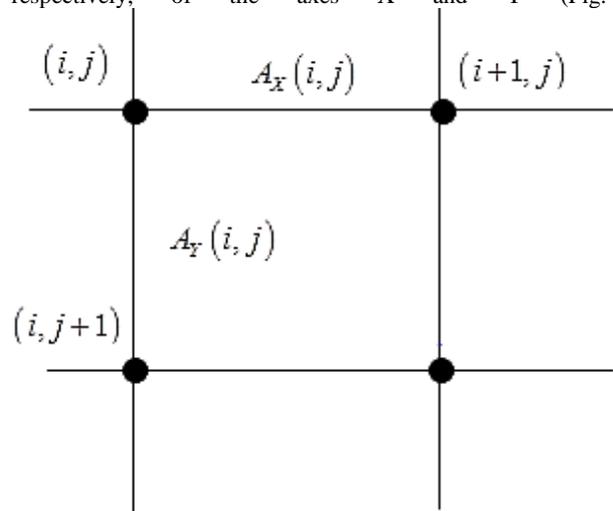


Fig. 3. The coefficients of a differencing template

We use the Seidel iterative method [21, 22] for the numerical solution of the system of difference equations. This method provides a sufficiently good rate of convergence and agrees well with the difference representation of the model with relative simplicity of implementation.

In Seidel's method, each element of the grid domain corresponds to an element of a two-dimensional array with indices equal to the node coordinates. In each iteration, the nodes are sequentially bypassed (we take a detour along the horizontal grid lines). For the current node, the refined approximation of the solution is calculated using a formula having the form for the adopted coefficient storage scheme

$$U_{(i,j)} = \frac{1}{Ax(i, j) + Ax(i-1, j) + Ay(i, j) + Ay(i, j-1)} [Ax(i, j) \cdot U(i+1, j) + Ax(i-1, j) \cdot U(i-1, j) + Ay(i, j) \cdot U(i, j+1) + Ay(i, j-1) \cdot U(i, j-1) + f(i, j)] \quad (13)$$

When calculating by (13), we use the approximations already updated in this iteration in neighboring nodes. The approximation obtained is written in place of the old value. The process ends when the norm of the relative change in the solution is sufficiently small

$$\frac{\|U^{(k)} - U^{(k-1)}\|}{\|U^{(k)}\|} \leq \varepsilon,$$

where $U^{(k)}$ и $U^{(k-1)}$ — solution vectors on the k -th and $(k-1)$ -th iterations, ε — given small positive number.

6. Results of modeling

Single threat modeling results implemented in MATLAB system [23–24] are presented below. Variants of the main program and functions implementing the iterative process are developed.

The problem is solved in a rectangular domain, with the coordinates of the lower-left corner of $x_1 = 0, y_1 = 0$ and the upper-right corner of $x_2 = 1$ and $y_2 = 1$. Grid pitch $h = 0.01$, permeability of the external environment $\sigma = 1.0$, coordinates of danger source $x_{Dan} = 0.5$ and $y_{Dan} = 0.5$, danger magnitude $u_{Dan} = 1.0$. Coordinates of the protected area $x_{Prot1} = 0.2, y_{Prot1} = 0.4, x_{Prot2} = 0.4, y_{Prot2} = 0.6$, permeability of the protected area $\sigma_{prot} = 1.0e-12$. Danger density within the protected area is zero. The decision was made up to $\epsilon = 10^{-6}$. The three-dimensional graph of the solution surface is shown in Fig. 4.

From the graph in Fig. 4 danger level is sharply reduced, but is not zero in the protected area with almost zero danger permeability

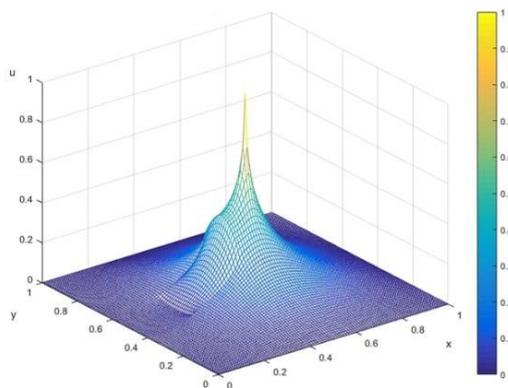


Fig. 4. Solution surface graph

Danger values can be clearly seen on the graph of the level lines (Fig. 5).

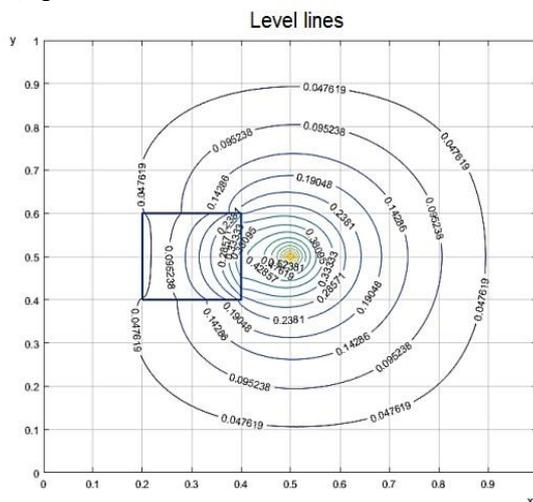


Fig. 5. Graph of lines for solving the problem with one danger source

The protected area is highlighted by a rectangle. The solution is obtained after 1979 iterations (Fig.6).

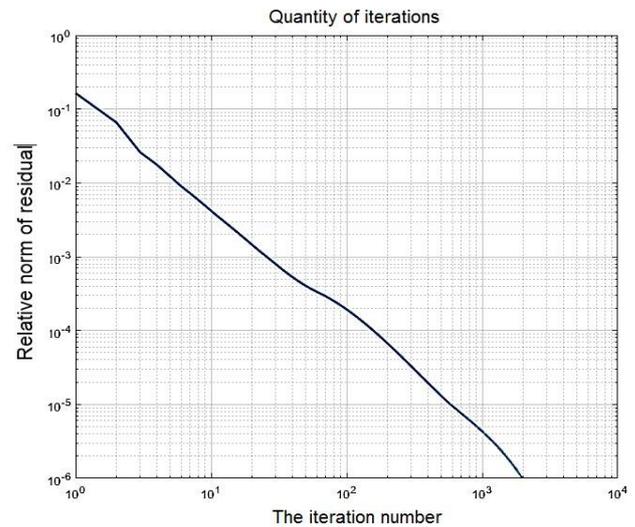


Fig. 6. The graph of dependence of the relative norm of the correction on the iteration number when solving the problem with one danger source

The results obtained should be considered as intermediate, more precisely, as initial, taking into account the limitations and assumptions presented above. Next, we must solve the problem of finding statistics for solving similar problems for various initial and boundary conditions and for other classes of functions. The final solution, in this case, should not be obtained, but adopted on the basis of certain heuristic rules. The results prove the principal possibility of applicability of the author's approach, connected with the combination of heuristic formalization and mathematical modeling with the subsequent decision-making on the configuration of the requirements for the object's protection system.

7. Conclusion

The proposed author's approach is the first stage in the solution of the global issue of creating an automated system for managing the level of aviation security of objects with a distributed structure (airports). A heuristic method was developed in combination with heuristic procedures, which excludes all previously uncontrollable difficulties of formalization from the process of mathematical modeling. The adequacy of the mathematical model, the reliability and accuracy of the results obtained is solved within the system of decision-making on the formation of requirements for technical means of protection on the basis of the principle of acceptability of aviation security level. At this stage, the transition from the intensity of the distribution of security threats by the object topology to the performance indicators of the protection system and its elements is carried out. This transition is the prerogative of further scientific development of the authors.

8. References

1. Y. Wang, H. Sun, J. Zhu, B. Zhu. Optimization Model and Algorithm Design for Airline Fleet Planning in a Multi-airline Competitive Environment. Mathematical Problems in Engineering. Article ID 783917, 13 (2015).
2. F. L. Edwards, D. C. Goodrich. Introduction to Transportation Security. — CRC Press, 388 (2012).
3. J. Price, J. Forrest. Practical Aviation Security: Predicting and Preventing Future Threats. — Butterworth-Heinemann, 598 (2016).
4. G. Tamasi, M. Demichela. Risk assessment techniques for civil aviation security. Reliability Engineering & System Safety, Vol. 96, Issue 8, 892-899, (2011).

5. L. Elisov, N. Ovchenkov, R. Fadeev. Introduction to the theory of aviation security / [ed. L. Elisov]. — Yaroslavl: Filigree, 320 (2016).
6. S. J. Farlow. Partial Differential Equations for Scientists and Engineers. — Wiley, 448 (1982).
7. Safety Management Manual (PMU). DOC 9859. AN / 474. Second edition. — ICAO, Montreal, 240, (2009)
8. V. Volynsky, Yu. Volynsky-Basmanov, Yu Mikhailov. On the issue of the definition and quantification of threats to the implementation of acts of illegal interference into the activities of civil aviation facilities. — Moscow: Transport Safety and Technology, No. 1 (24), 24–29, (2011).
9. L. Elisov, N. Ovchenkov. On the issue of structural modeling of subjects of transport security. Scientific Bulletin of the Moscow State Technical University No. 197. — Moscow: MSTUCA, 58–63, (2013).
10. L. Elisov, N. Ovchenkov. Some issues of grid and neural network modeling of airport security management problems. Scientific Bulletin of MSTUCA, **20**, No. 03. — Moscow: MSTUCA, 21-29, (2017)
11. J. R. Buchanan, Z. Shau. A First Course in Partial Differential Equations. — World Scientific Publishing, 624 (2017).
12. J. D. Logan. Applied Partial Differential Equations. — Springer, 289 (2014).
13. S. Chapra. Numerical Methods for Engineers. — McGraw-Hill Science, 994 (2009).
14. G. F. Pinder. Numerical Methods for Solving Partial Differential Equations. — Wiley, 320 (2018).
15. G. D. Smith. Numerical Solution of Partial Differential Equations: Finite Difference Methods. — Clarendon Press, 350 (1986).
16. S. Mazumder. Numerical Methods for Partial Differential Equations: Finite Difference and Finite Volume Methods. — Academic Press, 484 (2016).
17. S. Rao. The Finite Element Method in Engineering. — Butterworth-Heinemann, 782 (2017).
18. G. E. Forsythe, W. R. Wasow. Finite-Difference Methods for Partial Differential Equations. — Dover Publications, 464 (2004).
19. R. S. Varadhan. Lectures on Diffusion Problems and Partial Differential Equations. — Springer, 315 (1981).
20. A. Samarskii, P. Vabishchevich. Computational heat transfer. — Moscow: Librocom, 784, (2014).
21. C. T. Kelley. Iterative Methods for Linear and Nonlinear Equations. — Society for Industrial and Applied Mathematics, 180 (1987).
22. V. Gorbachenko. Computational linear algebra with examples on MATLAB. — St. Petersburg: BHV Petersburg, 320, (2011).
23. H. Moore. MATLAB for Engineers. — Pearson, 688 (2017).
24. J. Li, Y.-T. Chen. Computational Partial Differential Equations Using MATLAB. — Chapman & Hall, 378 (2008).