DELTA 3D WIRE PRINTER FOR BUILDING OBJECTS – THEORETICAL PREREQUISITES FOR PROTOTYPE DESIGN

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Abstract: With the use of 3D printing technology, layer by layer extrusion is possible printing of construction objects, but printers represent a large size and mobility limited metal construction. The main reason for the large size of constructions for existing structures of 3D printers for building objects are large strains of bending moments that construction must take. In a new scheme of 3D printer called “Delta Wired 3D Printer”, the large stress created from bending moments are transformed in normal stress from tension. At this time, the printer has been developed as a conceptual project is therefore theoretically necessary to identify efforts in wires which is to be suspended extruders (load). For creating a large scale working 3D printer model is necessary to create a prototype. Through a theoretical study of the basic design parameters can be made a real printer prototype project. By using the theory of similarity can be define a unit of prototype error as to how much it will respond to the real model. This is also the main purpose of this publication.

Keywords: DELTA WIRED 3D PRINTER, PRINTING CONCRETE MIXTURES, PRINTING PROTOTYPE DESIGN

1. Introduction

Creating the delta wired 3D printer prototype is a first step towards designing a real working model. In previous publications [8, 9, 10] relationships have been established by which the structural elements and the deformation of the extruder can be determined. In order to create a prototype of 3D printer, it is necessary to examine its relation to the real model in terms of structural elements and dimensions [1, 2]. The creation of a prototype, as well as the adequate connection between it and the real model, will allow for a preliminary analysis of the possibility of making such a printer used for printing of construction.

The advantages of the delta wired 3D printer are listed in [8], one of the most important advantages being the unloading of the bending tension construction by using three load-bearing wires loaded with normal stresses only.

2. Prerequisites and means for solving the problem

In order to achieve mathematical connectivity between parameters from the real object and the prototype, it is necessary to determine the basic strength and geometric parameters of the structure. In determining the load distribution [9], the dependencies were used in the supporting clones:

$$F_i = Q \cdot K_{a_i}$$

Where, $F_i$, $F_K$ and $F_C$ represented force in supporting clones, $Q$ – weight of loads (extruder) and $K_{a}$, $K_{b}$, $K_{c}$ – load factors for the respective clone.

The dependencies on load factors are as follows [9]:

$$K_{a_i} = \frac{\cos(\beta_i) \cdot \cos(\gamma_i) \cdot \cos(\phi_i) \cdot \sin(\phi_i) + \cos(\beta_i) \cdot \cos(\gamma_i) \cdot \cos(\phi_i) \cdot \sin(\phi_i)}{C}$$

$$K_{b_i} = \frac{\cos(\alpha_i) \cdot \cos(\gamma_i) \cdot \cos(\phi_i) \cdot \sin(\phi_i) + \cos(\alpha_i) \cdot \cos(\gamma_i) \cdot \cos(\phi_i) \cdot \sin(\phi_i)}{C}$$

$$K_{c_i} = \frac{\cos(\alpha_i) \cdot \cos(\beta_i) \cdot \cos(\phi_i) \cdot \sin(\phi_i) - \cos(\alpha_i) \cdot \cos(\beta_i) \cdot \cos(\phi_i) \cdot \sin(\phi_i)}{C}$$

Where for C is represent:

$$C = \begin{vmatrix}
\cos(\alpha_i) \cdot \sin(\phi_i) & -\cos(\beta_i) \cdot \sin(\phi_i) & \cos(\gamma_i) \\
\cos(\beta_i) \cdot \sin(\phi_i) & \cos(\alpha_i) \cdot \sin(\phi_i) & \cos(\gamma_i) \\
\sin(\phi_i) & \sin(\beta_i) & \sin(\gamma_i) 
\end{vmatrix}$$

From which it is clear, the factors that affect to the load factors are the following angles:

$$K_{a_i}; K_{b_i}; K_{c_i} = f(\alpha_i; \beta_i; \gamma_i; \phi_i; \psi_i; \theta_i)$$

Where $\alpha_i$, $\beta_i$ and $\gamma_i$ are angles between horizontal plane and respectively wire, $\phi_i$, $\psi_i$ and $\theta_i$ are angles between wires projected in horizontal plane.

If the principle of similarity is used - the pillars height, the distances between them and the current load coordinate (X, Y and Z) are proportional to the prototype, then it can be claimed that the angles that determine the value. The load factors will be the same. This will lead to equal values of the factor determining the load in the wires.
From the analysis of the dependencies determining the load in the wires it is clear that for each individual model the weight of the load (the extruder) is important. If we denote the weight of the extruder of the real model of the 3D printer with \( Q \), and the weight of the extruder of the prototype with \( Q' \) then the scale between them we have:

\[
PS_Q = \frac{Q'}{Q} < 1
\]  

Where, \( PS_Q \) is called prototype scale for weight.

For examination full deflection of extruder in [10] the following dependencies were used.

\[
\Delta l = \Delta l_a + \Delta l_b + \Delta l_c
\]

The complete deflection of the extruder \( \Delta l \) is equal to the sum of the deformations in each of the supporting clone \( \Delta l_a, \Delta l_b \) and \( \Delta l_c \). For clone A the dependence is as follows [10]:

\[
\Delta l_a = \sqrt{(la + \Delta la_{tens.w})^2 - (la \cdot \cos(\alpha_x) - \Delta la_{bend})^2} - la \cdot \sin(\alpha_x) + \Delta la_{comp}
\]

Where, \( la \) – length of wire in clone A, \( \Delta la_{tens.w} \) – deformation from tensile in wire A, \( \Delta la_{bend} \) – distortion by bending in pillar A, \( \Delta la_{comp} \) – deformation of compression in pillar A. For the other two clones the dependencies are similar.

We are considering deformation from tensile in wire A for real printer:

\[
\Delta la_{tens.w} = \frac{Fa.la}{E_w.A_w} = \frac{K_{fa}.Q.la}{E_w.A_w} = \frac{K_{fa}.Q.la}{\pi.d^2} \frac{\pi.d^2}{4}
\]

Where, \( E_w \) – modulus of elasticity for wires material, \( Pa \), \( A_w \) – cross section area for wires, \( m^2 \), \( d \) – diameter of wire, \( m \);

For prototype deformation in wire A by tensile, we have to write:

\[
\Delta la_{tens.w}' = \frac{Fa.la'}{E_w.A_w'} = \frac{K_{fa}'.Q.la'}{E_w.A_w'} = \frac{K_{fa}'.Q.la'}{\pi.d^2} \frac{\pi.d^2}{4}
\]

Where, \( \Delta la_{tens.w}' \) – modulus of elasticity for prototype wires material, \( Pa \), \( A_w' \) – cross section area for prototype wires, \( m^2 \), \( d' \) – diameter of prototype wire, \( m \);

If we divide prototype deformation \( \Delta la_{tens.w} \) on deformation for real printer \( \Delta la_{tens.w}' \) we can get the deformation scale of the prototype protrusion for tensile strains in the PS_{la_{tens.w}}. Where, if the \( K_{fa}' = K_{fa} \) is replaced, at the geometric scale of the prototype \( PS_Q = const = \frac{la'}{la} \), and modulus of elasticity of wire materials are equal may be recorded:

\[
PS_{la_{tens.w}} = \frac{\Delta la_{tens.w}'}{\Delta la_{tens.w}} = \frac{K_{fa}'.Q.la'}{K_{fa}.Q.la} \frac{\pi.d^2}{4} = \frac{Q'.la'd^2}{Q.la.d^2} = \frac{PS_Q^2}{PS_Q'2} \left( \frac{d'}{d} \right)^2 = PS_Q.PS_Q' \left( \frac{1}{PS_{dw}} \right)^2
\]

Where, for \( d'd/d \) we can write is equal of \( PS_{dw} \) – prototype scale of wire diameter.

If we represent at tensile stresses in wires for the real model and the prototype to have normal operating conditions in both cases, they should be the same and should not exceed the limit of elasticity of the material. In this case, we can record:

\[
\sigma_{tens} = \frac{Fa}{A_w} = \frac{K_{fa}.Q}{A_w} = \frac{K_{fa}.Q}{\pi.d^2} \frac{\pi.d^2}{4} \text{ const};
\]

\[
\frac{K_{fa}.Q}{\pi.d^2} \frac{\pi.d^2}{4} \frac{\pi.d^2}{4} = \frac{Q'}{Q} \Rightarrow PS_{dw} = \sqrt{PS_Q}, or d' = d \sqrt{PS_Q}
\]

It is clear from the deduced dependence that in order to create the same conditions under which the prototype and the real model work, it is necessary to respect the ratios between the different scales.

For bending deformations in pillars for real model of 3D printer - \( \Delta la_{bend} \) can write:

\[
\Delta la_{bend} = \frac{K_{fa}.Q \cos(\alpha_z).d^3}{3.E.I}
\]

And for prototype:

\[
\Delta la_{bend}' = \frac{K_{fa}'.Q' \cos(\alpha_z').d^3}{3.E'.I'}
\]

Where, \( E \) and \( E' \) – modulus of elasticity for pillars material, respectively for real model and prototype; \( I \) and \( I' \) – moment of inertia for cross section of pillars, respectively for real model and prototype.

In the case pillars materials are same, that modulus of elasticity will be equal:

\[
PS_{la_{bend}} = \frac{\Delta la_{bend}'}{\Delta la_{bend}} = \frac{K_{fa}'.Q' \cos(\alpha_z').d^3}{K_{fa}.Q \cos(\alpha_z).d^3} = \frac{PS_Q^2.PS_G^3}{PS_{dw}^3} \cdot \frac{1}{I' \cdot I}
\]

Where, for ratio between moments of inertia for cross section of pillars \( I'/I \) can be write as \( PS_I \) and called prototype scale for pillars moment of inertia.

Looking at the maximum bending stresses at the base of the pillars and aligning them with those of the prototype, similar to the normal stresses in the wires examined before, we can write:

\[
\sigma_{max.bend} = \frac{M_{bend}}{W_{bend}} = \frac{K_{fa}.Q \cos(\alpha_z).d}{W_{bend}} \text{ const};
\]

\[
\frac{K_{fa}.Q \cos(\alpha_z).d}{W_{bend}} \frac{W_{bend}}{W_{bend}'} = \frac{Q'.d'}{d} \Rightarrow PS_w = PS_Q.PS_G, or W_{bend}' = W_{bend}.PS_Q.PS_G
\]

Where, \( W_{bend} \) and \( W_{bend}' \) - resistance moment of inertia for cross section of pillars, respectively for real model and prototype, \( PS_w \) – prototype scale for resistance moment.

From dependence (6) it remains to determine the component that records the pillars compression for which it can be written:

\[
\Delta la_{comp} = \frac{Fa.\sin(\alpha_x).d}{E.A_p} = \frac{K_{fa}.Q \sin(\alpha_x).d}{E.A_p}
\]

And for prototype:
If we substitute the value for the deformation of clone $A$ with the scale numbers, we have examined up to now we have:

\[
\text{If we compare the two dependencies, taking into account all the assumptions made above, we introduce a scale for the deformations from the compression of the pillars and designate it with PS_{A_{\text{comp}}}. Also, if for the relationship between the cross-sectional area of the pillars we write that PS_{A_{p}} = A_{p}' / A_{p}, then:}
\]

\[
PS_{A_{\text{comp}}} = \frac{\Delta la_{\text{comp}}'}{\Delta la_{\text{comp}}} = \frac{E'A_{p}'}{K_{F_{p}}Q\sin(\alpha_{2})d_{s}}
\]

\[
= Qd_{s}/A_{p}' = PS_{G_{p}}PS_{G_{p}}\left\{ \frac{1}{PS_{A_{p}}} \right\}
\]

The resulting relationship shows the relationship between the real deformation in clone $A$ of the 3D printer and the deformations of the prototype extruder and on the other hand the deformations of the real object. In order to visualize the results of the above dependencies, the following graphs have been developed. The scale between the prototype deformations and the real printer working area with the geometric parameters listed in Table 1 can be recorded:

\[
Ala = \sqrt{\left(Ala_{\text{scaled}}/PS_{G_{p}}PS_{G_{p}}\right)^{2} - \left(la_{s}/PS_{G_{p}}\cos(\alpha_{2}) - \Delta la_{\text{scaled}}/PS_{G_{p}}PS_{G_{p}}\right)^{2}} - \frac{la_{s}/PS_{G_{p}}\sin(\alpha_{2}) + \Delta la_{\text{scaled}}/PS_{G_{p}}PS_{G_{p}}}{PS_{G_{p}}PS_{G_{p}}}
\]

These dependencies can also be written for the other two wire clones. The relationship of $\Delta la'/\Delta la$ can be written as $PS_{A_{\text{comp}}}$ and for the other two branches $PS_{A_{b}}$ and $PS_{A_{c}}$. And for the total sag of the prototype extruder we have:

\[
\text{If we substitute the value for the deformation of clone A with the scale numbers, we have examined up to now we have:}
\]

\[
\text{The graphical change of PS}_{\Delta} \text{ is shown in Figure 4.}
\]

### 3. Solution of the examined problem

The theoretical dependencies revealed on the one hand the deformations of the prototype extruder and on the other hand the relationship between the deformations of the prototype and the deformations of the real object. In order to visualize the results of the above dependencies, the following graphs have been developed. The fig. 2 shows the variation of the deflections for prototype extruder with the following geometric and weight characteristics shown in Tabl.1.

| Table 1: Conceptual design parameter of Delta Wired 3D Printer for prototype and real model |
|:-------------|:----------|:----------------|:-------------|
| Height of pillars [mm] | 600  | 6000 | $PS_{S_{1}}$ | 0,1 |
| Wire diameter [mm] | 0,3 | 10 | $PS_{S_{2p}}$ | 0,03 |
| Weight of extruder [kg] | 0,3 | 300 | $PS_{S_{2w}}$ | 0,001 |
| Pillars cross section diameter x thickness [mm x mm] | 08,0 | 0256x8,0 | - | - |
| Pillars cross section area [mm²] | 50,3 | 6232,9 | $PS_{A_{p}}$ | 0,00807 |
| Pillars resistance moment for cross section [mm⁴] | 50,26 | 374754,3 | $PS_{W}$ | 0,000134 |
| Pillars moment of inertia for cross section [mm⁴] | 201,1 | 47968530 | $PS_{I}$ | 0,0000042 |

The scale between the prototype deformations and the real printer can be determined with the following dependence:

\[
PS_{\Delta_{1}} = \frac{Ala}{Ala'}
\]

Where, $PS_{\Delta_{1}}$ is prototype scale for extruder deflection. The graphical change of $PS_{\Delta_{1}}$ is shown in Figure 4.

### 4. Results and discussion

Through a theoretical analysis of the relationship between the geometric and operational parameters of a real model and a prototype of the delta wired 3D printer, the dependencies that can be used in the design for real model and the prototype of the printer are reached. Important for the operation of such a printer is the stability expressed by the deflection of the load.
The resulting dependencies and outcomes are applicable in the following cases:
- dependencies (10 and 14) may be used to create the prototype of the delta wired 3D printer to determine the dimensions of the design elements of the prototype so as to create the same operating conditions of the real model and the prototype;
- in order to establish the veracity of the deformation model considered, the prototype will be compared and the deformations measured in the course of its work will be compared with those obtained theoretically;
- by detecting real deformations in the prototype, the deformations that will be obtained in the real object, subject to certain scales, can be identified in practice by using the data obtained;
- it was found that the translational coefficients between the complete deformation of the prototype and that of the real model are constants but differ depending on the current coordinate of the extruder. In the zone of the ground top of fig.4, it is shown in green that prototype scale for extruder deflection takes values of $0.25 \div 0.3$. This means that the deformations of the real object will be $3.3 \div 4$ times bigger than the deformations of the prototype.

5. Conclusion

From the theoretical research on the way of working, the geometric elements and the scale that connects the prototype and the real model of the delta wired 3D printer, the necessary connectivity of the two models has been elucidated, and transistor dependencies have been established taking into account the geometric dimensions of the models. Through the research, real data derived from the prototype model can be transformed into data that will eventually be obtained when building a real model.

It turns out that if we build a prototype of a 3D printer on the scheme under consideration and we define a specific value for the load deflection at given coordinates. Then we create a working - real printer model with the scale shown in Table 1, we can guess what the real model deflection will be. This makes it easier to make decisions about the construction of the real model, as well as the possibility of predictability of future results.

6. References

[10] Vasilev, T., Analysis of the Deformations in „Delta Wired 3D Printer”, ANNALS of Faculty Engineering Hunedoara, Pending publication;