A SOLID BODY SURFACING MATHEMATICAL MODEL IN STRATIFIED INCOMPRESSIBLE FLUID UNDER THE ACTION OF BUOYANCY FORCE AND LIMITED MOTION CONTROL

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ВСПЛЫТИЯ ТВЁРДОГО ТЕЛА В НЕСЖИМАЕМОЙ СЛОИСТОЙ ЖИДКОСТИ ПОД ДЕЙСТВИЕМ ВЫТАЛКИВАЮЩЕЙ СИЛЫ И ОГРАНИЧЕННОГО УПРАВЛЕНИЯ ДВИЖЕНИЕМ

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Резюме: *This paper results are based on the mathematical model of the motion control of an autonomous solid body in stratified incompressible fluid which was presented by the authors at XII MTM Congress held in September 2015 and XIV MTM Congress held in September 2017. This paper presents an analytical mathematical model of a solid body, which surfaces in stratified viscous incompressible fluid, a difference scheme and its solution. The body is equipped with controlled rudders, wings of finite span, and does not have its own propulsion system. It is moved by the influence of the buoyancy force and wings lift effect. This body motion is considered to be plane-parallel motion. The mathematical model synthesis is based on the hydrodynamic equations.*

КЛЮЧЕВЫЕ СЛОВА: MATHEMATICAL MODEL, MOTION OF SOLIDS IN A FLUID, MOTION COTROL, BUOYANCY FORCE, ENSURING ACCESS TO THE GIVEN POINT, WINGS OF FINITE SPAN, WINGS LIFT, DIFFERENCE SCHEME

1. Introduction

The effectiveness of measurements and observations obtained in the study of the underwater world via underwater vehicles, in particular, unmanned, depends on minimizing the impact of these submersible crafts to surrounding underwater environment. First of all, it refers to moving devices, which movement is carried out by various power plants (screw propeller or other propulsion). Therefore, the reduction or removal of such effects is an important applied problem. It is obvious that the ideal situation would be the complete lack of engine. This means that movement control of such body can be carried out only by natural hydrodynamic forces, for instance, the Archimedes force (buoyancy) or an wing lift effect (the body can be equipped with some wings). Basic terminology and fundamental results for the body’s motion in continuum can be found in the classical books [1, 2, 5].

2. Assumptions

As an autonomous rigid body, the authors propose to consider a research submersible – a uniform sphere-shaped rigid body with two similar symmetrically located around the ball centre wings (fig. 1). Actually other modifications of mutual bracing of the sphere-shaped body and wings are possible. However, the proposed mathematical model can be taken as a basis for whole these alternatives.

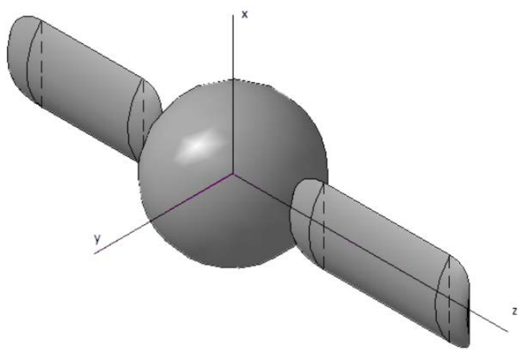

Fig. 1. *Schematic submersible craft image.*

The motion of submersible craft is assumed to happen in a limitless borehole bottom reservoir with an ideal incompressible non-conducting stratified liquid with viscosity effect. The viscosity is taken into account as a Stokes' drag force.

It is also assumed that each layer has own density, which is known. Furthermore, liquid in each layer can move rectilinearly and uniformly with known velocity along the horizontal axis, which is perpendicular to a wingspread.

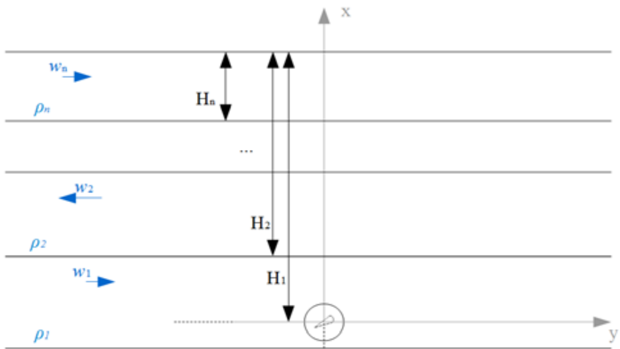

Fig. 2. *Stratified continuous medium figure.*

In this paper the authors consider plane-parallel motion of submersible craft case. At the initial time this body is located in stationary state at a predetermined depth (fig. 2).

For constructing the solution of such a problem in stratified liquid it is necessary to define the obtaining solution algorithm in a one-layer liquid for building a similar solution in stratified liquid.

3. Mathematical model

At the previous authors paper [3,4] mathematical model of the submersible craft plane-parallel motion based on Newton's second law (basic law of dynamics) was constructed. It allows controlling the body through wings angle of attack α modifications

In this paper the authors build more general model based on hydrodynamic equations

Fluid motion is described by a number of hydrodynamic equations: continuity equation, equation of continuum motion, energy conservation equation and constitutive equation.

In the case of an ideal incompressible fluid the complete description can be received by using continuity equation and equation of continuum motion, which are written as following forms:

(1) $div(v) = 0,$

(2) $\frac{dv}{dx} = F - \frac{1}{\rho} grad(p).$

Due to consideration plane-parallel problem, the equations can be presented as

$$(3)\quad \begin{cases} \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \\ \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial t} = F_x - \frac{1}{\rho}\frac{\partial p}{\partial x} \\ \frac{\partial v_y}{\partial t} + v_y \frac{\partial v_y}{\partial t} = F_y - \frac{1}{\rho}\frac{\partial p}{\partial y} \end{cases}$$

where v_x, v_y – submersible craft movement velocity in corresponding directions, p – fluid pressure, F – volume force per unit mass.

For analysis purposes original system is divided into two systems:

1) Sphere-shaped rigid body.
2) Finite-span wings in liquid. (In general other wings configuration options are possible. However, for simplicity the calculations it is assumed than wings are similar and symmetrically located around the ball center)

In this case the volume forces acting on the body (1) are gravity force and motion drag force, on the wings (2) in liquid – gravity force and motion drag force too. Wings mass is much less the body mass, so wings gravity force can be neglected.

Initial conditions for system (3) are supposed zero conditions $v_x = 0$ и $v_y = 0$ due to the fact that the submersible craft is stationary at the initial time. Setting boundary conditions at the border of the body is quite laborious, so known formulas are used to describe the wingы of a finite span [1, 2, 5]. Then the system (3) can be transformed to following view:

$$(4)\quad \begin{cases} \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \\ \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial t} = -2F_i cos\delta - \left(F_{drag}^{(1)} + 2F_{drag}^{(2)}\right)\cos\delta + \\ \quad +F_{arch} - F_g - 2F_{lift}\sin\delta \\ \frac{\partial v_y}{\partial t} + v_y \frac{\partial v_y}{\partial t} = -2F_i \sin\delta - \left(F_{drag}^{(1)} + 2F_{drag}^{(2)}\right)\sin\delta - \\ \quad - 2F_{lift}\cos\delta \end{cases}$$

Here F_{arch} – is the buoyancy force, $F_{drag}^{(j)} = C_X^{(j)} S^{(j)} \frac{\rho v^2}{2}$ – the head resistance force for a sphere (j=1) and wings (j=2), $F_{lift} = \rho v^2 S \frac{k\alpha}{1+\mu_0}$ – the wing lift, $F_i = \frac{\rho}{2} v^2 S \frac{\mu_0}{2k}\left(\frac{2k\alpha}{1+\mu_0}\right)^2$ – the induced drag force.

For solving partial differential equations system of the first order (4) the corresponding difference system is constructed (an explicit first-order accuracy scheme is used):

$$(5)\quad \begin{cases} (v_x)_{m,k}^n = (v_x)_{m,k}^{n-1} + \Delta t * \left(\frac{(v_x)_{m+1,k}^{n-1} - (v_x)_{m,k}^{n-1}}{\Delta x}(v_x)_{m,k}^{n-1} + \right. \\ \left. +b_1 - (b_2 \cdot \alpha^2 + b_3 + 2b_4)\left((v_x)_{m,k}^{n-1}\right)^2 - 2b_5\alpha (v_x)_{m,k}^{n-1}(v_y)_{m,k}^{n-1}\right) \\ (v_y)_{m,k}^n = (v_y)_{m,k}^{n-1} + \Delta t * \left(\frac{(v_y)_{m,k+1}^{n-1} - (v_y)_{m,k}^{n-1}}{\Delta y}(v_y)_{m,k}^{n-1} - \right. \\ \left. -(b_2 \cdot \alpha^2 + b_3 + 2b_4)(v_x)_{m,k}^{n-1}(v_y)_{m,k}^{n-1} - 2b_5\alpha\left((v_x)_{m,k}^{n-1}\right)^2 \right) \end{cases}$$

Here coefficients are defined as:

$b_1 = \frac{\rho g V - mg}{m}$, $b_2 = \rho S_{kp} \frac{2k\mu_0}{(1+\mu_0)^2} \cdot \frac{1}{m}$, $b_3 = c_{0\_sph} \frac{\rho \pi R^2}{2} \cdot \frac{1}{m}$, $b_4 = c_{0\_w} \frac{\rho S_w}{2} \cdot \frac{1}{m}$, $b_5 = \rho S_{kp} \frac{k}{1+\mu_0} \cdot \frac{1}{m}$.

The values of the coordinates *x(t)* and *y(t)* are calculated for each step:

(6) $\dot{x} = v_x, \quad \dot{y} = v_y$

4. *Numerical example*

Software called MATLAB R2016A is used for numerical solution

Motion of the submersible craft in STRATIFIED (double-layer) ideal incompressible viscous fluid with viscosity effect with differently directed shear flows in the line of horizontal axis $\overrightarrow{w_1} \uparrow\downarrow \overrightarrow{w_2}$.

Authors examine following cases of the variation law of attack angle:

1) α = 0.3 rad;
2) α = 0 rad;
3) α = – 0.3 rad.
4) in the bottom layer α = 0 rad, in the second layer α = 0.3 rad.

Then appropriate motion trajectories of the submarine craft can be calculated by solving the difference system (5) (fig. 3). The difference equation system is sequentially solved for each layer starting with the bottom layer. Its initial conditions are supposed zero conditions. For other layers initial conditions are recalculated depending on coordinates of inertia center of the submarine craft at the transitional point from layer to layer.

Shear flows velocities are embedded in equations (6):

(7) $\dot{x} = v_x, \quad \dot{y} = v_y + w.$

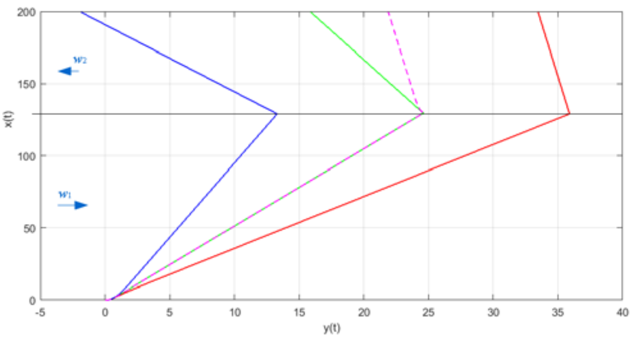

Fig. 3. *Motion trajectories of the submarine craft for different cases of the variation law of attack angle in double-layer fluid:*
green – α = 0;
Red – α = 0.3;
blue – α = -0.3;
magenta (dash-dot line) – in the bottom layer α = 0, in the second layer α = 0.3.

At these examples, the following values of quantities are offered. The diameter of the surfaced body (ball) is 1 meter; its mass is calculated like $m = 0.98\rho V$, where ρ is averaged body density. It is assumption to consider rectangular wings with wingspan 1 meter, aspect ratio of the wing 5 and relative maximum thickness 16 %. An initial immersion depth H_1 equals 200 meters. The second layer depth H2 is 70 meters. Liquid densities in different layers equals ρ_1= 1050 kg/m^3 and ρ_2=1025 kg/m^3. Shear flows have velocities $|\overrightarrow{w_1}| = 0.15$ m/s and $|\overrightarrow{w_2}| = 0.1$ m/s.

5. Conclusions

In this paper the authors build mathematical model of the submersible craft plane-parallel motion in ideal incompressible non-conducting stratified liquid with viscosity effect based on hydrodynamic equations. This model is more versatile than the models in previous authors' papers. [3,4],

The corresponding difference system is constructed for this model. It helps in finding applied problem numerical solution.

The numerical experiment shows that the transition from a simpler mathematical model [3,4] to a more general one (system (4)) does not significantly affect the result.

The simplicity of the model constructing and the speed of calculation in computational method for specific problem should cause usage one or another model.

6. Literature

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