

THE METHOD OF MATHEMATICAL MODELING OF PROCESSES TO IDENTIFY, ESTABLISH CHARACTERISTICS AND RELATIONSHIPS OF THE CONNECTIONS OF SOCIALLY SIGNIFICANT PHENOMENA

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Abstract: Prospects for progress in the creation of analytical systems, the purpose of which is the statistical determination of the trends in the spread of socially significant processes and related events, are critical because of the global geopolitical crisis. His imprints are imposed on almost all states, in conditions of which the significance of the influence of negative and positive social phenomena is determined by a steady increase. In this article, the method of mathematical modeling of the process of identifying, establishing characteristics and potential relationships between various processes that make up or have an effect on socially significant phenomena is delineated.

1. Introduction

The most important component of any socially-oriented automated system is to ensure the effectiveness of its management and use, which is the subject of many studies [1,3,8,12]. The accuracy of strategic decisions at various levels of the power vertical, the policies of financial corporations, the activities of organizations providing security and many other significant components of modern society depend on the systems of the specified designation. The proposed method is designed for initial conditions that presuppose the operation of specified and systematized information. The determination of the characteristics of a set of phenomena and the results obtained from them are the decisive condition for choosing the principles of algorithmization, as well as the methods of calculation at the stages of subsequent investigations. It should be noted that the prerequisites for taking the necessary measures, both for strengthening and easing any consequences of the phenomena of the processes of a designated character, are formed on the basis of the extent of the spread of such phenomena and the tendencies accompanying them. This conclusion can be assumed as one of the central in the context of the overall task of analysis and presented through membership functions. Depending on all interests and combinations of their manifestations, it becomes obvious that the definition of their characteristics is a fundamental step in the calculation.

2. Formal mathematical model of information system

Let us turn to an analysis of the elements of the phenomenon. When using a socially-oriented automated system containing a description of the parameters of the elements, the categorization and distribution of data occurs on the basis of confirmation of the criteria set by the evaluation system [2,4]. Each stage of gathering information and placing it into categories presupposes the rules of categorization, which are the initial and basic for the assessment of the phenomenon as a whole. For the purpose of this, a number of information objects provide for categories of translated information [8,10]. Intermediate, but inalienable, step is computational procedures for formalizing qualitative information in order to prevent a decrease in the quality of the completeness of the data coming from the experts and, as a consequence, the accuracy of the subsequent results. For this task, the fuzzy logic apparatus can be used, as well as for the task of formulating information incompleteness as the next important quality of the input data. In the future, this device can be used to unify and translate a large number of data in the whole system under consideration. For simplicity, we will designate each procedure for unification and translation as preliminary calculations for the main iterations, then the system can be divided into subsystems. A common bridge between the system and all subsystems is categorically-topological logic [2.9] formalizing the general structure of calculations in order to achieve the expediency of procedures for algorithmizing the

entire mathematical apparatus. Let us dwell in more detail on the subsequent procedures. Thus, the delineation and processing of information can be carried out by means of technical means, aimed at searching for information on a group of characteristics, performed by parsers, as the most convenient and easy to use in use. During procedures for analyzing the characteristics and prevalence of an event, it is difficult to determine their potential scale as a phenomenon. The detailing of the events accompanying such phenomena predetermines the prospect of taking into account the exact number of constituent elements and allows them to be correlated with the coefficient of significance of this or that determinable factor. Such an analysis can be carried out by means of logical and statistical categorization of the information databases induction, that is, one attribute for prevalence or increase in order to compare with the prevalence of the factor or group of factors. From this it follows that, provided that the details of fuzzy positions are detailed and the conditions of accuracy are satisfied while distinguishing the principles of behavior of socially significant phenomena, the conditions for examining the boundaries of the set of events can be specified [11]. The conclusion is obvious that their group assumes a systematic repetition, which can be treated as the phenomenon mentioned earlier, and the set of processes in the phenomenon presupposes the formation of the object of investigation. Then, the system itself can be considered as a relationship in the object of investigation, which implies the definition of the properties of the isomorphism of sets. However, in order to determine the behavior of the phenomena, additional calculations are necessary, and such problems can be handled by estimation through graph theory in applied tools of the spectrum of graphs on the subsequent delineation of clear and fuzzy cognitive maps. Let's address to algorithm of the basic calculations of cumulative influence of all possible factors on the basic indicators of stability of occurrence of events [13]. Initially, the formation of the system itself is due to a situational sampling of external events that are dominant for isomorphic data. Such a step is envisaged for further accuracy of operation, as well as for the establishment of events, the prevalence of which can be regarded as unambiguous in determining the nature of the events that constitute the essence of the phenomenon itself. For the purpose of this they can be interpreted as oriented graphs and, based on connectivity criteria from data of different influence classes, are translated into matrices by structured relations in the dependency relationships, further formulation can be represented by adjacency and reachability matrices, which not only makes the data unified for further iterations in the derivation of connectivity and strong connectivity in matrices, but also operations of disjunction and conjunction [6].

So, let the sets be given starting from the properties of isomorphism, then, depending on their categories, the graphs G_1, G_2, \dots, G_n .

For example, there are two graphs

$$G_1 = (V(G_1), E(G_1)) \quad G_2 = (V(G_2), E(G_2))$$

and the sets $V(G_1), V(G_2)$ do not intersect. Then the union of graphs G_1, G_2 is a graph with a set of vertices $V(G_1) \cup V(G_2)$ and a family of edges $E(G_1) \cup E(G_2)$. It is also possible to form a connection of graphs G_1, G_2 denoted by $G_1 \oplus G_2$ taking their union and joining each vertex of the graph G_1 to each vertex of the graph G_2 . Since the isomorphism conditions must be described in graphs, we define connectivity expressions (which exclude incoherence of graphs). We give a connection with the separating set R , when the ratio $R \subset V(G_n)$ of a graph G_n can be defined as a vertex connection $k(G_n)$, and the edge connection $\lambda(G_n)$ as $R \subset E(G_n)$ a graph G_n . The conditions for given sets can be considered valid for $\lambda(G_n) \geq k$, that is, the graph G is edge- k -connected and the minimal edge separating set in the graph G contains at least k edges for all data sets. In this case, the sets of graphs are oriented and multigraphs are excluded, and the selected connections are derived from the described conditions into a matrix description of structured relations in the following dependencies: strongly connected, one-sided, weakly connected (previously excluded disconnected relations also need to be retained for possible subsequent iterations) [5].

On the basis of the above, we can define the adjacency matrix of a graph G_n in the form of a square matrix $A = (M_n \times M_n)$, since the rows and columns are put in a one-to-one correspondence to the vertices of the set V . The value of the element (a_{ij}) of this matrix located at the intersection of the i -th row and the j -th column is determined by the rule:

$$a_{ij} = \begin{cases} 1, & \text{if } (v_i, v_j) \in G \\ 0, & \text{if } (v_i, v_j) \notin G \end{cases}$$

The reachability matrix of a given graph G_n is determined by the binary closure matrix with respect to the transitivity of the relation E (from the given adjacency matrix of the graph). In this case, information on the existence of paths between the digits of the digraph is preserved. In the formation of the reachability matrix expressions, the condition for using disjunction operations and conjunction of adjacency matrices is valid for subsequent derivation to connectivity matrices and strong connectivity. Depending on the set in digraphs, we obtain the matrix expression by attainability $M^* = M \vee M^2 \vee \dots \vee M^n$. We derive matrices for strong and weak connections of the following form: in matrices of strong connection $S_1(G_n) = \{s1_{ij} : i = 1, \dots, n; j = 1, \dots, n\}$ and connection $S_2(G_n) = \{s2_{ij} : i = 1, \dots, n; j = 1, \dots, n\}$ we have

$$s1_{ij} = \begin{cases} 1, & \text{if } v_i \text{ achievable from } v_j \text{ and} \\ & v_j \text{ achievable from } v_i \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$s2_{ij} = \begin{cases} 1, & \text{if } \exists \text{ route connecting} \\ & v_i \text{ and } v_j \text{ and} \\ & \text{if } \exists \text{ route connecting} \\ & v_j \text{ and } v_i . \\ 0, & \text{otherwise} \end{cases}$$

Then we obtain the matrix disjunction of the choice $S = S_1(G_n) \vee S_2(G_n)$.

The subsequent choice by disjunction involves the translation of an array of digraphs of the matrix description of connectivity into matrices by weight functions: $A = \{(a_{ij}), i = 1, \dots, n; j = 1, \dots, n; a_{ij} = f(V_i, V_j)\}$,

where V_i, V_j are the vertices of the graph, $f(V_i, V_j)$ is the weight function.

In the future, weighted values can be translated into the cognitive map $G_n = (V, E)$, where the set of vertices V - is the totality of all the elements of the event, examined by the given array and obtained by weight functions, and E the connection between the elements V taking into account the cause-effect relationships between them and their mutual influence.

2.1. Mathematical model for describing the characteristics of an event with a fuzzy cognitive map

As a result, we have a cognitive map of the event in the form of a matrix representation $A = \|a_{kl}\|$ whose elements a_{kl} reflect the direct influence of the l -th element on the k -th element, where the sign a_{kl} shows the "direction" of the effect, its absolute magnitude is the degree of such influence, and the zero value corresponds to the absence of influence. The essential values are determined in dynamics $y_k^t = (y_k^0, \dots, y_k^t)$ - the vector of the dynamics of the k -th element up to the time t , where $k \in M$, $Y(t) = (y_1(t), \dots, y_m(t))$ is the vector of the values of the elements at time t , $Y_t = (Y(0), Y(1), \dots, Y(t))$ is the trajectory dynamics of the event up to the time t , $t = 0, 1, 2, \dots$. Then we have a matrix relation of the determined influence in the dynamic characteristics of the cognitive positions sought by the conditions

$$s_{ij} = \begin{cases} 1, & \text{if } \exists \text{ route connecting} \\ & v_i \text{ and } v_j \text{ and} \\ & \text{if } \exists \text{ route connecting} \\ & v_j \text{ and } v_i . \\ 0, & \text{otherwise} \end{cases} \quad (2) \quad (5)$$

Using the obtained matrix relation, we can determine the detailed weights of digraphs (as the required conditions) and in order to verify the derived characteristics, we obtain an extended interpretation by means of the F-graphs of cognitive maps: $F = \{(V, E)\}$

$$F(x_i, x_j, e_{ij}) = \begin{cases} +1, & \text{if the increase (decrease)} \\ & \text{of } x_i \text{ entails an increase} \\ & \text{(decrease) } x_j \\ -1, & \text{if the increase (decrease)} \\ & \text{of } x_i \text{ entails am decrease} \\ & \text{(increase) } x_j \end{cases} \quad (6)$$

$i, j = 1, 2, \dots, k$.

In the weighted signed digraph, the values of the weight coefficients of the arcs are given, in general form (4) $F(x_i, x_j, e_{ij}) \omega_{ij}$. In particular, the normalized exponent of the intensity of the influence of the previously presented characteristic function with the properties can be used objectively ω_{ij} to the

situation: $1 \leq \omega_{ij} \leq 1$, $\omega_{ij} = 0$, if x_i it does not depend on x_j and the effect is absent, $\omega_{ij} = 1$ with the maximum positive influence x_i on x_j , $\omega_{ij} = -1$ with the maximum negative influence x_i on x_j .

An important variety of cognitive positions for the system are formalized from clear cognitive maps with an unclear relational description. In this case, the elements of the system are perceived as unstructured or semi-structured, and the graphs are set by fuzzy parameters, and the arcs are given by the relations of their influence on the cause-effect relationships. The main purpose of implementing the fuzzy map device is to exclude the possibility of output beyond the matrix values (or ranges of values in the future). In the final combined versions of formalization, a mixed model of the matrix relation of the determined influence in the dynamic characteristics of cognitive positions is derived. Returning to the given expressions, we can define: provided that the binary relations on the set are fuzzy, the relations between the concepts (as nodes) are represented by fuzzy sets in powers of belonging. The set of relations of influence between all concepts is given by a matrix of fuzzy relations. At the same time, the fuzzy relation is considered as a fuzzy map of the fuzzy set by the influencing results, which, under system modeling, makes it possible to represent these concepts under external influence. In this case, it is fair

$$\begin{aligned} G &= G_1 \cup G_2, \quad G_1 = (V(G_1), E(G_1)), \\ G_2 &= (V(G_2), E(G_2)) \Rightarrow \\ G &= (V(G_1) \cup V(G_2), E(G_1) \cup E(G_2)) \end{aligned} \quad (7)$$

For a simplified interpretation, we translate the associated set into an adjacency matrix, where the element ω_{ij} of the given matrix is at the intersection of row i and column j to estimate the effect of the elements x_i on x_j . With expert intervention, the values can be translated into percentages. For further transformations, we can configure the cognitive maps in the model, provided by the conditions for sequential variation of the set by the balance of sign digraphs in models (the formal mathematical model of balance, is described conditionally for the purpose of regrouping). This is achieved by transfer into symbolic model delay, repeated description sheet, transfer to the binomial model taking into account the potential uncertainty certain experts sets (probability space graphs with N vertices if $N \rightarrow \infty$, where each pair of N is connected with a probability p and the result obtained by the number of edges be a random variable with an expected value $N \rightarrow \infty$ If G_0 - graph with vertices P_1, P_2, \dots, P_N , and n edges, the probability of obtaining it by means of this process will be $P(G_0) = p^n x(1-p)^{0.5N(N-1)}$. Different interpretations of vertices, edges and weights on edges, as well as various functions that determine the influence of links on elements, lead to the conclusion of a set of grouped elements, as a result of which the graphs are localized, weighed and added to the adjacency matrices with a subsequent iteration cycle. In the future, by calculating the spectrum of graphs, we can reconstruct the entire chain of events and further identify the phenomenon itself. Transforming spectra, we are dealing with invariance, even after groups of matrix permutations and transformations. The spectrum is a finite sequence of numerical invariants, its distinctive feature is the informative quality of the content of graph structures. This indicates that the essential information for us (or a significant part of it) is contained in the spectrum and in the end we can use its spectrum instead of digraph, since the finite sequence of numbers is easier to process in automatic mode, which proves applicability for the systems discussed.

In order to verify the results, the entire structure obtained is additionally checked for isomorphism properties and the data is entered into an intermediate covariance matrix for the likelihood estimation of algorithmic processes [7] (we exclude the description,

since system algorithms were not initially specified). The result of the study is the dependence of the change in cognitive maps in the fuzzy representation with further interpretation on the spectrum of graphs and subsequent processing of intermediate values of fuzzy positions.

3. Conclusion

On the basis of the research, a new approach is proposed to form the structure of a software analytical complex, processing socially significant information objects (events) based on the use of fuzzy cognitive maps. The conclusion is that a significant part of the required information is contained in the digraph spectrum and, ultimately, we can use its spectrum instead of the digraph, since the final sequence of numbers is easier to process in automatic mode, which proves applicability for the systems discussed. This approach provides the ability to describe in the language of set theory the relationship between modules (objects), programs, information system and business processes.

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