

# JUSTIFICATION OF VOLUME OF OUTPUT DEPENDING ON PRODUCT ENVIRONMENTAL CRITERION BY NONLINEAR MATHEMATICAL PROGRAMMING

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**Abstract.** The article provides the economical formulation, mathematical formalization and practical implementation of assortment planning of output product on the basis of financial profitability of each of the products; environmental profitability of each of the products; the period of receipt of funds for each sold product. Mathematical tools of formalization and implementation are nonlinear mathematical programming. The study was implemented on material of one of the key enterprises of the Republic of Bashkortostan.

**KEY WORDS:** ASSORTMENT OF OUTPUT, ENVIRONMENTAL FRIENDLINESS OF PRODUCT, NONLINEAR MATHEMATICAL PROGRAMMING.

## 1. Introduction

Among many practical challenges at enterprises there is always the problem of planning of goods and services. At the stage of promotion of goods or services into the market any it is difficult to propose any mathematical models allowing to justify a quantity of products and its assortment. In marketing it is proposed to use test-market sales, market research, participation in exhibitions and fairs in order to justify the volume of products.

In the period of strong sales there is a sufficient number of sales statistics that is why the construction of mathematical models of planning of goods assortment is already quite possible. Assortment planning is considered as periodically solved problem and the results of its solution determine the assortment strategy of the firm for a certain period.

The comparison of various strategies of assortment planning (maximum difference of prices between the price of wholesale supply and sale price of goods excluding penalty payments; maximum demand for previous sales, regardless of penalty payments; equal distribution of assortment; replenishment of previous sales; adaptive strategy focused on future demand taking into account environmental factor and amount of penalty payments and etc.) allows to propose ultimate environmental effect of assortment planning and percent decrease of expected profit.

## 2. Publication

The mathematical model applied in this study allows finding better solutions on planning of product assortment taking into account the identified above factors and limits in relation to the conditions of the existing market. The approach to this problem is based on the method of nonlinear mathematical programming implemented in the software program - Microsoft Excel.

Consider the enterprise producing and distributing  $i$ -type products ( $i = 1, \dots, N$ ). The manufactured types of products are still in demand at the market that is why it is necessary to focus on promotion of already producing types of products before launching a new product line.

Besides, the following indicators should be defined:

$j$  – investigated period ( $j = 1, \dots, J$ )

$P_{ij}$  – sale price of a unit of  $i$ -type product in the  $j$  period;

$C_{ij}$  – direct variable costs on production of a unit of  $i$ -type product in the  $j$  period;

$G_{ij}$  – penalties for production of a unit of  $i$ -type product in the  $j$  period;

$t_{ij}$  – period of return of funds for  $i$ -type product since the start of its production for the  $j$  period, measured in periods;

$k_{ij}$  – quantity of  $i$ -type product manufactured and distributed in the  $j$

period.

Taking into account the fact that in practice there are limits on volumes of output and sales of products of each type:

$$k_{ij \min} \leq k_{ij} \leq k_{ij \max}, \quad i = 1, \dots, N, \quad j = 1, \dots, J \quad (1)$$

$k_{ij \min}$  – minimum volume of output of  $i$ -type product in the  $j$  period (volume of output which the enterprise is to produce in order not to lose its market share),

$k_{ij \max}$  – maximum volume of output of  $i$ -type product in the  $j$  period (this indicator is defined by the firm's opportunities to sale the end product and also production capacity).

Maximum profit can be achieved by distribution of the available current assets by types of products in accordance with financial profitability but due to environmental profitability.

Thus, the profit per unit of product (without penalties) is

$$S_{Mij} = P_{ij} - C_{ij} \quad (2)$$

Financial profitability per unit of product is calculated as follows

$$M_{ij} = \frac{P_{ij} - C_{ij}}{C_{ij}} \quad (3)$$

Unit financial profitability of product unit is

$$R_{ij} = \frac{P_{ij} - C_{ij}}{C_{ij} t_{ij}} \quad (4)$$

The term of unit financial profitability is introduced due to the need to compare products with various values of profitability and period of return of funds for the manufactured products.

The term of environmental profitability, like financial profitability, is introduced. But in the latter case instead of profitability at cost, profitability of environmental payments, penalties for harmful products is calculated.

Unit environmental profitability per unit of products is

$$E_{ij} = \frac{P_{ij} - G_{ij}}{G_{ij} t_{ij}} \quad (5)$$

The effectiveness of environmental activities is defined by aggregate indicator of total profitability, combining various proportions of financial and environmental profitability.

$$RE_{ij} = (1-a) \cdot R_{ij} + a \cdot E_{ij} \quad (6)$$

where  $a$  – a coefficient of environmental friendliness of the production set depending on the selected production strategy,  $0 \leq a \leq 1$ .

The effective functioning of the enterprise requires to maximize accumulated financial profit obtained from the enterprise's principal activities during the period which is calculated as accumulated receipts in this period.

$$Z^{0j} = \sum_{j=j-g}^j \sum_{i=1}^N (P_{ij} - C_{ij} - G_{ij}) k_{ij} \rightarrow \max_{j: t_{i(j-g)}} \quad (7)$$

where  $g$  – variation limits  $t_{ij}, g = \{1, 2, 3, 4\}$

While the problem is considered as dynamic (calculated for  $j$  periods), the profit obtained in the  $j$  period ( $j = 1, \dots, J$ ) is reinvested. Minimum volume of output  $W_{min}^j$  for next period is initially provided:

$$W_{min}^j = \sum_{i=1}^N (C_{ij} + G_{ij}) * k_{ij min} \quad (8)$$

Costs for producing minimum volume of output in the coming period are deducted from the receipts at the beginning of the period  $Z^{0j}$ . Then, the funds remaining at the enterprise's disposal are

$$Z^j = Z^{0j} - W_{min}^j \quad (9)$$

The remaining  $Z^j$  is invested to produce more cost-effective types of products (first, the output with the highest unit aggregate profitability is manufactured, then, output with less profitability and etc.) for the current period

$$Z^j = \sum_{i=1}^N (C_{ij} + G_{ij}) k_{ij} \quad (10)$$

$$i: RE_{ij} \rightarrow \max$$

At this stage the objectives are identified.

At the beginning of the planning time interval  $[0, T]$  divided into  $j$  periods ( $j = 1, \dots, J$ ) the management of the enterprise provide financial resources in the amount of  $Z^{01}$  for production of  $i$  types products ( $i = 1, \dots, N$ ).

At the beginning of each period it is necessary to distribute the available financial funds by  $i$  types of products ( $i = 1, \dots, N$ ) due to the following criteria:

1) limits on output are defined by the formula 1.

Cost of minimum volume of output of  $i$ -type products in the  $j$  period is calculated by the formula 8;

2) the funds remaining at the enterprise's disposal at the beginning of the  $j$  period (besides the first period when they are set) are determined by the formula 2.9 and the accumulated receipts in the  $j$  period – by the formula 7;

3) distribution of the available funds is conducted in accordance with unit financial profitability of each product in each particular period and is calculated by the formula 10. Total profitability of product unit is determined by the formula 6, unit financial profitability of product unit – by the formula 4, unit environmental profitability of product unit is defined by the formulas 6, 4 and 5, consequently.

The result of funds distribution in the  $j$  period is the determination of the set of values  $\{k_1^j, \dots, k_N^j\}$  which ensures that the enterprise will obtain maximum profit due to unit financial profitability and unit environmental profitability;

4) fixed costs which the enterprise pays at the beginning of each month (except for the beginning of the planning ) are covered due to profit and are financial values;

5) the enterprise follows the strategy of reinvestment of profit. Such policy means that at the end of the period the net balance should be minimum (the latter is limited by that volumes of production are integer values)

$$Z^j - \sum_{i=1}^N (C_{ij} + G_{ij}) k_{ij} \rightarrow 0 \quad (11)$$

6) the indicators  $P_{ij}, C_{ij}, G_{ij}, t_{ij}, k_{ij min}, k_{ij max}$  change from period to period and are provided by Marketing Department (for a month ahead with greater accuracy, for a longer period there are predictive estimates which are corrected as production and sales plans are implemented).

Now, it is necessary to solve the problem of selection of output depending on environmental friendliness of products.

OJSC "Ufa varnish and paint plant" manufacturing eight

types of products: enamels ПФ-115, ПФ-226, ЭПП-751, XB-785, paints MA-15 and red lead MA-15, prime coating ГФ-021, varnish БТ-577 is the object of the investigation. The plant's objective is to distribute the funds optimally during the set time interval. The main criterion is to maximize profits taking into account the factor of environmental friendliness of the products in the long term. The problem is solved in stages using the following procedure.

1<sup>st</sup> stage. The table with indicators for each type of products in each period needed during analysis should be prepared (Table 1).

The indicator  $Z^{0j}$  – the enterprise's financial funds at the beginning of the period ( $Z^{01} = 7000$  roubles) is calculated.

**Table 1** – Calculation of profitability indicators

Item	Product sale price $P_{ij}$ , RUB in thousands/tonness	Cost price, $C_{ij}$ , RUB in thousands/tonne	Penalties payments, $G_{ij}$ , RUB in thousands/tonne	Return period of funds $t_{ij}$ , week	Unit financial profitability of product unit $R_i$ , %	Unit environmental profitability of product unit $E_i$ , %	Total profitability of product unit $RE_i$ , %
Prime coating ГФ-021	26.67	20.34	1.53	1	31.12	16.39	23.75
Enamel XB-785	36.70	21.74	2.67	2	34.41	6.36	20.38
Enamel ЭПП-751	34.00	22.17	2.52	2	26.68	6.25	16.47
Varnish БТ-577	13.54	8.46	0.65	3	20.02	6.65	13.33

Enamel ПФ-115	32.99	23.56	2.46	2	20.01	6.22	13.11
Enamel ПФ-226	31.24	22.46	1.25	3	13.03	8.02	10.53
Paint MA-15	28.44	22.14	1.91	3	9.49	4.62	7.05
Red lead MA-15	12.05	8.59	0.86	4	10.07	3.26	6.66

$$v = Z^j / (C_{ij} + G_{ij}) \quad (12)$$

2<sup>nd</sup> stage. The values of unit financial profitability  $R_i$  and unit environmental profitability  $E_i$  for each type of products are calculated. Products become ordered when total profitability  $RE_{ij}$  ( $a = 0.5$ ) decreases. Priorities of types of products are identified while total profitability decreases.

3<sup>rd</sup> stage. The value of accumulated variable costs needed for production of minimum quantity of each type of products is calculated. All accumulated variable costs of each type of products corresponding to minimum volume of output are added up and the indicator  $W_{min}^j$  is obtained. Thus, the value of the funds available at the enterprise's disposal  $Z^j$  is calculated. The value of the index  $i$  is 1.

4<sup>th</sup> stage. The remaining funds for each type of products having higher priority in accordance to limits on volumes of output are distributed iteratively, for example, the product  $i$ . In this case the number of units  $k_{ij}$  which can be produced is calculated if there is an identified earlier

amount of financial resources  $Z^j$ .

The following formula is used for calculation

**Table 2 – Calculation of indicators in the first period**

1st period						$Z^{01}$	7 000.00
Priority	Item	$RE_{il}$	$k_{il}$	$W_{min}^1$	$W^1$	$S^1$	$Z^1$
1	Prime coating ГФ-021	23.75%	39	196.87	853.09	1 280.16	5 365.69
2	Enamel XB-785	20.38%	30	390.62	732.42	1 688.20	4 512.60
3	Enamel ЭПН-751	16.47%	19	172.81	469.05	884.00	3 780.18
4	Varnish БТ-577	13.33%	32	72.85	291.39	541.60	3 311.13
5	Enamel ПФ-115	13.11%	54	624.38	1 404.86	2 573.22	3 019.74
6	Enamel ПФ-226	10.53%	9	71.12	213.35	374.88	1 614.87
7	Paint MA-15	7.05%	24	96.22	577.30	796.32	1 401.52
8	Red lead MA-15	6.66%	5	9.45	47.25	72.30	824.22
	Total:		212	1634.31	4588.71		

$RE_{il}$  - total profitability by products,  $W_{min}^1$  - costs on minimum volume of output,  $W^1$  - costs above minimum volume of output (the amount of distributed financial resources due to the criterion  $RE_{il}$  and limits on volumes of production),  $S^1$  - sales revenue by each type of products,  $Z^{01}$  - amount of financial resources at the beginning of the period,  $Z^1$  - funds gradually distributed by types of products, the last line shows the demand balance after distribution in the current period,  $Z^{02}$  - amount of financial resources obtained in the second period is defined as an amount of revenue by types of products and demand balance as of end of previous period.

7<sup>th</sup> stage. If  $j \leq J$ , the index  $j$  has the value  $j + 1$ . It is necessary to return to the 1<sup>st</sup> stage, otherwise, the procedure is completed.

The problem has been solved under the following conditions – fixed costs of 1000 roubles per month. Three strategies have been applied:

- 1) distribution of funds and production not including environmental factor, profit maximization;
- 2) distribution of funds due to environmental factor but excluding financial profitability;
- 3) usage of 50% proportion of financial and environmental profitability.

Then, the volume of output of  $i$ -type product is defined in accordance with the following equation. If

$$k_{ij \min} + v < k_{ij \max}, \quad (13)$$

the value  $k_{ij}$  has the value  $k_{ij \min} + v$ . Otherwise, the value  $k_{ij}$  has the value  $k_{ij \max}$ .

The value is calculated as follows  $(C_{ij} + G_{ij}) * k_{ij}$ .

The value  $Z'$  is defined by the formula

$$Z' = Z^j - (C_{ij} + G_{ij}) * (k_i - k_{i \min}) \quad (14)$$

If  $Z'$  is so low that this amount cannot be used for production of any product, or if it is zero, the 6<sup>th</sup> stage starts, otherwise,  $Z^j$  has the value  $Z'$ .

5<sup>th</sup> stage. The index  $i$  has the value  $i + 1$ . Then, it is necessary to return to the 4<sup>th</sup> stage.

6<sup>th</sup> stage. As a result of implementation of 4<sup>th</sup> and 5<sup>th</sup> stages all financial resources are distributed by types of products due to the criterion of total profitability (Table 2 shows the obtained data for the first period).

### Interpretation of results of solving the problem of output product depending on environmental friendliness of product.

While maintaining the strategy of profit maximization without other limits the enterprise receives higher profits in the long term as it invests the funds obtained from sales of products into production of types of products with the highest financial profitability.

If the enterprise follows the strategy of maximum environmental effect under the current limits, total sales revenue and profits decrease.

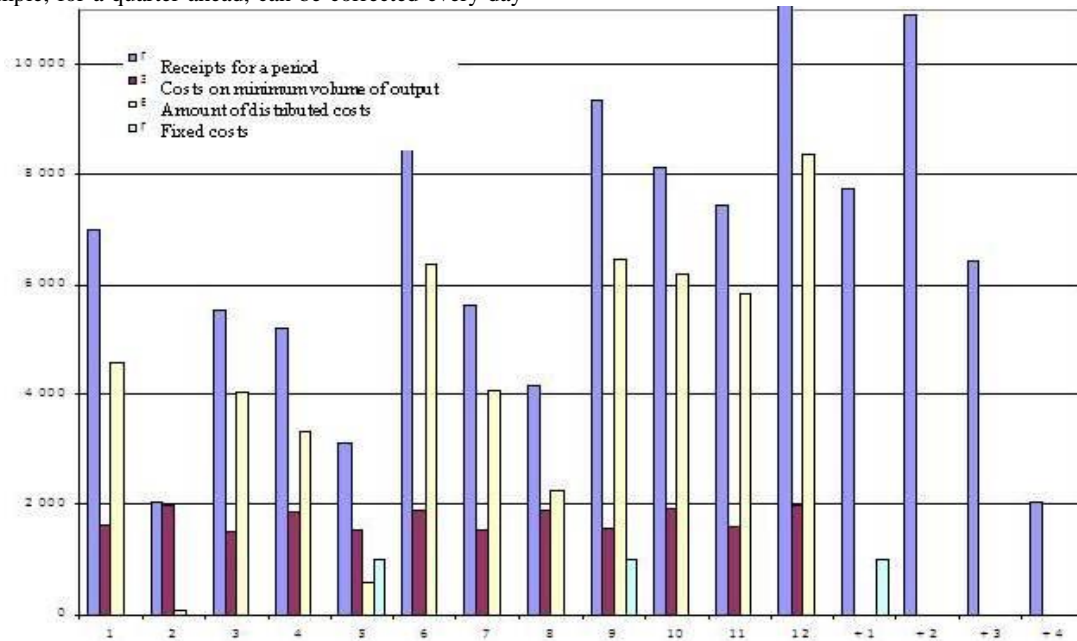
The strategy proves the need to manage output of products depending on the criterion of environmental friendliness when the enterprise sets an average profitability (total as a combination of financial and environmental in equal proportions of 50% and 50%) – as of end of investigated period the profit is 4 089 roubles. This amount is rather lower than the enterprise would have received if the first strategy was implemented. But in comparison with the second strategy it is much more (Fig. 1). It would be good to use the criterion of environmental friendliness as financial losses are slightly less and the production of greener products allows to define

that the enterprise follows the selected strategy of consistent improvement of the manufactured products and aims for maximum environmental friendliness under the current economical limits.

**Conclusion.** The advantages of this problem are that its solution allows automating the process of decision making in the sphere where there is difficulty of accurate forecasting of future prices for products, sales volume and etc. That is why projected values calculated, for example, for a quarter ahead, can be corrected every day

and the data for a month ahead are more likely to be calculated.

Thus, the solution of the problem allows managers of enterprises to use figures of profits, costs, work load of production facilities calculated for a week, a month ahead (for a quarter and a year ahead), to have the data on profitability of each type of products and its compliance with quality standards in order to distribute funds for advertising and promotion of products by types more effectively.



*Figure 1 – Dynamics of indicators of solving the problem of output of products depending on environmental friendliness of the product while using strategy of compromise, rubles in thousands.*

#### LITERATURE:

1. Bronshteyn A.M., Litvin V.A., Rusin I.I. Greening the economy: methods of regional governance. M.: Nauka, 1990. 120 p.
2. On environmental expertise: federal law of 23.11.1995 No. 174-Φ3 (edit. of 18.12.2006) // Russian Federation Code. 1995. No. 48. Art. 4556.
3. Gizzatullin R. Kh. Theoretical aspects of environmental function of modern Russian government / R.Kh. Gizzatullin // Legal policy and legal life. 2005. No. 1. Pp. 121-129.
4. Dorzhieva R. Ts. Environmental requirements: concepts and correlation with other requirements / R. Ts. Dorshievs // Environmental law. 2006. No. 5. Pp. 7-10.
5. Petrova T.V. Technical regulation as a part of legislative regulation of environment protection / T.V. Petrova // Environmental law. 2005. No. 1. Pp. 77-81.
6. Reymers N. F. Environmental management: glossary. – N.F. Reymers. M.: Mysl, 1990. 637 p.
7. Rodionova L.N., Mukhtarova E.G. Market orientation of methods of environmental management in the region. Economical vestnik UGNTU. Collection of scientific papers/ Edit. Rodionava L.N. and others. Ufa: pub. house of Fund of assistance in development of scientific investigations, 2001. 168p. Pp.105-110.