Generation of test sequences with a given switching activity

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Abstract: The proposed study is based on the universal method of quasi-random Sobol sequences generation, efficiently used for address test sequence formation. As the mathematical model, a modification of the economical method of Antonov and Saleev is used. The main idea of the suggested approach is the use of generating matrices with not necessarily the maximum rank for the procedure of generating test patterns. The proposed approach allows the generation of significantly more different sequences with different switching activities of the individual bits as well as the sequences itself. Mathematical expressions are obtained that make it possible to estimate the limiting values of the switching activity, both of the test sequence itself and the individual bits. Examples of the application of the proposed methods are considered.

Keywords: COMPUTER SYSTEM TESTING, ADDRESS TEST SEQUENCES, SWITCHING ACTIVITY, MULTIPLE TESTING, MODIFIED SOBOL SEQUENCES

1. Introduction

The testing of modern computing systems such as embedded systems, systems on a chip, and nets on a chip is important and is in demand at the present time [1, 2, 3]. There are numerous approaches and new solutions aimed to increase efficiency of modern computer system testing. Among them the deterministic tests play a crucial role, such as the counting sequences, Gray sequences, anti-Gray sequences, sequences with maximal Hamming distance, and many others [4, 5]. Such sequences are usually periodic and are often called enumeration sequences, de Bruijn sequences, or sequences of maximum length [5]. There are many kinds of maximum length sequences. Each of these test sequences is described by its unique algorithm that suggests a specific implementation and has characteristics that are common with the other sequences. As the general characteristic of a test sequence, the most commonly used one is the so-called switching activity, which affects the switching activity of the tested computer systems [4–8].

Address sequences as subset of periodic tests have been investigated within the framework of the memory built-in self-testing [9–13], and multi-run memory testing [14–16]. The main feature of this kind sequence is that it consists of the entire set of binary vectors including all possible 2^m binary combinations, where m is the address length in bit. The task of generating the desired address sequence can therefore be regarded as the generation of m-dimensional binary vectors in binary space.

There is no doubt that the efficiency of the test is a major design issue. To achieve higher efficiency, a characteristic like the switching activity often plays the crucial role. At the same time, the restriction on the set of the patterns (which is always 2^m) in address sequences may reduce the efficiency of the memory test procedures. To overcome this tradeoff the new approach as the extension of the idea of address sequences generation for a general case of the test sequences with specified values of switching activities is proposed and analyzed in this paper. The motivation for this work is to design an efficient test sequence generator, which allows to generate significantly more different sequences with different switching activities of the individual bits of an address and of the whole sequence itself.

2. Mathematical Model

To significantly reduce hardware overhead needed to generate many different address sequences, a mathematical model of a universal generator was considered and investigated [15]. An address sequence is an ordered sequence of m-bit binary vectors A(n) = a_{m-1}(n) a_{m-2}(n) a_{m-3}(n) ... a_1(n) a_0(n), a_i(n) \in \{0, 1\}, i \in \{0, 1, 2, ..., m – 1\} and n \in \{0, 1, 2, ..., 2^m – 1\} where the vectors take all possible values \{0, 1\}, and \{0, 1, 2, ..., 2^m – 1\}, respectively. As the basis of this model, a modified method of Sobol sequence generation is used [11, 16, 18]. According to this model, the nth element A(n) = a_{m-1}(n) a_{m-2}(n) ... a_1(n) a_0(n), of the Sobol sequence is formed in accordance with the following recurrence relation

A(n) = A(n-1) \oplus v_i, \quad n = 0, 2^m - 1, i = 0, m-1,

(1)

in which only one modified direction number v_i = \beta_{m-1}(i) a_{m-2}(i) a_{m-3}(i) ... \beta_2(i) \beta_1(i), i \in \{0, 1, 2, ..., m – 1\} is added to the previous element A(n-1) of the Sobol sequence [15]. The value of the index i of the direction number v_i that is used in expression (1) as a term depends on the so-called switching sequence T_{m-1} of the reflected Gray code [15, 18]. For example, for m = 4, the switching sequence has the form T_3 = 0, 1, 2, 0, 1, 2, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0. In this case, the switching sequence patterns are generated. The motivation for this work is to design an efficient test sequence generator, which allows to generate significantly more different sequences with different switching activities of the individual bits of an address and of the whole sequence itself.

constructed from m linearly independent binary vectors v_i = \beta_{m-1}(i) a_{m-2}(i) a_{m-3}(i) ... \beta_2(i) \beta_1(i), i = 0, m-1. Linear independence is a necessary condition to generate all entries of the address sequence. In this case the matrix (2) has the maximal rank.

To evaluate the properties of modified Sobol sequences A(n) = a_{m-1}(n) a_{m-2}(n) a_{m-3}(n) ... a_1(n) a_0(n) when used as a test sequence, the metric F(a_i), i \in \{0, 1, 2, ..., m – 1\}, which determines the number of switches (changes) of the jth digit a_i of the sequence A(n) has been proposed and analyzed in [15]. This metric F(a_i) is called switching activity [6, 8, 18] and determines the total number of switches of the jth digit of the test patterns A(n) when all 2^m patterns are generated.
3. Proposed method

As was shown earlier [11, 15–16, 18], the use of generating matrices (2) of maximum rank allows to generate a wide range of sequences having a maximum length of $2^n$. Such sequences were often called address sequence because they are composed from the whole set of non-repeating binary $m$-bit patterns. The requirement of maximizing the rank of the generating matrix $V$ allows to obtain all $2^n$ binary combinations in the generated sequence $A(n)$ (1), however, they impose several restrictions on the properties of such sequences [18].

The proposed solution is based on the extension of the mathematical model (1) in terms of generating matrix $V$, which in this case does not have to have the maximal rank. This modification will allow to generate a wider spectrum of test sequences with a desired switching activity than the known solutions [15, 18]. As the examples of such an approach, Table 1 contains some sequences $A(n)$ obtained according to (1) with multiple generating matrices $V$ for $m = 4$ with the ranks less than 4.

The first $(V_1)$ is the trivial solution to generate the sequence $A(n)$ with the maximum switching activity $F_{sw}(A) = 4$, which equals $m$ for general case. It should be mentioned that the sequence $A(n)$ consists from two patterns, namely arbitrary $A(0)$ and its negation, as presented in Table 1, where $A(0) = 0 1 0 0$ and $A(1) = 1 0 1 1$. The rank of the matrix $V_1$ equals to 1, matrix $V_2$ has rank 1, and matrix $V_3$ has rank 3. In all these cases the test sequence $A(n)$ is a periodic sequence with the period less than $2^n = 2^4$. At the same time the matrix $V_2$ with rank 2 used in (1) for test sequence generation provides the output sequence with the maximum period of $2^4$ but does not include all possible 4-bit binary combinations.

A brief analysis of the examples presented in Table 1, allows to make conclude that in a case of a random matrix $V$ with an arbitrary rank there are different types of test sequences $A(n)$ generated by equation (1). The type of test sequences called address sequence in $V$ and $V_1$ of Equation (1) are presented below (3).

\[
\begin{align*}
V &= \begin{pmatrix} 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 \\
1 & 0 & 1 & 1 \\
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1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 
\end{pmatrix}
\end{align*}
\]

First, let us formulate the property of the sequences $A(n)$ generated based on relation (1) using a random generating matrix $V$.

The switching activity $F_{sw}(A)$, for the $j$th $j \in\{0, 1, 2, ..., m-1\}$ digit $a_j$ of the sequence $A(n) = a_{m-1}(n) ... a_2(n) a_1(n) a_0(n)$ generated according to (1) takes values in the range from 0 to $2^m - 1$. The value of this switching activity $F_{sw}(A) = 0$ for the $j$th bit of the $A(n)$ generated according to (1) is provided by the $j$th all zero column in the generating matrix $V$, as it was shown in Table 1 (see $V_1$ and $V_2$). Maximal switching activity $F_{sw}(A) = 2^m - 1$ for the $j$th bit of the $A(n)$ corresponds to all ones within $j$th column in matrix $V$ as it is for the cases of $V_1$ and $V_3$, shown in Table 1.

The switching activity $F(A)$ of the sequence $A(n)$, takes the minimum value $F(A) = 0$ in the case of an all zero generating matrix $V$. The maximum value of the switching activity $F(A) = m \times (2^n - 1)$ is achieved with the all ones matrix $V$ (see $V_1$ in Table). The ranges of values of average switching activity values $F_{sw}(A)$ and $F_{sw}(A)$ are presented below (3).

\[
\begin{align*}
\min F_{sw}(A) = \min F(A) = (2^n - 1) = 0; \\
\max F_{sw}(A) = \max F(A)/2^{m-1} = 1; \\
\min F_{sw}(A) = \min F(A)/2^{m-1} = 0; \\
\max F_{sw}(A) = \max F(A)/2^{m-1} = m.
\end{align*}
\]

A wide range of possible values of switching activity (3), as well as the absence of many restrictions on their mutual relationship allows us to generate a significantly larger number of test sequences with specified values of switching activities.

4. Maximal Length Test Sequences

As the test sequence $A(n) = a_{m-1}(n) a_{m-2}(n) a_{m-3}(n) ... a_1(n) a_0(n)$, where $a(n) \in\{0, 1\}$, $i \in\{0, 1, 2, ..., m-1\}$, and $n \in\{0, 1, 2, ..., 2^m-1\}$, $m$-dimensional binary vectors in binary space were considered [18]. The generation of the specified test sequence has been regarded as the generation of $m$-dimensional binary vectors in binary space. The set of linearly independent vectors $v_i = \beta_{m-i}^* (\beta_{m-i}^*)^t \beta_{m-i}^* (\beta_{m-i}^*)^t ... (\beta_1^*)^t (\beta_1^*)^t i = 0, 1, 2, ..., m-1$, generates the $m$-dimensional binary vectors $A(n)$ through all their possible linear combinations [19, 20]:

\[
A'(n) = v_1^* \cdot b_1(n) \oplus v_2^* \cdot b_2(n) \oplus v_3^* \cdot b_3(n) \oplus ... \oplus v_{m-1}^* \cdot b_{m-1}(n).
\]

\[
B(n) = b_{0}(n) b_{m-2}(n) b_{m-3}(n) ... b_{1}(n) b_{0}(n); b_{0}(n) \in\{0, 1\}, i \in\{0, 1, 2, ..., m-1\}
\]

is any binary vector set consisting of all possible $2^n$ binary combinations. Then the vector space $A(n)$ formed according to (4) is of dimension $m$ and consists of all $2^n$ vectors, and that is why vectors $A(n)$ can be used as an address sequence [18]. For further investigations, the set of vectors $B(n)$ is regarded as the Linear sequence or simply the binary Up-counter sequence.

Application of the Gray code sequence in (4) was the productive idea for recursive generation of the quasi-random Sobol sequences according to (1) [21]. Relation between the output sequence $A'(n)$ and $A(n)$ generated based on linear sequence (4) and Gray sequence (1) can be described in terms of corresponding generation matrices $V'$ and $V$.

For these sequences $A'(n)$ and $A(n)$ the next statement is true.

Statement 1. Sequence $A'(n)$ generated as $m$-dimensional binary vectors according to (4) based on any generating $m$ by $m$ matrix $V^*$ can also be obtained from the recursive relation (1) ($A(n) = A'(n)$) with matrix $V$ obtained from (7), and vice versa.

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Then to get the sequence \( v^* \), are determined, and the sequence \( v_0 \) follows from next statement \([19]\).

Let \( v_0 \) and it is obvious that the sequence \( A'(n) \) generated based on \( V_3 \) is identical to the sequence \( A(n) \) generated according to \( V_3 \).

Statement 1 makes to allow the conclusion that the resulting sequence \( A(n) \) obtained from the recursive relation (1) is the linear combination of the binary sequences \( b(n) \in \{0, 1\} \), \( i \in \{0, 1, 2, \ldots, m - 1\} \) representing the counter sequence \( B(n) \). The sequence \( b(n) \) has period equal to \( 2^{n-1} \), that is why the period of \( A'(n) \) will be equal to the maximal period when linear sum (4) includes \( b_{m-1}(n) \), what follows from next statement \([19]\).

Statement 2. The sum \( A'(n) \) of the periodic binary counting sequences \( b(n) \in \{0, 1\} \), \( i \in \{0, 1, 2, \ldots, m - 1\} \) has the period \( 2^{n-1} \), where \( j \) is the maximal index of the nonzero values of \( v_j^* \).

Based on the last statement, it is easy to obtain a condition for the formation of a sequence \( A(n) \) of the maximal period which consists in the fulfillment of the inequality \( v_{m-2} \neq v_{m-1} \). This inequality follows from the following affirmation. According to statement the maximal length sequence \( A(n) \) will be obtained in a case when \( v_{m-2}^* \neq 0 \). Then to get the sequence \( A(n) = A'(n) \) the next condition should be accomplished \( v_{m-2} \oplus v_{m-1} \neq 0 \) (see (5)).

Table 1 contains two examples of \( A(n) \) with maximal period second period 2 for the case of two generating matrices \( V_2 \) and \( V_3 \), due to both matrices \( v_2 \neq v_3 \). There are three matrices \( V_1 \), \( V_2 \) and \( V_3 \) in Table 1 with \( v_2 = v_3 \), which correspond to the sequences with a period less than \( 2^4 \).

**5. Generation of Test Sequences with Specified Switching Activities**

Considering the wide range of applications of test sequences with specified values of switching activities \( A(n) \) \([5–10, 18]\), the problems of finding a generator for such sequences can be formulated as follows:

Find the generating matrix \( V(2) \), which provides the required values of \( F_{av}(A) \) and \( F_{aw}(a) \) of the sequence \( A(n) = a_{m-1}(n)a_{m-2}(n) \ldots a_2(n)a_1(n), a(n) \in \{0, 1\} \), \( i \in \{0, 1, 2, \ldots, m - 1\} \) and \( n \in \{0, 1, 2, \ldots, 2^{m-1} - 1\} \), where the desired values \( F_{av}(A) \) and \( F_{aw}(a) \) do not exceed their bounds (3). There are three cases of this design problem. The first case is the test sequence with a desired \( F_{av}(A) \) and the second is specified \( F_{aw}(a) \) for \( k \leq m \) digits \( a_{k-1}, a_{k-2}, \ldots, a_1, a \in \{0, 1, 2, \ldots, m - 1\} \) \( i = \Gamma_k \), of the sequence \( A(n) \). The third case is the union of the first two.

Let’s show the solutions for all those three types of problems using the algorithm of partition of a positive integer number \( F(A) = (2^m - 1) \times F_{av}(A) \), that is the basis for matrix \( V \) construction as it was shown in \([18]\).

**Algorithm for partitioning an integer into terms:**

Input data: An integer \( F(A) \) belonging to the range from 0 to \( m^2(2^m - 1) \); The terms should belong to the set of in integer values \( 2 \), where \( \{i \in [0, 1, 2, \ldots, m - 1]\) to obtain a partition of the integer \( F(A) \).

1. Initially, the sum of all terms 2 is formed, which is equal to the maximum \( m \)-bit binary number \( 2^m - 1 \).

2. The division operation \( F(A) \) by \( 2^m - 1 \) is performed. The resulting quotient \( q \) determines the minimum amount of use of each of the terms 2 in the partition of the integer \( F(A) \). If the remainder \( r \) of the division operation is zero, the quotient \( q \) is the number of uses of each of the terms 2 in the partition \( F(A) \), and at this step the partition algorithm ends. Otherwise, the next step is performed.

3. The remainder \( 0 < r < 2^m - 1 \) from the division operation performed in the previous step is represented in the binary code \( r = b_{m-1} \times 2^{m-1} + b_{m-2} \times 2^{m-2} + b_{m-3} \times 2^{m-3} + \ldots + b_0 \times 2^0 \), \( b_i \in \{0, 1\} \).

4. The partition of the integer \( F(A) \) into terms 2 is constructed, where \( \{i \in [0, 1, 2, \ldots, m-1]\) each of the terms is included in the partition \( q + b_{m-1} \) times. The quantity \( q + b_{m-1} \) determines the value of the Hamming weight \( w(V_i) \) of the binary vector \( v_i = \beta_{m-1}(0)\beta_{m-2}(1)\beta_{m-3}(1) \ldots \beta_0(0)\beta_0(1) \), which is the result of applying this algorithm.

**Problem 1.** Find the generating matrix \( V \) for the sequence \( A(n) \) for a given value of \( m \) and the desired value of \( F_{av}(A) \).

The solution of the problem 1 will be as follows: obtaining \( F(A) = \text{int}(2^m \times F_{av}(A)) \); partitioning the integer \( F(A) \) into terms \([18]\); obtaining the values of the row weights \( w(V_i) \) of the desired generating matrix \( V \) and finding the matrix with maximal rank corresponding to the given values of \( m \) and \( w(V) \). The matrix with maximal rank is chosen from the matrix with the restriction that \( v_{m-1} \neq v_{m-2} \), which guarantees the maximal length of the sequence \( A(n) \). The maximal rank of the matrix \( V \) is the requirement to get maximum number of binary combinations \( a_{m-1}(n)a_{m-2}(n)a_{m-3}(n) \ldots a_1(n)a_0(n) \) within \( A(n) \) \([19]\).

**Problem 2.** For a given value \( m \) find the generating matrix \( V \) for sequence \( A(n) \), in which specified \( F_{aw}(a) \) for \( k \leq m \) digits \( a_{k-1}, a_{k-2}, \ldots, a_{k-1}, a \in \{0, 1, 2, \ldots, m - 1\} \) \( j = 1, k \), are determined.

Initially, as in the case of Problem 1, the average values \( F_{aw}(a_{k-1}), F_{aw}(a_{k-2}), F_{aw}(a_{k-3}), \ldots, F_{aw}(a_0) \) of switching activities are represented as total values of the number of switching bits \( a_{k-1}, a_{k-2}, a_{k-3}, \ldots, a_0 \) of \( A(n) \). These values \( F_{aw}(a_0), F_{aw}(a_1), F_{aw}(a_2), \ldots, F_{aw}(a_{k-1}) \) are determined according to the relation \( F_{aw}(a) = \text{int} \{ F_{av}(A)(2^m - 1) \} \). Then, \( F_{aw}(a_0) \) is converted to a \( m \)-bit code represented in the binary number system \( F_{aw}(a_{k-1}) = F_{aw}(a_{k-2}) = \beta_{m-1}(0)2^{m-1} + \beta_{m-2}(0)2^{m-2} + \ldots + \beta_0(m-1)2^0 \). The values of all \( k \leq m \) columns of the matrix \( V \) are calculated, which provides the switching activities \( F_{aw}(a_{k-1}), F_{aw}(a_{k-2}), F_{aw}(a_{k-3}), \ldots, F_{aw}(a_0) \). The next step in solving Problem 2 is to randomly (equally likely and independently) generate the remaining columns of the binary matrix \( V \). From the resulting matrices, the matrix with the highest rank must be selected.

**Problem 3.** For a given value \( m \) find the generating matrix \( V \) for sequence \( A(n) \), in which specified \( F_{av}(A) \) for \( k \leq m \) digits \( a_{k-1}, a_{k-2}, a_{k-3}, \ldots, a_0 \), \( a \in \{0, 1, 2, \ldots, m - 1\} \) \( j = \Gamma_k \), are determined, and the switching activity \( A(n) \) equals \( F_{aw}(a) \).

The correct formulation of Problem 3 assumes that \( F_{aw}(a_{k-1}) + F_{aw}(a_{k-2}) + F_{aw}(a_{k-3}) + \ldots + F_{aw}(a_0) < F_{av}(A) \leq m \). At the initial stage, the solution of Problem 3 repeats the solution to Problem 2. Next, the steps of the procedure for solving Problem 1 are performed. The difference is the partition on the terms have to be performed on the
The partition of the integer $F^r(A)$ into terms $2^i$ is constructed, where each term is included in the partition $q + b_{m-1} = 1 + 0$ times. Since $r = 0000$, the terms $2^1, 2^2, 2^3$ are included in the partition of 15 only once. The value $q + b_{m-1}$ determines the Hamming weight of the rows of the desired matrix $V$, consisting of four rows and four columns, excluding the first and third columns. Then, randomly generated values of six two-digit binary vectors with Hamming weights equal to $w(v_i) = w(v_j) = w(v_k) = 1$, which will be determine the values of the remaining columns (except the first and third) of the desired matrix. For the matrixes thus obtained, the maximal of its rank is determined. One of the possible solutions of this example may be the matrix $V_5$, shown in Table 1.

### 6. Conclusion

The use of a modified mathematical model for the Sobol sequences generation has allowed to expand the capabilities of the test sequence generator in terms of a significant increase in the number of different types of such sequences. The paper describes a method for constructing a test sequences with given values of switching activity, of the test patterns and the activity of their bits. The essence of the method consists in the synthesis of the required generating matrix providing specified values of switching activity. The limitations of the previously proposed and investigated techniques associated with possible conflicting requirements for the values of the weights of the rows of the matrix and their linear independence are shown. The main idea that distinguishes the results obtained in this article is the use of arbitrary matrices for which the condition that ensures the maximum period of the test sequence is satisfied, it is necessary to use matrices with a maximum rank. In this case, not only the required values of the switching activity are provided, but also a larger number of test patterns in the sequence. Examples of the use of such sequences for the purpose of constructing a test pattern generator are considered.

### 7. References


