Improvement of signal quality in CDMA systems on the basis of analysis of correlation properties of pseudo-random sequences

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Abstract: *The article presents a continuation of researches in the field of improvement of signal quality in data transmission systems with code division multiple access (CDMA). It is discussed the correlation properties of pseudo-random sequences (PRS) used in the formation of noise-like signals in CDMA systems. The analysis, carried out in MATLAB environment, showed that correlation properties of the Walsh functions derivatives, used for generating PRS, have much better correlation characteristics than the original Walsh functions. Besides, these properties depend on the type of generating function of Walsh functions derivatives. It was justified the advantage of using these signals in the development of CDMA systems in order to improve the signal quality.*

Keywords: *CDMA SYSTEM, NOISE-LIKE SIGNALS, PSEUDO-RANDOM SEQUENCES, AUTOCORRELATION FUNCTION, CROSS CORRELATION FUNCTION, WALSH FUNCTION DERIVATIVE, GENERATING FUNCTION.*

1. Introduction

Sufficient experience in the use of broadband communication systems (BBCS) has confirmed their advantages, such as high resistance to narrowband interference, the ability to operate multiple subscribers in one communication channel, transmission secrecy, high resistance to multipath propagation.

For data transmission systems with code division multiplexing, in comparison with other types of systems, it is possible to reuse (multiple) the frequency resource due to the division of channels not by frequency or by time, but by "form", which allows the simultaneous operation of many subscribers in one and the same frequency band.

Such a system uses pseudo-random sequences (PRS) with specified correlation properties. PRS are widely used to generate noise-like signals (NLS) in communication systems such as DS-CDMA, GPS / Navstar, Glonass, and IEEE 802.11b wireless networks.

2. Theoretical preconditions

The correlation functions of complex noise-like signals are determined by the correlation functions (CF) of the manipulating sequences. Therefore, considering the CF of complex signals, it is sufficient to analyze the correlation functions of the manipulating sequences. Signal decoding on the receiving side is carried out by a correlation receiver, the basis of which is a correlator, which is a series-connected multiplier and an integrator that calculates the cross-correlation function (CCF) of the incoming signal with the PRS stored in the memory.

The most important parameter of the used pseudo-random sequences is their correlation properties [1]. Moreover, on the choice of binary code sequences, i.e., the noise immunity of the entire information system, depend their correlation properties. In addition, the code sequence must be well balanced, that is, the number of ones and zeros in it must differ by no more than one character. The last requirement is important to fulfill by excluding the constant component of the information signal.

The cross-correlation function of two signals $u(t)$ and $v(t)$ is equal to the scalar product $u(t)$ with a copy of $v(t)$ shifted by t_o as a function of the argument t_o:

$$R_{uv}(t_o) = \int_{-\infty}^{\infty} u(t)v(t-t_o)dt. \qquad (1)$$

The cross-correlation function between two discrete signals is calculated by the formula:

$$R_{uv}(n) = \sum_{j=-\infty}^{\infty} u_j\, v_{j-n}. \qquad (2)$$

However, based on the generalized Rayleigh formula, we can write:

$$R_{uv}(t_o) = \frac{1}{2pi}\int_{-\infty}^{\infty} U(w)V^*(w)\exp(jwt_o)dw. \qquad (3)$$

In asynchronous systems, to ensure orthogonality of signals for any t_0 the CCF should be zero. However, due to the linearity of the Fourier transform, this is possible only if $U(w)\cdot V^*(w) = 0$ on the entire frequency axis. Equality of CCF to zero means that two signals are orthogonal for any t_o only if their spectra do not overlap. However, this cannot be achieved in a multiple access system. The consequence of this is the occurrence of inter-user interference, i.e., nonzero response of the receiver of the *k*-th user to the signals of other users [3].

The Walsh functions [7, 8] can be referred to discrete signals with the best CCF structure. Walsh functions were developed in 1923 as a development of the system of Rademacher functions known by that time by adding new functions to it [3,7].

Walsh functions are formed from Rademacher functions using the following relationship:

$$wal_0(\theta) \equiv 1, wal_n(\theta) = \prod_{k=1}^{m}[rad_k(\theta)]^{n_k}, \qquad (4)$$

where n is the Walsh function number, n_k is the value (0 or 1) of the k -th bit of the Walsh function number n, written as m-bit binary Gray code.

Hence it is easy to see that the number of functions in the Walsh system turns out to be equal to $N = 2^m$, where m is an integer. Walsh functions take only two values: +1 and –1, which is a useful property when building circuits on binary digital elements (triggers).

The most common orthogonal system used in multichannel systems with channel division by code are Walsh - Hadamard systems (matrices) of order N = 4k, k is an integer, which are determined by the recurrence rule:

$$W_{2N} = \begin{bmatrix} W_N & W_N \\ W_N & -W_N \end{bmatrix}, \qquad (5)$$

where W_N is the Walsh - Hadamard matrix of order N, and it is assumed that $W_1 = 1$, or in sign form $W_1 = +$.

However, a feature of orthogonal codes is that the orthogonality of these codes is performed only at the "point", i.e., in the absence of time shifts. In real conditions, such situations are not met, orthogonality is violated, which in turn leads to an increase of the level of multiple access interference and the appearance of errors in the processing of input data. Therefore, various methods are used to eliminate these disadvantages.

In order to improve the properties of correlation functions (ACF and CCF), Walsh signal systems often construct so-called derivatives of signal systems [3, 7, 8].

A derivative is a signal, obtained as a result of element-wise (symbol-by-symbol) multiplication of two signals. A system composed of derived signals is called a derivative [3, 7, 8]. Let us consider the essence of the empirical method for constructing derived signal systems, when the Walsh system is used as the initial one (Fig. 1), where each line is a code sequence of the corresponding BPSK signal.

The original Walsh system – Y_{16}

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Walsh derivative system – Π_{16}

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\end{array}\right].
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Fig. 1 *The principle of constructing derivatives of Walsh functions*

The system of signals in Fig. 1, orthogonal at the point $t = T_s$ has, in general, poor properties of ACF and CCF, however, it is very simple from the point of view of signal generation and processing.

A signal is chosen as the generating signal so that the derived system has good correlation properties. As a rule, this is a signal with a good ACF, and it is found with the help of a computer or in another way. In [3], an example of obtaining the derivative of a function (Fig. 1) is given, when M is taken as the generating function - a sequence of the form {+ + + - - - + - - - + - - + -} or in the bipolar representation {+1 +1 +1 -1 -1 -1 +1 -1 -1 -1 +1 -1 -1 +1 -1}.

Obviously, the correlation properties of Walsh functions derivatives will depend on the type of the generating sequence. Let's carry out this analysis.

Consider the correlation properties of Walsh function derivatives if we take as generating functions M - sequence from [3] and de Bruijn sequence with the generating polynomial $f(x) = x^4 \oplus x^3 \oplus 1$. The PRS is obtained by the generator for de Bruijn sequence in the form of a shift register with nonlinear feedback under certain initial conditions (A = 1000) and in bipolar representation have the form: B_1 = +1+1+1-1+1+1-1-1+1-1+1-1-1-1-1+1.

In this case, the system of Walsh function derivatives will have the form:

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\Pi_{16}=\left[\begin{array}{cccccccccccccccc}
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Fig. 2 *Walsh function derivative with generating function in the form of de Bruijn sequence B_1.*

Consider the correlation properties of Walsh functions derivatives (for example, the 2nd and 12th), taken from Fig.1 [3]. The results of calculating the aperiodic (AACF) and periodic (PACF) autocorrelation functions of the selected Walsh function derivatives are shown in Fig. 3 - Fig. 6, respectively.

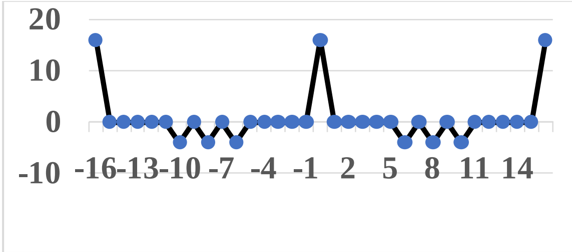

Fig. 3 *PACF of the 2nd derivative of the Walsh functions (Fig. 1)*

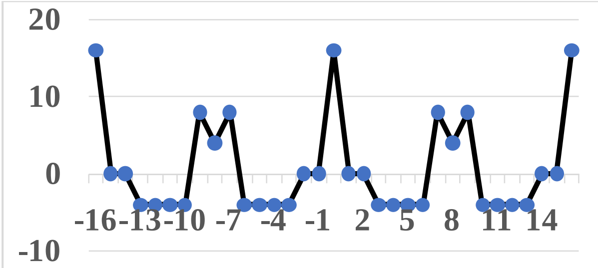

Fig. 4 *PACF of the 12th derivative of the Walsh function (Fig. 1)*

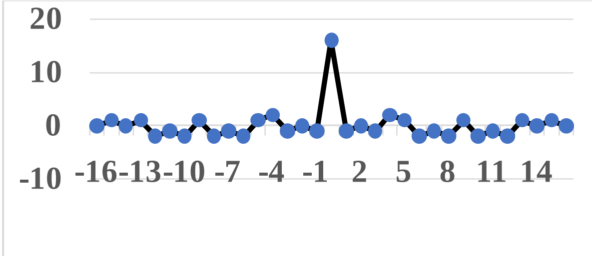

Fig. 5 *AACF of the 2nd derivative of the Walsh functions (Fig. 1)*

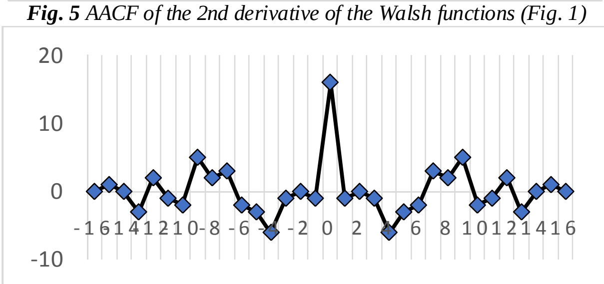

Fig. 6 *AACF of the 12th derivative of the Walsh function (Fig. 1)*

From Fig. 3 - Fig. 6 is seen that considered 16-bit Walsh function derivatives (Fig. 1) have not very good autocorrelation functions. To ensure synchronization during reception, it is necessary to eliminate the influence of side lobes (the suppression coefficient ranges from 2.67 for AAKF 12th and up to 8 for AAKF 2nd Walsh derivative). Similarly, for PACF - from 2 for 12th Walsh derivative and up to 4 for the 2nd Walsh derivative, i.e. the 2nd Walsh function derivative has the best correlation characteristics. For comparison, let us show what form the PACF of the original 2nd Walsh function has.

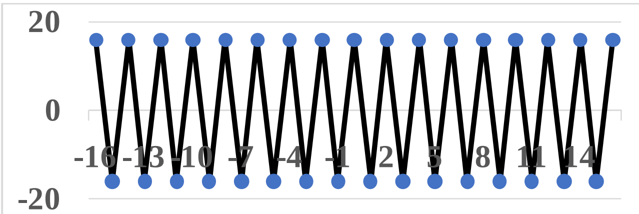

Fig. 7 *PACF of the 2nd original Walsh function (Fig.1)*

Comparative analysis of the PACF of the 2nd original Walsh function (Fig. 3) with the PACF of the 2nd derivative of the Walsh function (Fig. 7) shows that the derivatives of the Walsh function have much better correlation characteristics.

Let us consider the form of the cross-correlation function of the considered Walsh function derivatives. Graphical representation of the CCF is shown in Fig. 8.

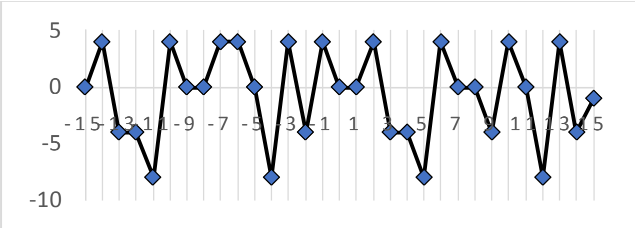

Fig. 8 *The graph of the CCF of the 2nd and 12th derivatives of the Walsh functions (Fig. 1)*

From Fig. 8 it follows that the CCF of Walsh function derivatives can be considered satisfactory, the CCF is more or less uniform, without extreme outliers, and can be used in code division multiplexing systems with the use of solvers that do not respond to CCF outliers within the specified limits.

Let us analyze the correlation properties of the 2nd and 12th derivatives of the Walsh functions with a generating function in the form of a de Bruijn sequence (Fig. 2). The results of calculating the aperiodic (AACF) and periodic (PACF) autocorrelation functions of the selected Walsh derivatives are shown in Fig. 9 and Fig. 8, respectively.

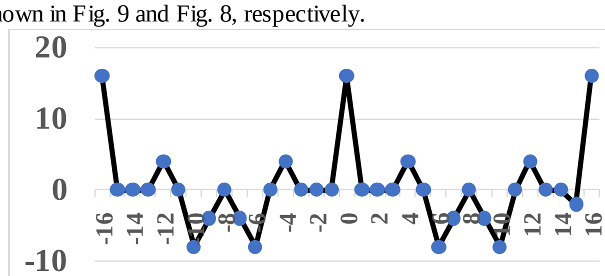

Fig. 9 *PACF of the 2nd derivative of the Walsh functions (Fig. 2)*

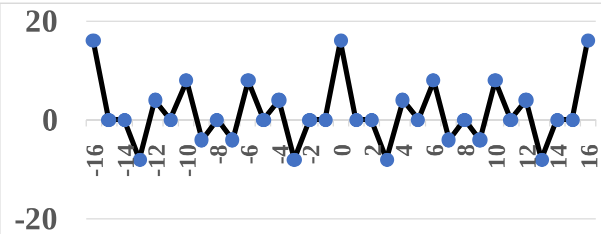

Fig. 10 *PACF of the 12th derivative of the Walsh function (Fig. 2)*

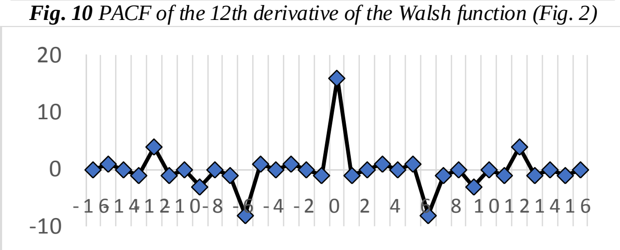

Fig. 11 *AACF of the 2nd derivative of the Walsh functions (Fig. 2)*

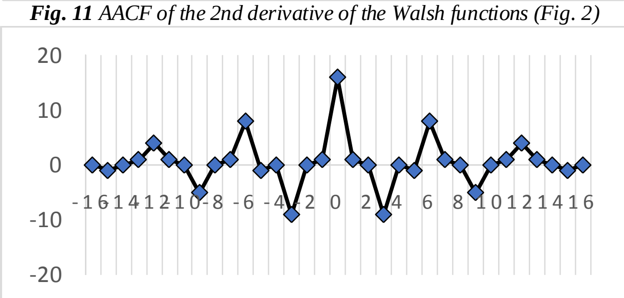

Fig. 12 *AACF of the 12th derivative of the Walsh function (Fig. 2)*

As can be seen from Fig. 9 - Fig. 12, the considered 16-bit derivatives of the Walsh function (Fig. 2) have not very good autocorrelation functions; to ensure synchronization during reception, it is necessary to eliminate the influence of side lobes (the suppression coefficient is the ratio of the amplitude of the maximum ACF peak to the maximum value of the amplitude of the side lobes ranges from 1.78 for AAKF and 2 for PAKF).

When considering the form of the cross-correlation of the considered Walsh function derivatives, was obtained graphical representation of the CCF, shown in Fig. 13.

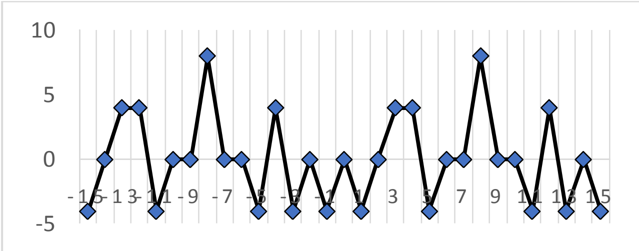

Fig. 13. The graph of the CCF of the 2nd and 12th derivatives of the Walsh functions (Fig. 2)

3. *Experimental data*

Let us analyze the correlation properties of considered above Walsh function derivatives in the MATLAB environment.

For this purpose, there was developed the program which allowed to calculate the aperiodic and periodic correlation functions of two pseudo-random sequences of the same length, given in bipolar form of arbitrary length. The program also allowed to calculate the cross-correlation function of these two sequences. The results of calculating the correlation characteristics are shown in Fig. 14, ..., Fig. 18 respectively.

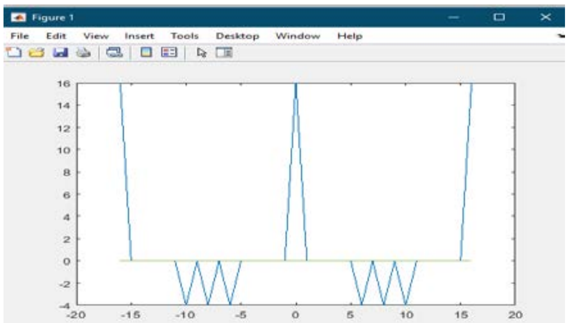

Fig. 14 *PACF of the 2nd derivative of the Walsh functions (Fig. 1)*

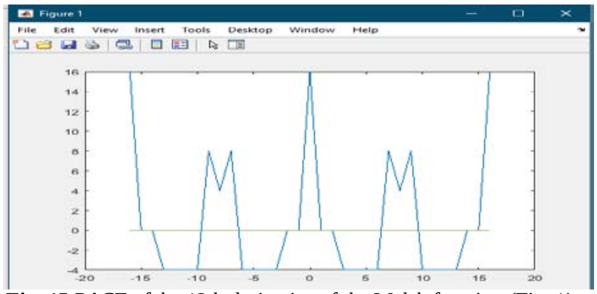

Fig. 15 *PACF of the 12th derivative of the Walsh function (Fig. 1)*

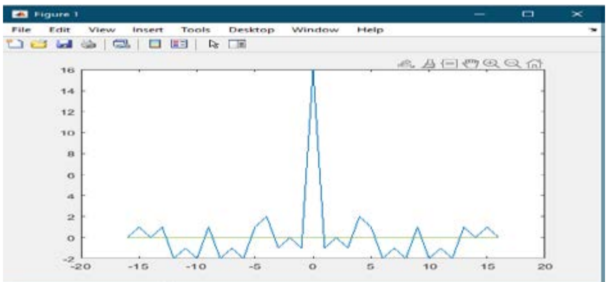

Fig. 1 *AACF of the 2nd derivative of the Walsh functions (Fig. 1)*

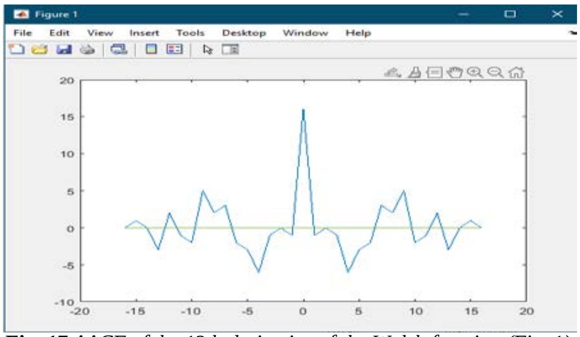

Fig. 17 *AACF of the 12th derivative of the Walsh function (Fig. 1)*

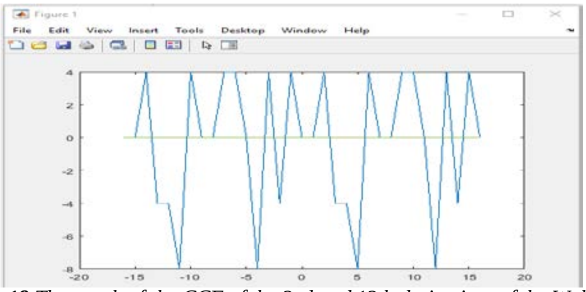

Fig. 18 *The graph of the CCF of the 2nd and 12th derivatives of the Walsh functions (Fig. 1)*

The analysis of Walsh function derivatives showed that in comparison with generating function of de Bruijn sequence, the generating function taken from [3] have better autocorrelation characteristics: the suppression coefficient lies within the limits from 2.67 for the 12th Walsh derivative to 8 for the 2nd Walsh derivative for AAKF and from 2 for the 12th Walsh derivative to 4 for the 2nd Walsh derivative for PACF. Thus, the 2nd derivative of the Walsh function has the best autocorrelation properties.

A comparative analysis of the correlation characteristics obtained analytically (Fig. 3 – Fig. 6, Fig.8) and by modeling in the MATLAB environment (Fig. 14 – Fig. 18) shows their identical form.

4. Conclusion

Analysis of the correlation properties of the studied broadband signals based on the Walsh functions derivatives allows to draw the following conclusions:

1) The correlation characteristics of Walsh functions, that are orthonormal, have good cross-correlation functions, which are equal to zero between two different Walsh functions. However, these functions have such properties only at the point at zero shift. In real conditions, especially in multipath propagation, orthogonality is violated and the cross-correlation function is nonzero. This leads to an increase in the level of multiple access interference and to errors in signal (channel) separation.

2) The correlation properties of the Walsh functions derivatives have much better correlation characteristics than the original Walsh functions.

3) The correlation properties of the Walsh functions derivatives depend on the type of the generating function. It is necessary to choose generating function that obtain the best correlation characteristics (in our case, the derivatives of the Walsh functions with the generating function taken from [2]).

4) The Walsh function derivatives (L = 16 bits) have a large amplitude of the central peak of the ACF, equal to the length of the sequence, but the amplitude of the side lobes slightly increases. This must be taken into account when designing multichannel systems to ensure reliable synchronization of receiving devices when exposed to noise.

5) The large length of the spreading code based on the Walsh functions derivatives allows distributing the signal energy over the spectrum, increasing the noise immunity of the system, providing good protection against unauthorized access, and improving electromagnetic compatibility with neighboring radio engineering systems. But it is necessary to provide special protection measures against sidelobe effects on the processing of input data.

6) The Walsh functions derivatives allow to obtain the required set of different pseudo-random sequences, which are needed for communication systems with code division channels (for example, 64-bit Walsh functions derivatives allow obtaining 64 different pseudo-random sequences, i.e., the same number of independent channels).

Search and design of PRS of arbitrary length is a real practical task. Obtaining all the sequences of the ensemble based on Walsh function derivatives with an analysis of the influence of generating function type on their correlation properties is necessary for the optimal use of considered sequences in the communication system. This, in turn, determines the quality of the communication system, the number of system subscribers, protection against unauthorized access, and, consequently, the economic aspects of systems.

At the same time, these sequences must have certain characteristics such as low threshold of cross-correlation and a pronounced peak of autocorrelation with minimal levels of side lobes, etc. The results can be used in the development of broadband communication systems and information transmission systems with protection from unauthorized access.

5. References

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