Simulation of light propagation in a photonic crystal fiber

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Abstract: *A group of numerical methods suitable for describing the propagation of light in a Photonic Crystal Fiber (PCF) are discussed. PCFs can be classified according to the mechanism of light propagation. The methodology of the analysis is briefly reviewed to understand how changing the different physical parameters of the fiber and the design affects its optical properties. By analyzing the systematic studies presented, a design with the desired optical fiber properties can be realized.*

PCFs can be made from just one material with two-dimensional photonic crystals or periodic arrays of air holes parallel to the fiber axis to form a shell and core shape. Several types of PCFs have been considered. The mechanism for trapping light in the fiber core is explained. The use of the wave methods for electromagnetic field analysis is discussed. The application of variation principles and weighted residual methods is shown. The advantages of the finite element method (FEM) are indicated. The vector FEM for obtaining the modes in photonic crystal fibers is considered.

Keywords: *OPTICAL WAVEGUIDE, NUMERICAL METHODS, LIGHT PROPAGATION, MODES, FINITE ELEMENT METHOD, PHOTONIC CRYSTAL FIBER*

1. Introduction

The optical fiber is made in the shape of a thin cylinder. In it introduces light that interacts with the material. Optical fibers are used for measurement of different parameters, for gain of signals, for transmission of information and in the manufacture of lasers. They are used widely in telecommunications as environment for transfer on information between two points with a high a speed. They work like optical sensors for measurement of physically values and are also used for many other purposes.

The analysis of the optical fibers and devices requires the carry out of experimental or numeric research. The experiment takes away time and requires large quantity of resources. Theoretical research is founded on numeric simulations. They use an adapted analytical or numerical method depending on the fiber under study. The goal is to be understood how the change on the different ones physical parameters of fiber affects the optical fiber properties and how to realize a design with desired properties.

2. Construction and features

The optical fiber is made with a shape of a thin dielectric cylinder - fig. 1.

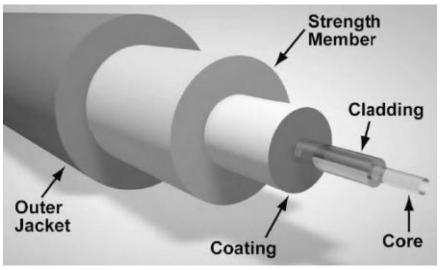

Fig. 1 *Structure of a silicon optical fiber*

It is built by a core with high index of refraction, in which propagates the light. The core is surrounded from cladding, with a little lower index on refraction from the core and which provides mechanical protection of the fiber from physical damage and protects the glass surface from moisture and dust particles.

The size of the core and difference between index of refraction of the core and of the cladding play main role at determination of the fiber properties. Important optical parameter on each transparent environment is the index on refraction n which is a ratio of speed of the light in the material v and speed of the light in vacuum c.

$$\boldsymbol{n} = \frac{\boldsymbol{v}}{\boldsymbol{c}} \qquad (1)$$

The change of the index on refraction by the length of the wave leads to the expansion of the duration of the pulse in time and in length (spatially) . The index on refraction on melted silicon dioxide strongly depends on the length of the wave . The change of the index of refraction at smaller wave length is too sharp, which leads to high value of the dispersion.

The propagation of light in an optical fiber can be explained by Snellius ' law. Some of the light rays traveling from the core to the cladding reach the core/cladding surface at an angle greater than the critical angle. This causes total internal reflection. These rays are reflected back into the core and do not pass into the cladding. This keeps the light in the core of the fiber. All other rays are refracted and exit the core.

In optical systems, the attenuation (loss of power) and the dispersion (broadening of the pulse) limit transmission possibilities. Their values should be small or zero so that their effect can be acceptable. The dispersion of silicon material is shown at fig. 2, and the internal losses - at fig. 3 [1].

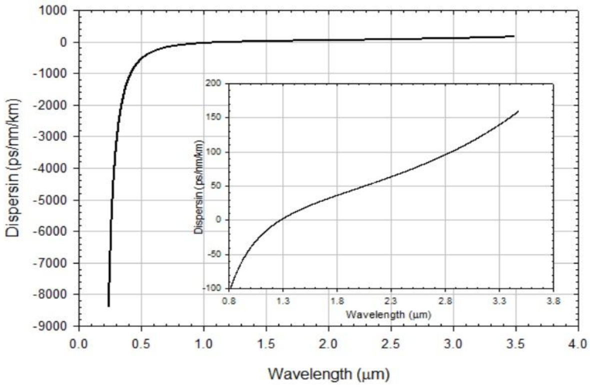

Fig. 2 *Dispersion of silicon material*

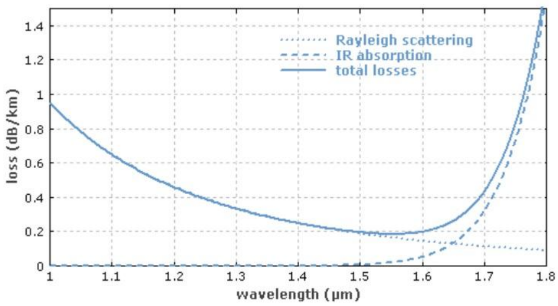

Fig. 3 *Internal losses of silicon material*

Some applications require a non-zero or greater variance value. This is the case for non-linear effects where non-zero variance is required to significantly affect the signal. Large dispersion values are required in specially designed fibers to compensate the dispersion of conventional telecommunication optical fibers. The dispersion must be controlled to suit the required application. Regardless of the manufacturing method, there are two types of optical fibers: single mode fibers (SMF) and multimode fibers (MMF).

The SMF core radius is much smaller than MMF . In SMF, only the fundamental mode can propagate through the core, while the number of modes in MMF is approximately (V^2 /2), where the V parameter is [2]:

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} \qquad (2)$$

In (2), a is the radius of the fiber core, λ is the wavelength of light, n_1 is the refractive index of the core and n_2 is the refractive index of the fiber cladding.

The design of PCF is different from conventional silicon fibers. PCFs can be fabricated from a single material with two-dimensional photonic crystals or periodic arrays of air holes parallel to the fiber axis to produce a shell-and-core shape [3]. At fig. 4 several types of PCFs are shown: (a) pure Si core fiber with cladding air holes; (b) fiber with air tubes in the core where light is trapped due to the bandpass effect; (c) the core is made of pure silica and the holes in the cladding are filled with a high-refractive-index liquid, and (d) hollow cylindrical multilayer fibers with a solid cladding - Bragg fibers.

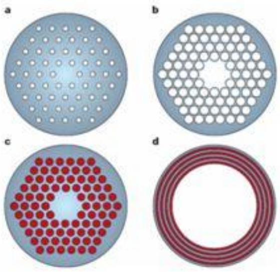

***Fig. 4** PCF fibers with different design*

At fig. 5 and fig. 6 are shown the cross-sections of helical PCF fibers with air tubes in the cladding with different structure.

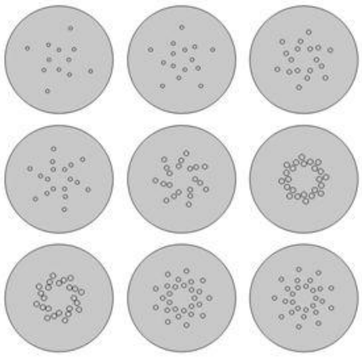

***Fig. 5** Cross section of ES-PCF fiber*

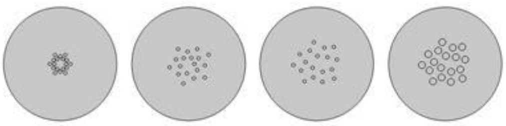

***Fig. 6** Cross section of ES-PCF, AS-PC, FS-PCF fibers*

Position and size of the holes in the cross-section of the PCF fiber control distribution and properties of the light. Their forms have different mathematically presentations.

Photonic crystal fibers can be classified into two main categories according to the mechanism of light propagation . The first are fibers with a high refractive index of the core. In them, light propagates similarly to conventional fibers. The second type of PCF are fibers with a low refractive index of the core, known as hollow core fibers / HC/ . The core of The HC fiber is made by a material with a lower refractive index than the cladding and the light is guided either by PBG - photonic band gap or anti-resonance effect. HC fibers are further sub-classified into HC-PBF, Bragg fibers, Kagome fibers and HC-NCF /Hollow-Core Negative Curvature Fiber/ - hollow-core fibers with negative curvatures at the core boundaries, where light is confined in a core with a lower refractive index than the cladding. HC-NCF fibers were created in 2010.

The number of modes supported by the PCF is determined similarly to conventional optical fibers by the V parameter. PCFs do not have a well-defined core and a constant refractive index value of the claddingl. The refractive index of the cladding is a function of wavelength, which changes depending on the interaction of the light with the two materials (core and air) of the cladding. At longer wavelengths, the light spreads inside the holes and causes the refractive index to change depending on the design. The formula for V_{PCF} is shown below [4].

$$\boldsymbol{V_{PCF}} = \frac{\boldsymbol{2\pi\Lambda}}{\boldsymbol{\lambda}}\sqrt{\boldsymbol{n_1^2 - n_{cl}^2(\lambda)}} \qquad (3)$$

where **Λ** is the aperture spacing, and $n_{cl}(\lambda)$ is the average refractive index of the PCF cladding.

In single mode, the guiding of light depends on the refractive index of the core and the average cladding index of the photonic crystal, which is a function of wavelength. In conventional optical fibers, there is a cut-off frequency above which the fiber is no longer single-mode . The single- mode operation region in PCFs is expanded because the average refractive index of the photonic crystal is wavelength dependent. At shorter wavelength, the mode field is more concentrated in the glass material. As the wavelength increases, the field penetrates more into the air holes. Therefore, at shorter wavelengths, the average cladding index is close to the material index. This results in a small index difference between the core and the cladding so that the value of V_{PCF} is small even at shorter wavelengths. Due to the dependence of the refractive index on the wavelength, the PCF can be designed to be single-mode when the condition is satisfied:

$$V_{PCF} < \pi \qquad (4)$$

3. Analysis methods

Analysis of the optical devices uses intensively analytical and numerical methods. Analytical methods apply to solving electromagnetic problems at simple constructions like slabs, grooved and ribbed waveguides. They give accurate solutions, but are limited and inapplicable for real cases involving complicated geometries and inhomogeneous areas.

The analysis of complicated optical waveguides structures with spatial inhomogeneity area is not possible. In that case is doing approximations and assumptions for problems which must be resolved with numerical methods. This brings known permissible error. The approximations which done classify the numerical methods in two basic classes.

The asymptomatic numerical methods require fundamentally approximation in Maxwell Equations. This is limiting the use on these methods for general cases - for example for large metal and conductive objects. Their advantages are the low computational resources and the high efficiency [5].

The second class numeric methods are full wave methods. At these methods the approximation does numerically and there is not initially physical approximation. These methods are classify additionally according to the used formula, integral or differential, according the domain in which they work, time or frequency, as some from the methods can be applied in both areas. They include three basic steps for solving of the problem. First the problem must be defined through a control equation that fetched from the equations of Maxwell. The second step is discretization on the domain into non-overlapping subdomains, for present arbitrary inhomogeneous dielectric waveguides, so that everyone subdomain or element to be homogeneous. The discretization leads to set of equations, which are described in matrix form. The last step requires decision of the matrix of unknown values using effective algorithms. Below are briefly described the most popular full wave methods.

The methods of the frequency area are appropriate for research on narrowband applications in stationary condition, while the methods of the time area are appropriate for broadband transient applications. The discretization in the integral formula includes the important surface of the problem, as by this way are decreasing computational resources. This however, leads to bad presentation of complicated structures and inhomogeneous materials. The integral formula can be used for research of distraction issues and open domain. In differentials formulas discretization includes the full volume. This requires more calculations, but can effectively presented inhomogeneous environments and complex structures.

The differential formulas they can be applied to a borderline problem.

There is no universal choice for the best method. Every method has advantages and disadvantages and on a case-by-case basis chooses the appropriate one. Every method however differs in the preliminary processing on the equation of Maxwell for obtaining the final formula.

Often encountered aspect at full waves methods is discretization of the systemic domain on subdomains. So the solution is up almost regardless from the system [6].

A representation of the E and H components of the z-axis wave propagation as a harmonic function of time was sought:

$$E(x,y,z) = E_{\nu}(x,y)\exp(-j\beta_{\nu}z) \quad (5)$$

$$H(x,y,z) = H_{\nu}(x,y)\exp(-j\beta_{\nu}z), \quad (6)$$

where E is the intensity on the electrical field, and H is the intensity of the magnetic field, ν is a mode index of relevant wave type [2].

In a 2D analysis for approximation of the distribution of the field they use triangular elements and polynomial approximation. The real wave function approximates with a linear function or with a square function. The polynomial approximation from higher range leads to more accurate results, however complicates the analysis and requires more memory. Broadly used are triangular elements from first range with 3 nodes.

The vector formulation of the FEM is used when the modes are hybrid and the longitudinal and transverse field components can be coupled. It can be output in different ways depending on the field components being used:

- transverse electric field components ;
- transverse magnetic field components;
- the two transverse field components;
- the two longitudinal field components;
- all six field components;
- all components of the electric field;
- all components of the magnetic field.

The last two ways are more efficient in terms of computing resources. However, the electric field formulation requires further integration to enforce the boundary conditions when the material changes at the boundary between the two elements. In the magnetic field formulation, the change in material at the boundaries does not require the imposition of a boundary condition since both materials are non-magnetic . Using six components requires more computing resources for storage and processing. The Rayleigh-Ritz procedure is used to obtain the matrix equation. First, the functional of the magnetic field vector formula is defined as:

$$F = \iint_{\Omega} (\nabla \times \boldsymbol{H})^{*} ([\varepsilon_r]^{-1}\nabla \times \boldsymbol{H}) d\Omega - \omega_0^2 \iint_{\Omega} \boldsymbol{H}^{*}.\boldsymbol{H} d\Omega^{*} \quad (7)$$

The domain is divided into small elements and the field in each element is defined as:

$$\boldsymbol{H} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{m}} N_i H_i \quad (8)$$

where *m is* the node number in the element, H_i is the magnetic field per node i , and Ni are the basis functions (shape functions). Equation (8) in matrix form is:

$$\boldsymbol{H} = [N]^T\{H\}_e \quad (9)$$

where T stands for transpose , $[N]^T$ is the matrix of the basis function a $\{H\}_e$ is column vector of field value in node.

Substituting (9) into (7) and applying the variational principle gives:

$$\iint_{\Omega} (\nabla \times [N]^T\{H\}_e^{*}[\varepsilon_r]^{\mathbf{-1}}.\nabla \times [N]^T\{H\}_{\boldsymbol{e}} - \omega^{\mathbf{2}}[N]^T\{H\}_e^{*}[N]^T\{H\}_{\boldsymbol{e}}) d\Omega = 0 \quad (10)$$

Writing it in matrix form gives:

$$[A]\{H\} - \omega_0^2[B]\{H\} = 0 \quad (11)$$

where ${\omega_0}^2$ is the principal value and {H} is the principal vector. $[A]$ is the complex Hermitian matrix and can be reduced to real symmetric in the case of lossy dielectrics. $[B]$ is a real symmetric matrix. The two matrices are defined as:

$$[A] = \iint (\nabla \times \boldsymbol{H})^{*}.\,\varepsilon^{-1}(\nabla \times \boldsymbol{H}) d\Omega \quad (12)$$

$$[B] = \iint \boldsymbol{H}^{*}.\,\mu.\mathbf{H} d\Omega \quad (13)$$

Some solutions of (11) may be spurious, not physical. They are obtained because some mathematical solutions of the eigenvalues of the equation do not satisfy the divergence condition (▽ **H**=0) automatically. The penalty function method is applied to eliminate or suppress false decisions. The purpose of the penalty function is to impose a bias-free constraint on the decision variables. Processing time increases slightly. The expression is:

$$\omega^2 = \frac{\iint (\nabla\times\boldsymbol{H})^{*}.\varepsilon^{-1}.(\nabla\times\boldsymbol{H})d\Omega + \left(\frac{\alpha}{\varepsilon}\right)\int(\nabla.\boldsymbol{H})^{*}(\nabla.\boldsymbol{H})d\Omega}{\iint \boldsymbol{H}^{*}.\mu^{-1}.\mathbf{H}d\Omega} \quad (14)$$

The formula includes the penalty term in the vector formulation that is used to find the mode solutions for the waveguides. The next problem is the discretization of the domain into smaller subdomains. The discretization of the domain or Networking is critical step in the FEM because the network determines computational requirements like time for storage and processing. M the cut plays important role for accuracy on the results. The process divides the domain on small ones subdivisions (elements). The elements must be adjacent and without distance between them, to cover the whole domain. There is different kinds elements dependent from the domain which must discretizes.

For a 1D problem, a linear element is used with two nodal points, one at each end of the element. In 2D, the element is a triangle or rectangle. A triangular element of the first order has three nodal points and a rectangular element has four. Higher order elements have more nodal points depending on the degree of the element and are more precise. Higher order elements lead to complex formulas because the degree of the interpolation function is related to the degree of the element. Triangles represent complex domains more accurately than rectangles because the boundaries can be more accurately approximated by triangles – Fig. 7.

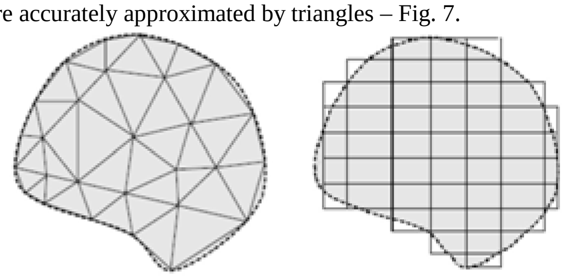

Fig. 7 *Comparison of meshes of triangles and rectangles*

The element is represented by an interpolation function that should approximate the field in the element with respect to the real physical problem. Compensating for the difference between accuracy and available resources is required. The error should be tolerable and the computer's processing time should be acceptable.

Most often, interpolation functions for representing elements are polynomials. A polynomial must contain all possible terms. The polynomial is unique and the orientation of the coordinates does not affect the shape function. It must have a number of members equivalent to the nodal points. There must be three terms in the first-order triangular element to form a complete polynomial.

PML (Perfectly Matched Layers) method is used to obtain the domain boundaries. PML allows unwanted unphysical reflected radiations to be absorbed at the boundaries. In PML, an artificial medium with a dielectric constant similar to the material is placed in the core domain. The impedance of both is identical, so there is no boundary reflection. The boundary absorbs waves traveling outward from the main domain at any angle and frequency.

The method on the moments (MoM), has an advantage at problems including open regions. Applies everything mostly in frequency district. This is the widest the one used method in antenna analysis. The mathematical formulation of electromagnetic fields with is made through discretization of the integral equation. MoM is computational effective but requires more complicated mathematics from others methods.

The finite differences method (FDM) is the widest used method at electromagnetic problems due to that it is clear and easy for application. The domain is split on network and the solution is for every point of the network. He applies the differential Maxwell equations, for brought out the wave equation and finds answer for the modes. The differential operator are replaced with operators for difference.

A similar method with the same concept is the method of the finite difference in the time domain (FDTD) [7]. It is used for the analysis of problems in the time area by directly discretization of the Maxwell equations. Both methods are very efficient, because they require a little operations on each point of the network [8]. The disadvantage of FDM is that the mesh must be uniform, to work well the method.

4. Conclusion

FEM, FDM and MoM are the main numerical methods used in electromagnetic analysis. For analysis of modern optical fibers and waveguides with a complex core and cladding structure is chosen one of these methods. At random borders of PCF structures, FEM is preferred because it makes a better approximation on their borders.

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