Application of perfect binary lattices for signal generation in multi-channel communication systems

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Abstract: *In multichannel data transmission systems noise-like signals, generated on the basis of derivatives of the Walsh function are used. The article examines the correlation properties of derivatives of Walsh functions with generating functions in the form of perfect binary lattices, which make it possible to provide the necessary data synchronization when receiving signals. Also, it was justified the advantage of using these signals in the development of CDMA systems in order to reduce the interference level of multiple access and to protect against unauthorized access.*
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1. Introduction

The main method of communication and navigation in the modern world is wireless information transmission. At the same time, the number of users of various communication networks is steadily growing, which requires the creation of a large number of independent channels within one communication system. In this regard, it is necessary to use the best methods of organizing communication with limited frequency resources and growing demands on the quality and quantity of transmitted information. Under these conditions, the most efficient systems are Code Division Multiple Access (CDMA) [1, 2, 3, 5, 6].

One of the quality indicators of transmitted signals is its noise immunity. The signal energy is the main parameter for its noise immunity, that is, the greater this value, the lower the signal-to-noise ratio is needed for reception with acceptable number of errors. Consequently, noise-resistant reception is possible when the required energy of the useful signal is provided in the optimal receiver by:

• increasing signal processing time;

• increasing the amplitude of the signal (the magnitude of its spectral components);

• expansion of the signal spectrum.

The first method leads to a decrease in the resulting data transfer rate, and the second requires an increase in transmitter power, which are not acceptable in mobile communications. The third method involves the use of signals with wide bands of occupied frequencies, much larger than the band of the information signal. Such properties have noise-like signals (NLS). This explains the high noise immunity and secrecy of CDMA communication systems compared to other standards (FDMA, TDMA, FDMA/TDMA, etc.).

The signal energy in a CDMA receiver is brought to the required level not by increasing the power of base station transmitters, but by summing up a huge number of low-power spectral components of the received signal. This makes such a signal not only difficult to decode, but also difficult to detect.

It should be noted that the information itself can be introduced into the wideband signal in several ways. For example, a narrowband signal is multiplied by a pseudo-random sequence (PRS) with period T, consisting of N bits of each duration τ_0. In this case, the NLS base is numerically equal to the number of PRS elements. [1, 2, 5, 6].

2. Theoretical preconditions

The reception of the NLS, as, indeed, of any other signals, is carried out using optimal receivers that minimize the probability of error. It is known that the structure of the optimal receiver depends on the type of modulation, as well as on how many signal parameters are known at the receiving point (coherent or non-coherent reception, etc.). However, in any case, the optimal receiver includes a correlator or a matched filter and a solver.

The receiver compares the received code sequence with its exact copy stored in memory. When it detects a correlation between them, it switches to the information receiving mode, establishes synchronization, and starts the operation of decoding useful information. Any partial correlations can lead to false triggering and disruption of the receiver, that’s why the code sequence must have good correlation properties.

The dominant role in choosing the type of PRS for the formation of NLS in data transmission systems is played, first of all, by the mutual and autocorrelation characteristics of the signal ensemble, its volume, and the ease of implementation of devices for generating and "compressing" (convolution) signals in the receiver [1, 3, 4 - 6, 11].

It is known that for the correct processing of input signals, the receiving and transmitting devices must work synchronously and in phase. Synchronization of the receiving device is carried out using a correlator that calculates the ACF (auto-correlation function) of the input signal, which for discrete signals is calculated by the formula:

$$\boldsymbol{R_u(n)} = \sum_{j=-\infty}^{\infty} \boldsymbol{u_j\, u_{j-n}}, \qquad (1)$$

where *n* is an integer, positive, negative or zero.

There are periodic (PACF) and aperiodic (AACF) auto-correlation characteristics of signals.

Discrete signals with the best ACF structure include Barker signals (codes), M-sequences [1, 4, 6].

Recently, in the native and foreign literature, much attention has been paid to the application of perfect binary lattices (PBL) [2, 8, 9] in various radio engineering problems, for example:

- for antenna aperture synthesis;
- to build perfect time-frequency codes;
- to obtain noise-like signals for data transmission systems.

A perfect binary lattice is a two-dimensional matrix sequence:

$$\boldsymbol{H(N)} = \left|\left|\boldsymbol{h_{i,j}}\right|\right|, \boldsymbol{i, j} = \overline{\boldsymbol{0, N-1}},\ \boldsymbol{h_{i,j}} \in \{\boldsymbol{-1, 1}\}, \quad (2)$$

which has an ideal two-dimensional periodic auto-correlation function, the elements of which are equal to:

$$\boldsymbol{R(m,n) = PACF(m,n)} = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} \boldsymbol{h_{i,j}\, h_{i+m,j+n}} = \begin{cases} \boldsymbol{N^2, at\ m = n = 0} \\ \boldsymbol{0, with\ other\ m\ u\ n'} \end{cases} \qquad (3)$$

where $\boldsymbol{m, n} = \overline{\boldsymbol{0,\ N-1}}$.

In accordance with the rules for the formation of perfect binary lattices, which are set out in [2], a perfect binary lattice of order N = 8 with a dimension of eight symbols, shown in Fig.1, is constructed.

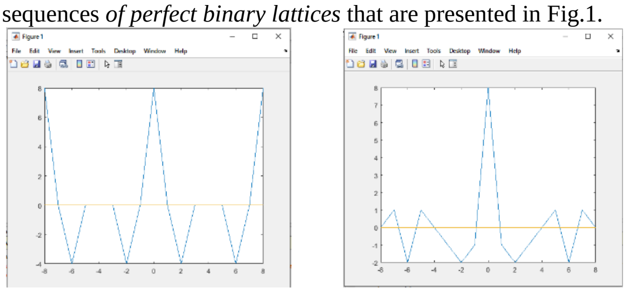
$$C = \begin{bmatrix} C_1 = + & - & - & - & + & + & - & + \\ C_2 = - & - & - & - & - & + & - & + \\ C_3 = + & - & + & + & + & + & + & - \\ C_4 = - & + & + & + & - & - & + & - \\ C_5 = + & - & - & - & + & + & - & + \\ C_6 = + & + & + & + & + & - & + & - \\ C_7 = + & - & + & + & + & + & + & - \\ C_8 = + & - & - & - & + & + & - & + \end{bmatrix}$$

Fig. 1 *Ensemble of perfect binary lattices*

Fig.1 shows that the sequences C_1, C_5 and C_8 coincide with each other, as well as the sequences C_3 and C_7. And the sequences C_1 and C_4, C_2 and C_6, C_4 and C_5, C_4 and C_8 are inverse to each other, i.e., they have the same correlation characteristics.

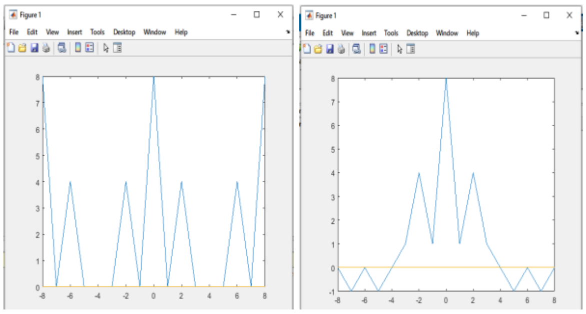
3. Experimental data

Consider the correlation characteristics of some of the sequences *of perfect binary lattices* that are presented in Fig.1.



a) b)

Fig. 2 PACF (a) and AACF (b) sequences C_1, C_4, C_5 and C_8 of perfect binary lattices



a) b)

Fig. 3 PACF (a) and AACF (b) sequences C_1, C_4, C_5 and C_8 of perfect binary lattices

From Fig.2 and Fig.3 it follows that the correlation characteristics of the sequences C_1, C_2, C_4, C_6, C_8 have correlation characteristics somewhat worse than those of the Barker codes and M sequences - the suppression coefficient (the ratio of the amplitude of the main ACF lobe to the maximum amplitude of the side lobes) is equal to only 2 for all sequences.

However, the power of binary perfect lattices is much greater than that of Barker codes and M-sequences, and such a suppression coefficient makes it possible to reliably distinguish such signals on the background of noise.

In data transmission systems with division of channels (signals) according to the waveform, all subscribers can work simultaneously in the same frequency band, i.e., signal spectra overlap. In order for the signals (channels) not to influence each other, they must be orthogonal. In this case, the cross - correlation function (CCF) of two discrete signals, equal to their scalar product, is equal to zero:

$$\boldsymbol{R_{uv}(n) = \sum_{j=-\infty}^{\infty} u_j\, v_{j-n} = 0}. \qquad (4)$$

However, based on the generalized Rayleigh formula, the scalar product of two signals in the time domain, up to a factor of $1/2\pi$, is equal to the scalar product of their spectral densities:

$$\boldsymbol{(u, v) = R_{uv}(n) = \frac{1}{2\pi}(U(\omega), V(\omega) = 0}. \qquad (5)$$

The zero CCF means that two signals are orthogonal for any τ only if their spectra do not overlap. However, in multi-channel code division systems, this cannot be achieved, since all subscribers operate in the same band. The consequence of this is the occurrence of multiple access interference, i.e., non-zero response of the receiver of the *k-th* user to the signals of other subscribers, and the level of this interference depends on the number of subscribers simultaneously working at the moment [6].

Therefore, it is necessary to choose such code sequences for which the CCF is minimal. Moreover, to solve the reception problem, only the correlation properties of the signal are important, and not their shape.

Walsh functions [7] can be attributed to discrete signals with the best structure of the cross-correlation function. The number of distinct functions in the Walsh system is $N = 2^m$, where m is an integer. Walsh functions take only two values: +1 and -1, which is a useful property when building circuits on binary digital elements.

The orthogonal system, which is most often used in multichannel code division systems, is the Walsh-Hadamard system (matrix) of order $N = 2^k$, k is an integer, which is determined by the recursive rule:

$$W_{2N} = \begin{bmatrix} W_N & W_N \\ W_N & -W_N \end{bmatrix}, \qquad (6)$$

where W_N is the Walsh-Hadamard matrix of order N, assuming W_1 =1, or in the signed form W_1 = +.

Should be noted, that a feature of orthogonal codes - the orthogonality property of these codes is satisfied only at the “point”, i.e., in the absence of time shifts. In real conditions, this property of orthogonality is violated, which in turn leads to an increase in the level of multiple access interference and the appearance of errors in the processing of input data. Therefore, various methods are used to eliminate these shortcomings.

In order to improve the correlation properties (ACF and CCF) of Walsh functions, the so-called derivative systems of signals are often built [2, 6, 11].

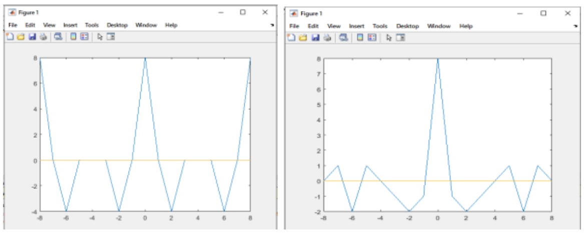
A derivative signal is a signal that is obtained as a result of element-by-element (symbol-by-symbol) multiplication of two signals - the original and generating one. A system composed of derivative signals is called a derivative.

As a generating signal, such a signal is chosen so that the derivative system has good correlation properties. As a rule, this is a signal with a good ACF. Let us consider the correlation properties of derivatives of Walsh functions, if we take a matrix of ***perfect binary lattices*** (Fig. 1) ***as generating functions***, which is multiplied row by row by the corresponding Walsh matrix (Fig. 4).

$$W_{\mathbf{deriv}} = \begin{bmatrix} + & - & - & - & + & + & - & + \\ - & + & - & + & - & - & - & - \\ + & - & - & - & + & + & - & + \\ - & - & - & + & - & + & - & - \\ + & - & - & - & - & - & + & - \\ + & - & + & - & - & - & - & - \\ + & - & - & - & - & - & + & - \\ + & + & + & - & - & + & - & - \end{bmatrix}$$

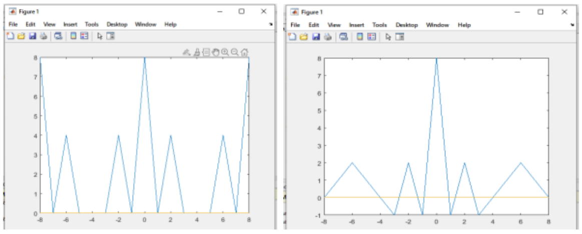
Fig. 4 *Derivative matrix of Walsh functions with* ***perfect binary lattices*** *as generating functions*

The correlation characteristics of some sequences from the derivative matrix of Walsh functions (for example, 1st and 4th) were investigated in the Matlab environment. The results of the analysis of aperiodic (AACF) and periodic (PACF) autocorrelation functions of the selected derivatives of the Walsh functions *with perfect binary lattices as generating functions* are presented in Fig.5 and Fig.6, respectively.



a) b)

Fig. 5 PACF (a) and AACF (b) of the 1st derivative of the Walsh function with perfect binary lattices as generating functions



a) b)

Fig. 6 PACF (a) and AACF (b) of the 4th derivative of the Walsh function with perfect binary lattices as generating functions

From Fig.5 and Fig.6 it follows that the characteristics of the autocorrelation functions of the derivatives of the Walsh functions are similar to the autocorrelation characteristics of ***perfect binary lattices*** and coincide with the conclusions made above.

As noted above, for multichannel systems with channel division by waveform, to reduce the interference of multiple access, the signals used in such systems must have certain cross-correlation characteristics.

Let us consider the cross-correlation characteristic of the 1st and 4th derivatives of the Walsh function, which were studied above (Fig. 7).

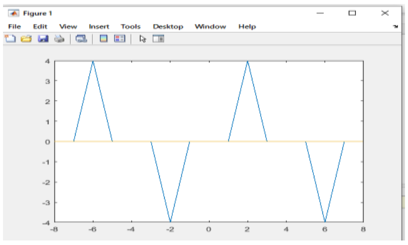


Fig. 7 CCF of the 1st and 4th derivatives of the Walsh functions with perfect binary lattices as generating functions

As can be seen from the CCF plot (Fig. 7), the considered derivatives of the Walsh function have satisfactory mutual correlation properties - the characteristic is more or less uniform with the same level of outliers (± 4). The considered derivatives of the Walsh function with *perfect binary lattices* as generating functions can be used in code division systems with the use of decision devices that do not respond to CCF emissions within the specified limits.

4. Conclusion

An analysis of the correlation properties of the studied broadband signals based on the derivatives of the Walsh functions with generating functions in the form of *perfect binary lattices* allows us to draw the following conclusions:

1. The correlation characteristics of Walsh functions that are orthonormal have good cross-correlation functions - the cross-correlation function between two different Walsh functions is zero. However, these functions have such properties only at a point (at zero shift). In real conditions, especially with multipath propagation, orthogonality is violated and the cross-correlation function of these functions is nonzero. This leads to an increase in the level of interference of multiple access and to errors in the separation of signals (channels).
2. The correlation properties of derivative Walsh functions with generating functions in the form of *perfect binary lattices have much better correlation characteristics than the original Walsh functions.*
3. The derivatives of the Walsh function (L = 8 bits) have a large amplitude of the central peak of the ACF, equal to the length of the sequence, but the amplitude of the side lobes slightly increases. However, the suppression factor is equal to 2 for all sequences. This makes it possible to reliably distinguish such signals on the background of interference and ensure reliable synchronization of receiving devices.
4. The large length of the spreading code based on the derivative of the Walsh functions allows:
   - spreading the signal energy over the spectrum,
   - increasing the noise immunity of the system,
   - providing good protection against unauthorized access,
   - improving electromagnetic compatibility with neighboring radio systems.
5. When using the derivatives of the Walsh functions, it is necessary to take into account the increase in the level of the side lobes of the ACF and emissions of the VCF, i.e., take special measures to improve reception synchronization and eliminate the impact on channel separation when the level of multiple access interference increases.

The obtained results can be used in the development of broadband communication systems and information transmission systems with protection from unauthorized access.

5. References

1. Л.Е. Варакин, *Системы связи с шумоподобными сигналами*. (М.: Радио и связь, 1985).
2. М.И. Мазурков, *Системы широкополосной радиосвязи*. (О.: Наука и техника, 2009).
3. В.Е. Гантмахер, Н.Е. Быстров, Д.В. Чеботарев, *Шумоподобные сигналы. Анализ, синтез, обработка* (СПб.: Наука и Техника, 2005).
4. W. Solomon, Golomb and Guang, Gong. *Signal Design for Good Correlation*. (Cambridge University Press, 2005).
5. В.П. Ипатов, *Широкополосные системы и кодовое разделение сигналов. Принципы и приложения*: Пер. с англ. (М.: Техносфера. 2007).
6. А.А. Бессарабова, М.Д. Венедиктов, В.И. Ледовских, *Разделение каналов по форме в широкополосных системах передачи информации*. (Воронеж: Воронеж. гос. техн. ун-т, 2006).
7. М.С. Беспалов., В.А. Скляренко, *Функции Уолша и их приложения*. Уч. пособие. (Владимир, Изд-во ВлГУ, 2012).
8. М.И. Мазурков, *Метод защиты информации на основе совершенных двоичных решеток* /М.И. Мазурков, В.Я.Чечельницкий, П.Мурр. (Радиоэлектроника (Изв. вузов), 2008 — Т.51, № 11).
9. М.И. Мазурков, В.Я. Чечельницкий, *Свойства полного класса совершенных двоичных решеток на 36 элементов.* (М.: Радиоэлектроника, 2004, №6).
10. В.Я. Чечельницкий, М.Ю. Герасименко, *Взаимокорреляционные свойства совершенных двоичных решеток эквивалентных классов*. (Одесса, Изд-во ОПУ, 2005).
11. Sestacova T., Sorochin Gh., Jdanov V. *Analysis of the correlation properties of direct and inverse composite Walsh functions*. In: Engineering. Technologies. Education. Security 2021: proc. of the IX Intern. scient. conf., Borovets, June 07-10, 2021. Bulgaria, 2021, vol. 2, pp. 83-87. Disponibil: http://techtos.net/sbornik/2021/2-2021.pdf