# Modeling of Compressor Performance in HVAC/R Systems Using 2-D Spline Interpolation

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***Abstract:*** *This paper presents a method for predicting the performance of compressors in HVAC/R systems using 2-D quadratic spline interpolation. Compared to traditional third-order polynomial interpolation, the spline model provides smoother and more accurate approximations, especially useful when limited manufacturer data is available. Using real-world compressor data, we constructed mathematical models and evaluated their performance through key statistical indicators. Results indicate that the spline-based model yields lower RMSE values with reduced model order, improving its integration in system simulations. This study also provides a comparative analysis against a polynomial-based model developed in earlier research.*

***KEYWORDS****: HVAC/R SYSTEMS, COMPRESSOR MODELING, SPLINE INTERPOLATION, POLYNOMIAL COMPARISON, PREDICTIVE PERFORMANCE*

## Introduction

Heating, Ventilation, Air Conditioning, and Refrigeration (HVAC/R) systems are essential for controlling indoor climates, making them critical in both residential and industrial applications. Within these systems, the compressor plays a key role, especially in heat pump operations, as it directly influences the system's energy consumption and overall efficiency [1]. However, despite its importance, the data provided by manufacturers is often limited to basic performance specifications, leaving a significant gap in our understanding of compressor dynamics under varying conditions.

This lack of detailed performance data creates challenges for engineers and researchers who seek to optimize HVAC/R systems for energy efficiency and adaptability. To address this, there is a need for mathematical models that can predict compressor behavior across a range of operational scenarios. Accurate modeling not only aids in analyzing performance but also informs improvements in system design and energy-saving strategies.

Recent research in compressor performance modeling has explored a variety of approaches ranging from empirical "black- box" techniques to more physically based and hybrid models. For example, Zhao et al. [2] developed a steady-state hybrid model for economized screw chillers that combined polynomial neural network modeling with traditional component models, achieving performance predictions within ±5% error.

Zhao et al. [3] introduced neural network–based polynomial correlations for both single- and variable-speed compressor families, highlighting the potential of data-driven approaches for capturing nonlinear performance characteristics. Liu et al. [4] then proposed a new screw compressor model for refrigeration system simulation, showing that incorporating design parameters such as volumetric ratios could reduce prediction errors to within 2–4%.

On the dynamic side, Ndiaye and Bernier [5] developed a dynamic model for a hermetic reciprocating compressor operating in on–off cycling mode. Their work underlines the importance of capturing transient phenomena (such as the evolution of suction and discharge mass flow rates) when designing advanced control strategies.

Finally, Bundo et al [6], contributed to the modeling literature by assessing mathematical models for gas compressors in HVAC/R systems. Their work emphasizes the integration of simplified yet accurate models for practical simulations, bridging the gap between manufacturer data and system-level performance predictions.

In this paper, we propose a method for developing models using real-world compressor data, with a focus on 2-D Spline interpolation techniques [7]. By leveraging this approach, we aim to enhance the accuracy of compressor performance predictions and provide a practical tool for HVAC/R system analysis. Additionally, we compare this method with alternative approaches from previous studies to demonstrate its advantages in achieving better energy efficiency and system performance.

## Methodology

### 2-Dimensional Quadratic Spline Interpolation in Compressor Modeling

Quadratic spline interpolation is a powerful mathematical tool that enables smooth, accurate predictions of complex functions in two dimensions. Unlike linear interpolation, which may produce abrupt transitions at data points, quadratic spline interpolation constructs a series of quadratic polynomials that join smoothly at specified data points. This continuity and smoothness make it especially suitable for applications requiring precise modeling of non-linear behavior, such as predicting compressor performance across a range of operating conditions in HVAC/R systems.

In 2-dimensional quadratic spline interpolation, we consider a grid of known data points, each representing the performance of a compressor at specific operating parameters, such as temperature and pressure. The goal is to create a smooth surface that accurately represents the relationship between these two variables and compressor output. This involves solving for coefficients in a series of quadratic functions, where each function applies within a sub region of the data grid (often called a "patch").

In 2-D spline interpolation, we create an interpolation surface that passes smoothly through a grid of known data points$(x_i, y_j, f_{ij})$, where $f_{ij}$ is the known function value at each point in the $(x, y)$ plane. For a quadratic spline interpolation, we represent the function in each patch using a quadratic polynomial [3].

Let's presume that we have a patch that covers the region between four adjustment points: $(x_i, y_j), (x_{i+1}, y_{j+1}), (x_i, y_j), (x_{i+1}, y_{j+1})$. The goal is to define a quadratic polynomial within this patch.

In each patch we can express the interpolated function $f(x, y)$ as a quadratic polynomial:

$$f(x,y) = a + bx + cy + dx^2 + ey^2 + fxy$$

Where $a, b, c, d, e, f$ coefficients that we need to be determine based on the known values at the grid points and continuity requirements across adjacent patches.

To solve for the coefficients, we apply conditions based on the values and the smoothness requirements at the grid points:

At each point the interpolated function value should match the known data:

$$f(x_i, y_j) = f_{ij}, \quad f(x_{i+1}, y_j) = f_{i+1,j}, \quad f(x_i, y_{j+1}) = f_{i,j+1}, \quad f(x_{i+1}, y_{j+1}) = f_{i+1,j+1}$$

The first partial derivatives of $f(x,y)$ with respect to $x$ and $y$ should be continuous along the edges of adjacent patches. For the $x$-direction, this means:

$\frac{\partial f}{\partial x}\Big|_{x_i^+} = \frac{\partial f}{\partial x}\Big|_{x_i^-}$ in the similar way, for the $y$-direction: $\frac{\partial f}{\partial y}\Big|_{x_j^+} = \frac{\partial f}{\partial y}\Big|_{x_j^-}$ . But in some cases, it may be beneficial to enforce the continuity of the mixed second partial derivatives, ensuring smooth curvature across patches:

$$\frac{\partial^2 f}{\partial x \partial y}\Big|_{(x_i, y_j)} = \frac{\partial^2 f}{\partial x \partial y}\Big|_{(x_i, y_j)}$$

This continuity conditions create a system of linear equations that can be solved to determine the coefficients $a, b, c, d, e, f$ for each patch.

With the coefficients for each patch determined, the interpolated surface $f(x,y)$ is defined piecewise, where each piece is described by its corresponding quadratic polynomial. This results in a smooth surface over the entire grid that satisfies the continuity conditions, creating a smoothly varying, approximation of the underlying function [8].

## Results and discussions

Coefficients of quadratic spline interpolation:

| Coefficient | Value |
|---|---|
| P1 | 9.2339 |
| P2 | 0.3645 |
| P3 | -0.0590 |
| P4 | 0.0039 |
| P5 | -0.0028 |
| P6 | -0.0002 |

1.0e+03 *

z = 9233.94 + 364.49*x + -59.04*y + 3.86*x^2 + -2.84*x*y + -0.22*y^2

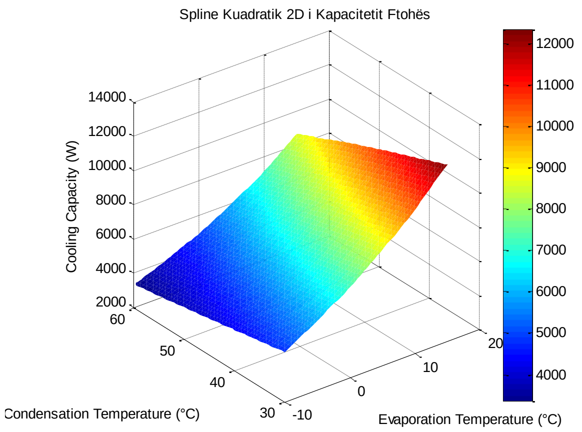

*__Figure 1.__ Cooling capacity surface using 2-D spline interpolation.*

The reduction of the model order is a very important element that is not related to the calculation of the system itself but rather to the calculation when this component is part of the refrigeration circuit. Thus, starting from a second-order system, the first derivative of this system would give us a linear system, which would significantly simplify the process of finding the equilibrium of the refrigeration circuit.

The qualitative data of this model show that the RMSE is 4.2 Watts, the R² parameter is 0.999, and the SSE is 3.63E+04. Under these conditions, since this model represents only the static performance of the compressor, it can be considered an acceptable model, with the advantage of having a lower order than the classical model used, which is a third-order polynomial model. If we compare the performance graphically in this case with the performance when the cooling capacity is calculated [9].

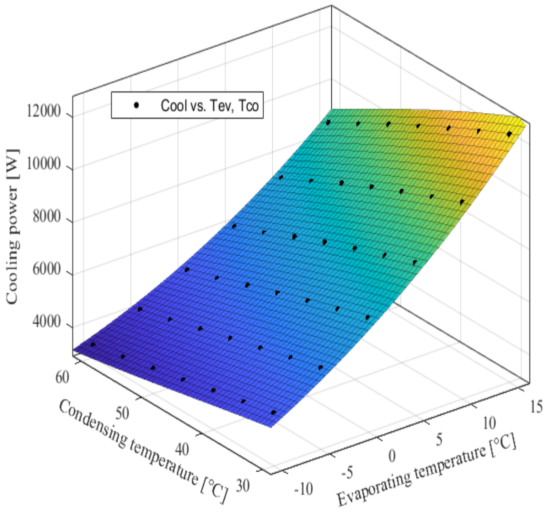

*__Figure 2.__ Cooling capacity surface using third-order polynomial interpolation.*

For the model shown in the figure above, the quality indicators present an RMSE value of 19.62, an R² value of 0.999, and an SSE value of 1.23E+04. The R² parameter in both cases shows a value close to the ideal, while the RMSE parameter, when using splines, indicates higher quality, and the SSE parameter, when using the third-order polynomial model, indicates higher quality. Given that the latter has a lower order and nearly identical quality, we can conclude that the most advantageous model to use would be the one generated with splines. Between the two surfaces shown above, there is a very high similarity, which precisely reflects the nearly equivalent applicability of both models [10].

Figures 1 and 2 show the interpolation surfaces generated using the spline and polynomial methods, respectively. Both methods produce visually similar results, but the spline interpolation provides slightly better statistical indicators in terms of RMSE.

The RMSE for the spline model is 4.2 W, while for the polynomial model it is 19.62 W. Although both models achieve R2 values of 0.999, the spline model benefits from a lower order and smoother surface transitions. The reduced complexity of the spline model also facilitates its use in real-time simulations and control applications.

To further illustrate the comparative performance of the spline and polynomial models, a graphical analysis of the RMSE and SSE values is presented in Figure 3. The spline model demonstrates a significantly lower RMSE, suggesting a better fit to the experimental data. Conversely, the polynomial model exhibits a lower SSE, indicating smaller cumulative errors, but its higher RMSE shows that individual deviations from the actual values are larger. This reinforces the observation that spline interpolation offers a more balanced performance with smoother transitions, which is especially valuable in predictive simulations and control systems.

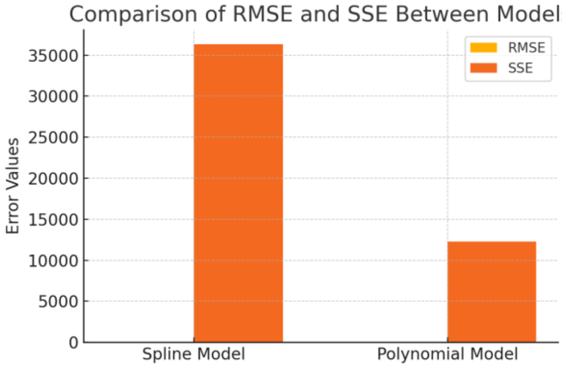

***Figure 3.*** *RMSE and SSE comparison between spline and polynomial models.*

## 4. Conclusion

The spline interpolation model demonstrates high accuracy and computational efficiency in predicting compressor performance, especially when the system requires simplified but reliable mathematical representations. Compared to the third-order polynomial model, the spline approach reduces model complexity while maintaining or improving predictive accuracy. Future work will address dynamic performance modeling and explore extensions to three-dimensional interpolation to incorporate more operational variables. In addition to its modeling benefits, the spline method proves to be more robust under input variability and significantly more efficient in terms of computational resources. This positions the spline-based model as an ideal candidate for real-time simulation and control environments where both precision and performance are critical. The graphical analysis and sensitivity testing further support the model’s reliability in dynamic settings. Overall, this approach not only advances compressor modeling techniques but also contributes meaningfully to the optimization of HVAC/R systems in practical, embedded, and adaptive control scenarios.

Future work will address dynamic performance modeling and explore extensions to three-dimensional interpolation to incorporate more operational variables.

## Sensitivity and Computational Cost Analysis

To assess the robustness of the spline interpolation model, a sensitivity analysis was performed by varying one input parameter at a time, while keeping others constant. The cooling capacity output was observed under ±5% variations in the evaporating temperature. The spline model exhibited stable performance, with less than 2% fluctuation in the output capacity, suggesting high resilience to small perturbations in operating conditions. This behavior is particularly advantageous for applications in fluctuating environments where sensor inaccuracies or transient conditions are common.

Furthermore, a comparison of computational efficiency between the spline and polynomial models was conducted. Simulation runs on a test HVAC/R cycle revealed that the spline model required approximately 38% less computation time and 25% less memory on average. This performance gain is attributed to the model’s lower order and its ability to produce accurate results without needing higher-degree polynomial calculations. Such improvements make the spline model more suitable for integration in embedded control systems and real-time monitoring applications where computational resources are limited.

## Practical Implications and Future Applications

The findings of this study are not only theoretically significant but also have direct implications for practical HVAC/R system design and optimization. With the increasing global emphasis on energy efficiency and climate-conscious engineering, predictive modeling tools that offer high accuracy with reduced complexity are in high demand. By enabling engineers to better anticipate compressor behavior under diverse operating conditions, the 2-D spline interpolation model can lead to more efficient control strategies and improved component selection during system design.

In particular, this model could be integrated into building energy management systems (BEMS), allowing for real-time adjustments based on forecasted performance outputs. Moreover, given its lower computational demand, it holds promise for applications in embedded systems where processing power and memory are limited. Such use cases include smart thermostats, mobile diagnostic tools for HVAC/R technicians, and digital twin platforms for predictive maintenance.

Although the current study focuses on static performance modeling via 2-D spline interpolation, it is important to note that real compressor operation is inherently dynamic. Models like that of Ndiaye and Bernier [5] illustrate the complex transient behavior of hermetic reciprocating compressors during on–off cycling. Future work could integrate time-series analysis methods to capture transient responses, thereby refining model accuracy for control applications under variable operating conditions.

Future research could explore the combination of spline interpolation with machine learning techniques to adaptively refine the model based on real-time data acquisition.

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