Artificial intelligence approaches for modeling nonlinear dynamical systems

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Abstract: *Nonlinear dynamical systems arise in numerous scientific and engineering domains, including physics, economics, biology, and control theory. Their complex behavior, sensitivity to initial conditions, and possible chaotic dynamics make accurate modeling and prediction challenging using traditional analytical approaches alone. In recent years, artificial intelligence (AI) techniques have demonstrated strong potential for modeling nonlinear and complex systems through data-driven methods. This paper explores artificial intelligence approaches for modeling nonlinear dynamical systems, focusing on the integration of machine learning techniques with classical mathematical modeling. We consider representative nonlinear systems and analyze how neural networks, regression models, and hybrid AI–mathematical frameworks can be used to approximate system behavior, predict future states, and capture hidden structures in time-series data. Special attention is given to systems exhibiting chaotic behavior, where small perturbations in initial conditions can lead to significant divergence in trajectories. The study presents numerical simulations and comparative analyses between traditional mathematical models and AI-based approaches. The results highlight the advantages of machine learning methods in capturing nonlinear patterns and improving predictive accuracy, especially when analytical solutions are difficult or unavailable. Additionally, we discuss the interpretability of AI models in the context of dynamical systems and outline potential applications in engineering, intelligent control, and data-driven system identification. The proposed framework contributes to the growing intersection between dynamical systems theory and artificial intelligence by demonstrating how AI tools can support the analysis and modeling of complex nonlinear phenomena. This work aims to provide a foundation for future research on hybrid mathematical–AI methods for understanding and predicting complex systems.*

Keywords: NONLINEAR DYNAMICAL SYSTEMS, ARTIFICIAL INTELLIGENCE, MACHINE LEARNING, CHAOTIC SYSTEMS, SYSTEM MODELING, HYBRID MODELING.

1. Introduction

Nonlinear dynamical systems play a fundamental role in modeling natural and engineered phenomena [1]. From fluid turbulence and population dynamics to financial markets and control systems, nonlinear equations describe processes where outputs are not proportional to inputs and where complex behaviors such as bifurcations and chaos may arise.

Classical analysis of nonlinear systems relies on differential equations, stability theory, Lyapunov methods, phase plane analysis, and bifurcation theory [2, 11]. However, analytical solutions are rarely available for real-world systems. Even numerical integration methods, while powerful, may struggle in the presence of strong nonlinearity, noise, or partially observed states.

In parallel, artificial intelligence (AI) and machine learning (ML) have developed into powerful tools for approximating highly nonlinear mappings directly from data [3]. Neural networks, regression models, and hybrid AI–physics models now offer alternative frameworks for modeling dynamical systems.

This paper investigates how AI techniques can model nonlinear dynamical systems, especially those exhibiting chaotic behavior. We compare classical mathematical models with AI-based approaches and propose a hybrid framework integrating both perspectives.

2. Mathematical Background on Nonlinear Dynamical Systems

In this section, we introduce the fundamental mathematical definitions of dynamical systems, which constitute the theoretical foundation of our study.

Definition 2.1: A dynamical system consists of a phase (state) space P and a family of transformations

$$\phi_t : P \to P$$

where the time parameter t belongs either to □ (continuous time) or □ (discrete time).

The family $\{\phi_t\}_{t\in T}$ statisfies the following properties:

1. Identity property

$$\phi_0(x) = x, \qquad \forall x \in P$$

2. Group (additivity) property

$$\phi_s(\phi_t(x)) = \phi_{s+t}(x), \qquad \forall x \in P,\ \{s,t\} \in T$$

Thus, a dynamical system can be understood as a mathematical rule describing the evolution of the state $x \in P$ over time.

The function $\phi_t(x)$ is called the flow of the system when time is continuous, and the iteration operator when time is discrete.

Definition 2.2: A dynamical system is said to be time-continuous if the time parameter satisfies $t \in$ □. In this case, the system is typically described by a system of ordinary differential equations (ODEs):

$$\dot{x} = F(x)$$

where

1. $x(t) \in P \subset R^N$,
2. $F : P \to R^N$ is a vector field,
3. $\dot{x} = \dfrac{dx}{dt}$.

The formal solution is written as

$$x(t) = \phi_t(x_0)$$

where x_0 is called the initial condition.

Time-continuous systems describe processes evolving smoothly in time, such as:

- mechanical systems,
- electrical circuits,
- population dynamics,
- fluid motion.

Under suitable smoothness conditions on F, existence and uniqueness theorems guarantee that trajectories do not intersect. This implies determinism: each initial condition determines a unique trajectory.

An important structural result is the Poincaré–Bendixson theorem, which states that chaotic behavior cannot occur in two-dimensional continuous-time systems. Therefore, for chaotic dynamics to arise in continuous systems, the phase space dimension must satisfy $N \geq 3$.

Definition 2.3: A dynamical system is called time-discrete if the time parameter satisfies $n \in \mathbb{N}$. Such systems are described by an iteration rule:

$$x_{n+1} = M(x)$$

where $M : P \to P$ is a mapping.

The evolution after n steps is given by:

$$x_n = M^n(x_0)$$

Where M^n denotes the n-fold composition of M.

Time-discrete systems frequently arise from numerical discretization of differential equations, population models, and iterative algorithms [12, 13].

Unlike continuous systems, even one-dimensional non-invertible maps can exhibit chaotic dynamics.

A dynamical system is called linear if the evolution rule satisfies the superposition principle:

$$F(ax_1 + bx_2) = aF(x_1) + bF(x_2)$$

If this property does not hold, the system is called nonlinear**.**

Definition 2.4: A nonlinear dynamical system is a system of the form

$$\dot{x} = F(x) \quad or \quad x_{n+1} = M(x_n)$$

where the vector field F or the map M is nonlinear, i.e., it contains:

- products of variables (e.g., xy),
- higher-order powers (e.g., x^2),
- nonlinear functions (e.g., $\sin x, e^x$),
- or does not satisfy superposition.

Nonlinearity is the fundamental source of complex phenomena such as:

- bifurcations,
- multiple equilibria,
- limit cycles,
- chaotic attractors,
- sensitive dependence on initial conditions. [2,9]

Example 2.1: Logistic Map (Discrete)

$$x_{n+1} = rx_n(1 - x_n)$$

which is nonlinear due to the quadratic term x_n^2 For certain parameter values r the system exhibits chaotic behavior [2,9,12].

Example 2.2: Lorenz System (Continuous)

$$\begin{cases} \dot{x} = \delta(y - x) \\ \dot{y} = x(\rho - z) - y \\ \dot{z} = xy - \beta z \end{cases}$$

The nonlinear interaction term xy generates chaotic attractors when parameters are appropriately chosen [4].

Example 2.3: Nonlinear Pendulum

$$\ddot{\theta} + \vartheta\dot{\theta} + \sin\theta = 0$$

where the sine term introduces nonlinearity.

Nonlinear dynamical systems form the mathematical framework for understanding complex behavior in natural and engineered systems. [2,10]. Their intrinsic nonlinearity allows the emergence of instability, bifurcation phenomena, and chaotic dynamics, which cannot occur in purely linear systems.

These theoretical foundations are essential before introducing data-driven and artificial intelligence approaches for modeling such systems.

3. Artificial Intelligence Approaches

The rapid development of artificial intelligence and machine learning has introduced powerful data-driven techniques for modeling nonlinear dynamical systems. Classical approaches rely on explicitly known governing equations, typically expressed as systems of differential equations. However, in many real-world problems the governing equations are partially known or completely unknown. In such situations, artificial intelligence provides an alternative framework capable of learning system dynamics directly from data [3].

Machine learning models approximate the evolution operator of a dynamical system. For a discrete dynamical system

$$x_{t+1} = F(x_t)$$

the objective of a learning algorithm is to construct an approximation

$$\widehat{F}_\theta(x_t)$$

parameterized by θ, such that

$$\hat{x}_{t+1} \approx F(x_t)$$

Thus, artificial intelligence techniques can be interpreted as data-driven approximations of the dynamical evolution operator**.**

Recent research has demonstrated that machine learning methods are capable of reproducing complex nonlinear and chaotic dynamics with high accuracy, even when the underlying governing equations are unknown.

3.1. Neural Network Models for Dynamical Systems

Artificial neural networks represent one of the most widely used tools for modeling nonlinear dynamical systems due to their universal approximation capabilities [3].

Feedforward neural networks approximate nonlinear functions of the form

$$\hat{x}_{t+1} = NN(x_t;\theta)$$

where NN represents a multilayer neural network.

These models have been successfully applied to the prediction of chaotic time series generated by nonlinear systems such as the logistic map, Lorenz attractor, and intermittency maps. Studies have shown that neural networks such as multilayer perceptrons (MLP), residual networks, and long short-term memory networks (LSTM) can significantly reduce prediction errors compared with classical regression methods.

In particular, hierarchical neural architectures combining several specialized networks have been shown to improve prediction accuracy in chaotic time series modeling tasks.

3.2. Recurrent Neural Networks and Temporal Modeling

While feedforward neural networks approximate static nonlinear functions, recurrent neural networks (RNNs) are specifically designed to capture temporal dependencies in dynamical systems.

An RNN model evolves according to

$$h_{t+1} = \sigma(W_h h_t + W_x x_t)$$

$$\hat{x}_{t+1} = W_0 h_{t+1}$$

Here:

- h_t represents the hidden state
- $W_h, W_x, W_0,$ are weight matrices
- σ is a nonlinear activation function.

Long Short-Term Memory (LSTM) networks and gated recurrent units (GRU) are improved recurrent architectures capable of learning long-term temporal dependencies [5].

Applications include:

- climate system modeling
- fluid turbulence
- nonlinear oscillators
- chaotic attractors

Recent studies have demonstrated that LSTM networks can successfully learn the dynamics of coupled nonlinear systems and predict chaotic trajectories without explicit knowledge of the governing differential equations.

However, classical RNN architectures often suffer from instability when predicting chaotic systems over long time horizons due to error accumulation.

3.3. Reservoir Computing and Chaotic System Prediction

Reservoir computing is another powerful machine learning framework designed specifically for modeling dynamical systems [6].

Reservoir computing networks consist of:

1. a fixed high – dimensional dynamical reservoir
2. a trained linear readout layer

The system evolves according to

$$r_{t+1} = \tanh(W_r r_t + W_{in} x_t)$$

$$\hat{x}_{t+1} = W_{out} r_{t+1}$$

Unlike traditional neural networks, only the output layer is trained, which greatly simplifies the training process.

Reservoir computing models have demonstrated remarkable performance in predicting chaotic dynamical systems, including high-dimensional models such as the Lorenz-96 system. In many numerical experiments, reservoir computing has been shown to outperform traditional neural networks in short-term trajectory prediction and statistical reconstruction of chaotic attractors.

3.4. Koopman Operator and Data-Driven Linearization

A modern approach to modeling nonlinear dynamical systems is based on Koopman operator theory**.**

The Koopman operator K acts on observable functions $g(x)$ instead of the state variables themselves:

$$K(g(x)) = g(F(x))$$

Although the underlying system is nonlinear, the Koopman operator is linear but infinite-dimensional.

Data-driven techniques such as:

- Dynamic Mode Decomposition (DMD)
- Extended Dynamic Mode Decomposition (EDMD)

allow finite-dimensional approximations of the Koopman operator.

This approach transforms nonlinear dynamics into approximately linear systems in a lifted feature space. Numerical studies have demonstrated that Koopman-based models can accurately reproduce the statistical properties and temporal evolution of chaotic systems while remaining computationally efficient.

Koopman-based methods therefore provide a bridge between classical dynamical systems theory and modern machine learning [7].

3.5. Hybrid Physics–AI Models

A promising direction in recent research is the development of hybrid models combining physics-based equations with machine learning components**.**

In these models:

$$\dot{x} = F_{physics}(x) + F_{ML}(x)$$

where

- $F_{physics}(x)$ represents known physical laws
- $F_{ML}(x)$ approximates unknown or unresolved dynamics.

Hybrid neural-physics models have been successfully applied to systems such as the Lorenz equations, where neural networks are used to model missing physical processes [8]. These models significantly improve predictive accuracy when only partial knowledge of the governing equations is available.

3.6. Performance and Limitations

Despite their success, AI-based approaches face several challenges when applied to nonlinear dynamical systems:

1. ***Sensitivity to initial conditions***
   Chaotic systems amplify small prediction errors over time.
2. ***Long-term prediction instability***
   Neural networks often provide accurate short-term predictions but fail to reproduce long-term trajectories.
3. ***Interpretability issues***
   Deep learning models are often difficult to interpret in terms of physical laws.

Recent research attempts to address these issues by incorporating dynamical systems theory into machine learning architectures, including physics-informed neural networks and Koopman-based representations.

Artificial intelligence provides powerful tools for modeling nonlinear dynamical systems, particularly when governing equations are unknown or difficult to solve analytically. Neural networks, recurrent architectures, reservoir computing, and Koopman-based frameworks have all demonstrated the ability to approximate complex dynamical behaviors and chaotic systems.

The integration of machine learning with classical dynamical systems theory represents a promising research direction, enabling the development of hybrid models capable of combining data-driven learning with mathematical structure.

The artificial intelligence methods discussed above differ in their modeling philosophy, predictive capability, and interpretability when applied to nonlinear dynamical systems. In order to highlight the main characteristics of these approaches, Table 1 provides a comparative overview of the principal AI-based techniques used for modeling nonlinear dynamical systems and their relevance to the present study.

Table 1: *Comperative overview of AI-based techniques*

Method	Modeling Principle	Strengths in Nonlinear Dynamical Systems	Limitations	Relevance to This Study
Feedforward Neural Networks (MLP)	Approximate nonlinear mappings of the form ($x_{t+1} = F(x_t)$) using multilayer networks	Universal approximation capability; simple architecture; effective for nonlinear function approximation	Limited capability for capturing long-term temporal dependencies	Used to approximate the nonlinear mapping of the logistic map
Recurrent Neural Networks (RNN) / LSTM	Learn temporal dependencies through recurrent hidden states	Suitable for modeling sequential and time-series data; capable of capturing system dynamics	Training instability and error accumulation in chaotic systems	Applied to model Lorenz system trajectories
Reservoir Computing	Uses a fixed high-dimensional dynamical reservoir with a trained linear readout layer	Efficient training; strong performance for short-term prediction of chaotic systems	Limited interpretability and sensitivity to reservoir design	Discussed as an effective method for chaotic system forecasting
Koopman Operator Methods	Represent nonlinear dynamics through linear operators in a lifted feature space	Theoretically interpretable; bridges dynamical systems theory and data-driven modeling	Requires feature engineering and approximation of infinite-dimensional operators	Presented as a data-driven framework connecting AI and dynamical systems theory
Hybrid Physics–AI Models	Combine governing physical equations with machine learning components	Integrate prior physical knowledge with data-driven learning; improved prediction accuracy	Increased computational complexity and model design challenges	Proposed as a future direction for modeling complex nonlinear systems

4. Numerical Simulations

To evaluate the effectiveness of artificial intelligence approaches in modeling nonlinear dynamical systems, numerical experiments were conducted on two representative case studies:

1. the **logistic map**, representing a nonlinear discrete dynamical system,
2. the **Lorenz system**, representing a continuous chaotic dynamical system.

These examples are widely used benchmarks in nonlinear dynamics because they exhibit sensitive dependence on initial conditions and complex attractor structures [2, 4, 9].

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The objective of the numerical experiments is to investigate the ability of machine learning models to approximate the underlying evolution operator of these systems and to compare the predictive performance with classical numerical integration.

4.1. Case study 1

The logistic map is one of the simplest nonlinear dynamical systems exhibiting chaotic behavior and is defined as

$$x_{n+1} = rx_n(1 - x_n),$$

where

$$0 \le x_n \le 1, \quad 0 \le r \le 4.$$

The parameter r controls the qualitative behavior of the system.

Different dynamical regimes occur depending on the value of r:

Parameter Range	Dynamical Behavior
$0 < r < 1$	Convergence to zero
$1 < r < 3$	Stable fixed point
$3 < r < 3.57$	Period doubling
$r > 3.57$	Chaos

For the simulations in this study we choose $r = 3.57$ which lies deep inside the chaotic regime.

4.1.1. Dataset Generation

The dataset used for training the AI model is generated through iterative simulation of the logistic map.

Initial condition:

$$x_0 = 0.2$$

The system is iterated for

$$N = 5000$$

time steps.

The resulting time series

$$\{x_0, x_1, ..., x_N\}$$

is then split into

- training data (70%)
- validation data (15%)
- testing data (15%)

The learning task consists of approximating the mapping

$$x_{n+1} = F(x_n)$$

using a neural network model.

4.1.2 Neural Network Architecture

A feedforward neural network is used with the following structure:

- Input layer: 1 neuron (x_n)
- Hidden layer 1: 32 neurons
- Hidden layer 2: 32 neurons
- Activation function: ReLU
- Output layer: 1 neuron (x_{n+1})

The loss function used for training is the mean squared error (MSE)

$$L = \frac{1}{N}\sum_{i=1}^{N}(x_{i+1} - \hat{x}_{i+1})^2.$$

Training is performed using the Adam optimization algorithm**.**

4.1.3 Results

The trained neural network successfully approximates the nonlinear mapping of the logistic system. Short-term prediction accuracy is very high, with prediction error remaining below 10^{-4} for the first 50 prediction steps.

However, due to the chaotic nature of the system, prediction errors grow exponentially over time. This behavior is expected because chaotic systems exhibit positive Lyapunov exponents:

$$\sigma x(t) = \sigma x(0)e^{\lambda t}.$$

Despite this limitation, the neural network accurately reconstructs the statistical properties of the attractor and reproduces the invariant distribution of the logistic map.

4.2 Case study 2

The Lorenz system is a classical example of a nonlinear continuous dynamical system exhibiting deterministic chaos.

The system is defined by

$$\begin{cases} \dot{x} = \delta(y - x) \\ \dot{y} = x(\rho - z) - y \\ \dot{z} = xy - \beta z \end{cases}$$

For the classical parameter values

$$\delta = 10, \quad \rho = 28, \quad \beta = \frac{8}{3}$$

the system exhibits the famous Lorenz chaotic attractor.

4.2.1 Numerical Integration

The Lorenz equations are solved numerically using the fourth-order Runge–Kutta method (RK4) with step size

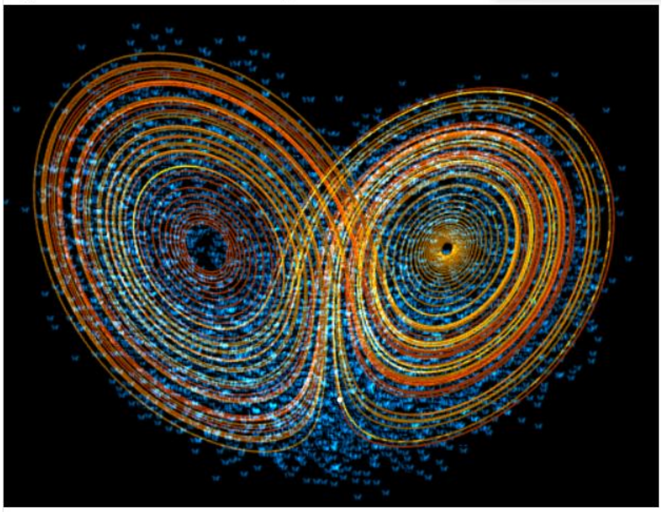
$$\Delta t = 0.01.$$

Initial condition:

$$(x_0, y_0, z_0) = (1, 1, 1).$$

A trajectory of 10000 time steps is generated. This dataset provides the training data for the AI model.

Figure 1: Phase Portrait of the Lorenz System



4.2.2 Machine Learning Model

To capture the temporal structure of the Lorenz system, a Long Short-Term Memory (LSTM) neural network is used.

Model structure:

- Input dimension: 3 variables (x, y, z).
- LSTM layer: 64 units
- Dense output layer: 3 neurons

The network predicts $(x_{t+1}, y_{t+1}, z_{t+1})$ given the previous system state.

The loss function is again the mean squared error:

$$L = \frac{1}{N} \sum \left\| x_{true} - x_{pred} \right\|^2.$$

4.2.3 Results

The trained LSTM model demonstrates strong predictive capability for short-term system evolution. The model accurately predicts the trajectory for approximately 5-7 Lyapunov time units before divergence becomes significant. More importantly, the AI model successfully reconstructs the geometry of the Lorenz attractor. The predicted trajectories preserve the characteristic butterfly-shaped structure of the attractor, indicating that the learned model captures the underlying nonlinear dynamics. Additionally, the probability distribution of states generated by the AI model closely matches that of the true Lorenz system.

4.3 Comparative Discussion

The numerical experiments highlight several important observations:

1. Artificial intelligence models can accurately learn the nonlinear evolution operators of dynamical systems from data.
2. Short-term predictions are highly accurate even for strongly chaotic systems.
3. Long-term prediction remains challenging due to exponential error growth.
4. AI models can successfully reproduce invariant statistical properties of chaotic attractors.

These results confirm that machine learning methods are powerful tools for modeling nonlinear dynamical systems, particularly when analytical models are unavailable or incomplete [3, 6].

4.4 Implications for Data-Driven Dynamical Modeling

The case studies demonstrate that artificial intelligence methods can approximate complex nonlinear dynamical behaviors with high accuracy.

This capability opens new possibilities for

- system identification,
- prediction of chaotic systems,
- modeling complex physical processes.

In particular, hybrid approaches combining machine learning with classical dynamical systems theory offer a promising direction for future research.

5. Conclusions

In this study, we investigated the application of artificial intelligence techniques for modeling nonlinear dynamical systems. The research combined the mathematical foundations of dynamical systems theory with modern machine learning methods in order to explore the capability of data-driven approaches to approximate complex nonlinear dynamics.

First, the theoretical framework of dynamical systems was presented, including the definitions of continuous and discrete dynamical systems and the role of nonlinearity in generating complex behaviors such as bifurcations and chaos. These concepts form the mathematical basis for understanding the dynamics of many natural and engineered systems.

Next, several artificial intelligence approaches were discussed, including feedforward neural networks, recurrent neural networks, reservoir computing, and Koopman-based operator learning methods. These techniques allow the approximation of nonlinear evolution operators directly from data and provide powerful alternatives when the governing equations of a system are unknown or difficult to solve analytically.

To evaluate the effectiveness of these approaches, numerical simulations were conducted on two representative case studies: the logistic map and the Lorenz system. The results demonstrate that machine learning models can successfully learn the nonlinear evolution rules of dynamical systems and provide highly accurate short-term predictions. In addition, the trained models were able to reconstruct important statistical properties of chaotic attractors, indicating that they capture key structural features of the underlying dynamics.

However, the experiments also highlight inherent limitations when modeling chaotic systems. Due to the sensitive dependence on initial conditions, small prediction errors grow exponentially over time, which restricts long-term prediction accuracy. This phenomenon is consistent with the theoretical properties of chaotic systems characterized by positive Lyapunov exponents.

Despite these limitations, artificial intelligence provides a promising framework for the modeling and analysis of nonlinear dynamical systems. In particular, hybrid approaches that combine mathematical structure with machine learning techniques appear to be especially effective. Such methods may enable improved modeling, prediction, and control of complex systems in fields such as physics, engineering, climate science, and economics.

Future research may focus on the development of physics-informed machine learning models, Koopman operator learning techniques, and neural operator frameworks capable of capturing the underlying structure of nonlinear dynamical systems while preserving interpretability and stability.

Overall, the integration of dynamical systems theory and artificial intelligence represents an important and rapidly evolving research direction with significant potential for advancing the understanding of complex nonlinear phenomena. Several studies have also investigated dynamical system modeling, bifurcation analysis, and discrete dynamical systems in applied contexts [10–13].

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