

APPLICATION OF MATHEMATICAL CRITERIA FROM THE OPERATIONS RESEARCH THEORY IN MANAGEMENT DECISION MAKING

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An introduction

The decision is the core of the management - it is its main characteristic. Without management decision, there is no real management. Many authors see the decision making as the most important management function. Moreover - in the literature there is even the so called "School of the decision", whose adherents not only say that decision making is the most important act in the management, but that the process of management is limited to the process of making decisions.

Essentially the management is a basic tool to effect and system of activities, related to:

- Alteration in the behavior of people and teams;
- Amendment in the organizational structure of manufacture and the management;
- Reallocation of resources;
- Modification of the conditions for the system functioning, as well the relations between its elements.

The management decision has a social purpose and consequences. It represents the choice of a possible way to achieve the objective under certain conditions for its implementation. It should not be seen as something accidental, arbitrary. Prerequisite for its acceptance always is the presence of a problem, ie discrepancy between desired and actual state of the controlled object, which is an obstacle to its effective functioning and development.

Each issue arises and exists in a particular environment under the influence of various factors - internal and external. This is essentially problem situation. Deciding on the problem situation provides steady state of controlled object.

In order to take management decisions the following four conditions should be satisfied:

1. Existence of a problematic situation;
2. Alternativeness (eligibility) in the outcome of this situation.
3. Be a governing body or a person who is authorized to make a decision.
4. Existence of set of criteria to make an optimal choice.

Managerial decision-making, as essential part of the management process, has informational nature. Decision-making is made and accepted as a result of the complex work with the information, i.e. the decision-making process depends on the organization informational status. The informational status reflects the subject awareness level in the process of decision making.

The process of decision-making has an intuitive, logical or rational nature. The intuitive decisions have several disadvantages [1]:

- They are not methodically prepared, they are improvised;
- They are too subjective, based on personal experience, acumen and talent of the director, his logic and inner conviction;
- Are not sufficiently motivated;
- Development is based on limited and often unverified information;
- The used information concerns the past period and the same can not be directly applied in the future, because the management situations are unique;

Despite the shortcomings of intuitive decisions we must consider the reasons why the share of these solutions is too high.

Making intuitive decisions is not preferable, but it would be justified in at least two cases:

- a) When the time available is not enough to study all the relevant information and the problem is urgent, it is better to make a decision, than to do nothing;
- b) Where it is considered that the potential losses from one intuitive solution are less than the cost of a scientifically justified decision.

Decision based on logic is a choice, driven by knowledge or experience of those who take them. For this type of decisions a person uses knowledge gained from situations occurred earlier. He predicts the results of the alternatives in the current new situation. Based on common sense, chose an alternative that has brought success in the past. In complex situations, the reasoning of the head may be insufficient. In this sense, the head may consciously or unconsciously not use the new features, new ideas into new areas.

Rational decisions are different from the previous two in that they do not depend on intuition and past experience of the head. For a good managerial decision there are a number of requirements:

- Should be scientifically reasoned;
- Clear and precise objectives, activities, resourcing of decisions should be formulated;
- There should be many alternatives;
- Freedom in decision making by managers;
- Does not contradict the principle of delegation of authority;
- And others.

1. Models and methods for decision making

The decision making is necessary for the implementation of the company management functions. The process of decision-making is a central element in the management theory. The decision methods and models support the managers to make informed and objective decisions in a complex environment.

Decision making models enables the managers to develop scenarios for the company future and to determine the potential consequences of alternative decisions. When the manager wants to compare various alternatives and to choose the optimum, the modeling is the one of the best suitable approaches [2, 5].

Complex systems, processes and tasks are difficult to handle without disclosure, clarification and resolution of the conflicts. In order to solve conflicts the same should be described and further look for a solution compromise. The corresponding class models describe how to resolve conflicts using the game theory. As group of models they occupied considerable place in the modern science and practice.

Some experts believe that the compromises have a place in all situations involving decision making. Others have argued that there shouldn't be conflicts in some cases, for example between the development of technology and the human individual improvement. Making appropriate decisions in the company's managing activities is closely related to some key features of the decisions, such as:

- Selecting objectives and goals;
- Considering the right planning horizon;
- Taking into account the dynamics of the system;
- Assuring self-consistency;
- Selection of certain criteria;
- Correct calculation of the cash-flows;
- Considering the general economic effect.

Decisions have different levels and failure to consider their hierarchy may lead to serious errors. The hierarchy of decisions

should be consistent with the hierarchy of the system structure, i.e. decisions at the highest level are the responsibility of the manager of the system. Second-level decisions should be taken only by deputy heads, etc. As a rule, any manager who deals with all matters, ignoring the jurisdiction, is doomed to failure. His main task should be to bring order in the control subsystem of the company.

2. Means supporting processes of knowledge use

Some of the most popular tools and technologies which allow efficient use and knowledge search include [2]:

- **Business Intelligence (BI) Tools** are among the most advanced software solutions that enable the users to process large volumes of complex data. The main purpose of these technology solutions is to automate and integrate as many steps and functions to cover more data for analytical processing.
- **Expert Systems (ES)** are computer programs that similar to people can find different decisions and who can offer specific advice or answers to the user. There are various expert systems and they can be distinguished according to their scope. There are expert systems to interpret data and signals, ES to diagnose, ES for design, planning and scheduling, for interpretation, for decision support and control of complex systems or ES to support the education.
- **Decision support systems** These are software technologies that enable managers and experts in decision making by examining and processing stored data. These are means supporting decision making using various techniques such as best practices, expert systems, simulations, tools for statistical analysis. They are usually based on the processing of information stored in corporate databases.
- **Simulation based systems.** These are computer programs that mimic reality through simulation of complex processes. Simulation is an excellent tool for the study of scenarios of "what-if" type in an interactive format, as they can present complex processes in an easy to understand manner. "Serious games" can be seen as a kind of simulation, acquisition of knowledge and the opportunity to gain experience.
- **Visualization Tools** help to remember and understand complex structures of knowledge.
- **Enterprise Resource Planning (ERP), Enterprise Resource Managing (ERM) and Customer Relationship Management (CRM)** - These are

applications that assist the management process with important knowledge about the organization, the employees, suppliers or the customers. They allow structuring the main assets of knowledge in the organization in the knowledge process concept. One of the main challenges facing the packages is their ability to integrate many different types of knowledge and represent them to the users in a meaningful way.

3. Using Excel for decision making

One of the main activities related to decision-making in each firm is the investment process. Investing is a process for assessing, analyzing, comparing and selecting investment projects in order to maximize the income or the value of the company (industry). One of the main steps of investment is the valuation of the investment projects, which could be seen from different points of view. The wide range and high level of generality of the economic indicators determines the importance of the economic aspect of the evaluation in the prioritization of the investment projects. In a competitive economy, discounting and considering the risk is necessary modernization of the existing methodologies to assess the economic efficiency of the performance of an economic project. It is desirable that such methodologies are supported by the use of appropriate software, such as MS Excel, which would facilitate their practical use. Evaluation of long term investment projects is usually associated with assessing the credibility of unique, non-recurring and non-stationary processes [6]. This complicates the use of the classical probabilistic methods for uncertainty assessment. To evaluate the reliability of the unique events in recent years mainly two groups of methods have been developed: based on "subjective probability" and the term "fuzzy set" [7].

Even if it is possible in some way to estimate the risk of an investment, in the general case, this problem is solved in terms of uncertainty. In all such cases, the conditions for one or another outcome in given situation depend on the objective reality, which in the decision theory is called "nature". Such situation is called "games with Nature". "Nature" in the theory of statistical decisions is seen as disinterested player whose "behavior" is unknown, but it does not contain elements of consciously counter our plans.

Consider the following situation: Let's have M possible strategies $A_1, A_2, A_3 \dots A_M$ [3, 4] and n possible states of the environment $P_1, P_2, P_3 \dots P_N$, which we call "strategies of the nature". Our result a_{ij} for each pair of strategies A_i, P_j is given by the matrix shown in Table

Table 1

	P_1	P_2	P_3	P_4	P_5	P_n
A_1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{1n}
A_2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{2n}
.....
A_m	a_{m1}	a_{m2}	a_{m3}	a_{m4}	a_{m5}	a_{mn}

The objective is to choose a strategy for the player A, which is the most beneficial. There are various criteria known from the operations research how to define the best strategy. The author of this paper has chosen three of them.

1. Wald's Maximin Criterion

The decision-theoretic view of statistics advanced by Wald had an obvious interpretation in terms of decision-making under complete ignorance, in which the maximin strategy was shown to

be a best response against nature's minimax strategy. Wald's criterion is extremely conservative even in a context of complete ignorance, though ultra-conservatism may sometimes make good sense (Wen and Iwamura, 2008). The Maximin criterion is a pessimistic approach. It suggests that the decision maker examines only the minimum payoffs of alternatives and chooses the alternative whose outcome is the least bad. This criterion appeals to the cautious decision maker who seeks insurance that in the event of an unfavorable outcome, there is at least a known minimum payoff.

This approach may be justified because the minimum payoffs may have a higher probability of occurrence or the lowest payoff may lead to an extremely unfavorable outcome [8]

2. Savage's Minimax Regret

The Savage Minimax Regret criterion examines the regret, opportunity cost or loss resulting when a particular situation occurs and the payoff of the selected alternative is smaller than the payoff that could have been attained with that particular situation. The regret corresponding to a particular payoff X_{ij} is defined as $R_{ij} = X_j(\max) - X_{ij}$ where $X_j(\max)$ is the maximum payoff attainable under the situation S_j . This definition of regret allows the decision maker to transform the payoff matrix into a regret matrix. The minimax criterion suggests that the decision maker looks at the maximum regret of each strategy and selects the one with the smallest value. This approach appeals to cautious decision makers who want to ensure that the selected alternative does well when compared to other alternatives regardless of the situation arising. It is particularly attractive to a decision maker who knows that several competitors face identical or similar circumstances and who is aware that the decision maker's performance will be evaluated in relation to the competitors. This criterion is applied to the same decision situation and transforms the payoff matrix into a regret matrix.

The Minimax Regret criterion focuses on avoiding the worst possible consequences that could result when making a decision. Although regret is an emotional state (a psychological sense of loss) which, being subjective can be problematic to assess accurately, the assumption is made that regret is quantifiable in direct (linear) relation to the rewards R_{ij} expressed in the payoff matrix. This means that an actual loss of, say, an euro (an accounting loss) will be valued exactly the same as a failure to take advantage of the opportunity to gain an additional euro (an opportunity loss, which is disregarded in financial accounting). In other words, the Minimax Regret criterion views actual losses and missed opportunities as equally comparable.

Regret is defined as the opportunity loss to the decision maker if action alternative A_i is chosen and state of nature S_j happens to occur. Opportunity loss (OL) is the payoff difference between the best possible outcome under S_j and the actual outcome resulting from choosing A_i given that S_j occurs. Thus, if the decision alternative secures the best possible payoff for a given state of nature, the opportunity loss is defined to be zero. Otherwise, the opportunity loss will be a positive quantity. Negative opportunity losses are not defined. Savage's Minimax Regret criterion is formally defined as:

$OL_{ij} = (\text{column } j \text{ maximum payoff}) - R_{ij}$ - for positive-flow payoffs (profits, income)

$OL_{ij} = R_{ij} - (\text{column } j \text{ minimum payoff})$ - for negative-flow payoffs (costs)

where R_{ij} is the payoff (reward) for row i and column j of the payoff matrix R .

Opportunity losses are defined as nonnegative numbers. The best possible OL is zero (no regret), and the higher OL value, the greater the regret.

Minimax Regret decision rule is defined as:

1. Convert the payoff matrix $R = \{ R_{ij} \}$ into an opportunity loss matrix $OL = \{ OL_{ij} \}$.

2. Apply the minimax rule to the OL matrix.

3. Hurwitz's Optimism – Pessimism Criterion

The most well-known criterion is the Hurwitz criterion, suggested by Leonid Hurwitz in 1951, which selects the minimum and the maximum payoff to each given action x . The Hurwitz criterion attempts to find a middle ground between the extremes posed by the optimist and pessimist criteria. Instead of assuming total optimism or pessimism, Hurwitz incorporates a measure of both by assigning a certain percentage weight to optimism and the balance to pessimism. However, this approach attempts to strike a balance between the maximax and maximin criteria. It suggests that the minimum and maximum of each strategy should be averaged using a and $1 - a$ as weights. a represents the index of pessimism and the alternative with the highest average selected. The index a reflects the decision maker's attitude towards risk taking. A cautious decision maker will set $a = 1$ which reduces the Hurwitz criterion to the maximin criterion. An adventurous decision maker will set $a = 0$ which reduces the Hurwitz criterion to the maximax criterion.

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A weighted average can be computed for every action alternative with an alpha-weight α , called the coefficient of realism. "Realism" here means that the unbridled optimism of Maximax is replaced by an attenuated optimism as denoted by the α . Note that $0 \leq \alpha \leq 1$. Thus, a better name for the coefficient of realism is coefficient of optimism. An $\alpha = 1$ denotes absolute optimism (Maximax) while an $\alpha = 0$ indicates absolute pessimism (Maximin). The α is selected subjectively by the decision maker.

Selecting a value for α simultaneously produces a coefficient of pessimism $1 - \alpha$, which reflects the decision maker's aversion to risk. A Hurwitz weighted average H can now be computed for every action alternative A_i in A as follows:

$H(A_i) = \alpha (\text{row maximum}) + (1 - \alpha) (\text{row minimum})$ - for positive-flow payoffs (profits, revenues)

$H(A_i) = \alpha (\text{row minimum}) + (1 - \alpha) (\text{row maximum})$ - for negative-flow payoffs (costs, losses)

Hurwitz decision rule is followed:

1. Select a coefficient of optimism value α .

2. For every action alternative compute its Hurwitz weighted average H .

3. Choose the action alternative with the best H as the chosen decision ("Best" means $\text{Max} \{H\}$ for positive-flow payoffs, and $\text{Min} \{H\}$ for negative-flow payoffs).

4. Example

In the presented work a specific example is discussed. Using MS Excel some of the basic financial parameters are calculated and with the above stated criteria for decision-making are arranged projects, which are proposed for selection [1, 5].

Example: Five investment projects A, B, C, D and E with the corresponding income and expenses are offered to an electricity company. How can we order them according to their attractiveness if we consider period of 13 years and 14% discount rate? Net Present Value – NPV and Index Probability – IP have been calculated to make the selection. The input data and the results of calculations (in million) are shown in the table below (Table 2, Table. 3).

VALUATIONS OF THE PROJECT - EXAMPLE

Table 2

	Option A million euro	Option B million euro	Option C million euro	Option D million euro	Option E million euro
Capital costs	15	17	18	14	16,5
Annual operating costs					
Fuel	4,2	4,2	4,2	4,2	4,2
Personnel	1,6	1,5	1,4	1,7	1,2
Operation & Maintenance	0,6	0,5	0,6	0,4	0,3
Overheads	1,0	1,0	1,0	1,0	1,0
Total annual cost	7,4	7,2	7,3	6,7	6,7
Annual income	10,0	10,1	10,4	9,8	9,6

Table 3

	Option A million euro	Option B million euro	Option C million euro	Option D million euro	Option E million euro
Annual income	10,0	10,1	10,4	9,8	9,6
Annual operating costs	7,4	7,2	7,3	6,7	6,7
Annual profit from operating activities	2,6	2,9	3,2	2,5	2,3
Present value 13 years (rate14%)	15,19	17,00	18,7	14,61	13,44
Capital costs	15	17	18	14	16,5
Net Present Value	0,19	0,00	0,7	0,61	-3,06

Table 3 shows that Project E should be skipped, about Project B we should be indifferent, and the remaining projects are ordered, according to their attractiveness, as follows C, D and A .

However, if there is a limit on the initial investment, the above shown table will change. Assume investment limit of 32 million. For the selection of possible project groups according their attractiveness the following steps were executed:

1. All possible groups of projects with investment sum within the limit were drawn. Of the 31 possible groups of projects, the program leaves 8 for the further analysis. For those clusters of projects NPV and IP was calculated;
2. The obtained eight portfolios were analyzed using the method, "Monte Carlo" (implemented in Excel) []. As result we get randomly distributed NPVs. The simulation is run 7000 times. The distribution of the input quantity "Annual profit from operating activities"

is considered Normal (Gaussian), with expectation equal to the input value (the probability of reaching it equals to 50%, for option A for example this is 2.6 million)

3. An evaluation matrix containing the following absolute values is created for each portfolio
 - The minimum value of NPV;
 - The maximum value of NPV;
 - The probability that NPV is positive;
 - The index of profitability.

The first three are calculated using the "Monte Carlo" method and are represented in (Table 4). The values in the matrix are used as ranking criteria.
4. Creating evaluation matrices with the scores according to the criteria Wald (Table 5), Savage (Table 6) and Hurwitz (Table 7).
5. Output of the final ranking is shown in Table 8.

Table 4

Evaluation matrix (absolute numbers)					
Nº	Option	NPVmin	NPVmax	NPV>0	IP
1	A	-20,105	22,03	53,8	1,013
2	Б	-21,08	21,05	51,9	1
3	B	-19,06	23,08	56,6	1,039
4	Г	-19,78	22,35	54,6	1,044
5	A+Б	-17,55	24,59	58,5	1,004
6	A+Г	-17,28	24,85	62,8	1,03
7	Б+Г	-17,21	24,93	63	1,018
8	Б+Г	-16,8	25,95	67	1,041

Table 5

Evaluation Matrix (balls)					
Wald					
№	Option	NPVmin	NPVmax	NPV>0	IP
1	A	2	1	1	3
2	Б	0	0	0	0
3	B	3	3	2	9
4	Г	2	2	1	10
5	A+Б	6	5	3	1
6	A+Г	6	5	5	7
7	Б+Г	6	6	5	4
8	B+Г	7	7	7	9

Table 6

Evaluation Matrix (balls)					
Savage					
№	Option	NPVmin	NPVmax	NPV>0	IP
1	A	5	6	6	7
2	Б	7	7	7	10
3	B	4	4	5	1
4	Г	5	5	6	0
5	A+Б	1	2	4	9
6	A+Г	1	2	2	3
7	Б+Г	1	1	2	6
8	B+Г	0	0	0	1

Table 7

Evaluation Matrix (balls)											
Hurwitz for different α											
№	Option	$\alpha=0,025$	$\alpha=0,05$	$\alpha=0,1$	$\alpha=0,3$	$\alpha=0,4$	$\alpha=0,5$	$\alpha=0,6$	$\alpha=0,7$	$\alpha=0,9$	$\alpha=0,95$
1	A	0,93	0,98	1,09	1,50	1,71	1,92	2,13	2,33	2,75	2,85
2	Б	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
3	B	2,35	2,51	2,85	4,18	4,85	5,52	6,19	6,86	8,20	8,53
4	Г	1,47	1,69	2,13	3,88	4,75	5,63	6,50	7,38	9,13	9,56
5	A+Б	1,03	1,15	1,40	2,37	2,85	3,34	3,83	4,31	5,29	5,53
6	A+Г	5,10	5,14	5,23	5,58	5,76	5,94	6,11	6,29	6,64	6,73
7	Б+Г	4,15	4,20	4,31	4,76	4,99	5,21	5,43	5,66	6,11	6,22
8	B+Г	7,06	7,12	7,23	7,70	7,93	8,16	8,39	8,62	9,09	9,20

The final ranking is shown in table 8 where the economical evaluation criteria used in table 3 are also considered. It is obvious that in the top five one and the same projects taking place and the packet B+G is the leading packet.

In this test program in Table 8 shows the final ranking, and is included in ranking if used method of economic evaluation used in the table 3. It is seen that in the first five involved the same projects as the first place beckons for package C + D.

Table 8

FINALLY RANKING

№			CRITERIA				
			Wald	Savage	Hurwitz		
	NPV	IP			$\alpha=0,4$	$\alpha=0,5$	$\alpha=0,6$
1	B+Г	Г	B+Г	B+Г	B+Г	B+Г	B+Г
2	A+Г	B+Г	A+Г	A+Г	A+Г	A+Г	Г
3	B	B	B+Г	B	B+Г	Г	B
4	Г	A+Г	B	Г	B	B	A+Г
5	B+Г	B+Г	Г	B+Г	Г	B+Г	B+Г
6	A	A	A+Б	A	A+Б	A+Б	A
7	A+Б	A+Б	A	A+Б	A	A	A+Б
8	Б	Б	Б	Б	Б	Б	Б

Conclusion

Following the theoretical analysis and the obtained results from the example, it can be concluded that the proposed method for ranking projects and decision-making in the selection of investment projects, in a good degree adapt existing methodologies (the closeness of the results obtained by different criteria). On the other hand, if the decision maker prefers to take account of risk by adding a risk premium to the discount rate or required rate of return in Excel and have the opportunity to do so. Also, the decision maker has the choice whether to use the Software Product only to obtain random values of economic indicators alone and make the final selection or use of such information for prior ranking of portfolios.

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