ON EXPERIENCE OF MODELING OF HYDRODYNAMIC PROBLEM USING COMBINED NEURAL NETWORK AND DYNAMIC MODEL

C.Sc. Doc. Yasinskiy I.
Ivanovo State Power University – Ivanovo, Russia
e-mail: igor2266@yandex.ru

Abstract: *Solution of gas-dynamics problems containing different fine structures (shock waves, boundary layers, traces, jets, etc.) as well as areas within which the flows are described by different models of turbulence by traditional grid methods is quiet difficult. These tasks are encouraged to address, using neural network technology in the analysis and grid-analytical form by stochastic optimization. The proposed method is demonstrated on the problems of mixing jets. Numerical experiments have shown several advantages of this approach: reducing the cost of computer time, good accuracy, the connection of patchy pattern of flow in a single unit.*

Keywords: turbulence model, random search, neural network interpolation, combining simulation and dynamic models, non-grid methods for field tasks.

1. Introduction

The use of neural network technology can provide substantial assistance in solving the problems of continuum mechanics, containing different fine structures: boundary layers, shock waves, traces, separation points, the coverage of different turbulence models [1].

The essence of the proposed computing technology is convenient to demonstrate on the problems of mixing of supersonic streams. Such problems arise, for example, in developing high-power gas lasers stirring [2]. On fig.1, 2 the schemes are shown with the longitudinal and cross-reactive blowing environments and their respective designs. Obviously, the flow patterns are very complex. They include various fine structures of shock waves, boundary layers and their interaction fields. In addition, you can specify the subfields in which various semi-empirical turbulence models are preferred.

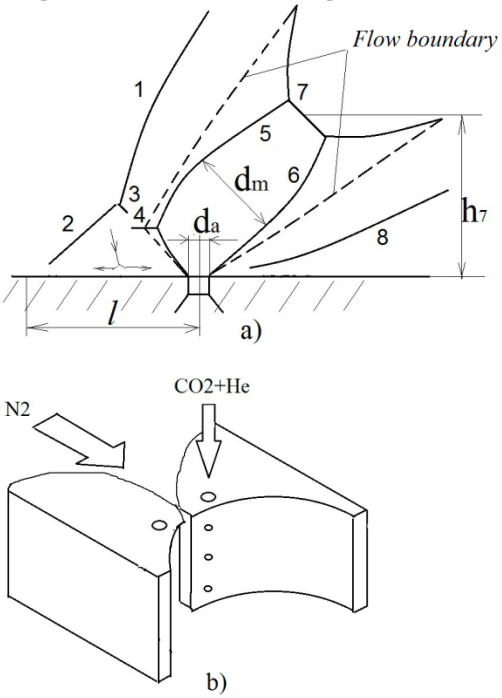

Fig.1. *Cross blowing of the reacting environments: a – scheme; b – constructive solution*
1 – main jump; 2 – skew jump; 3 – closer jump; 4 – jump in place of abrupt break; 5, 6 – barrel jump (inside flow); 7 – central jump (Mah's disk); 8 – jump, formed by the flow, gone around jet, inleaking on the wall; l – length of the separation zone; h_7 – distance to the Mah's disk center; d_m – diameter of the barrel jump; d_a – diameter of the injection hole.

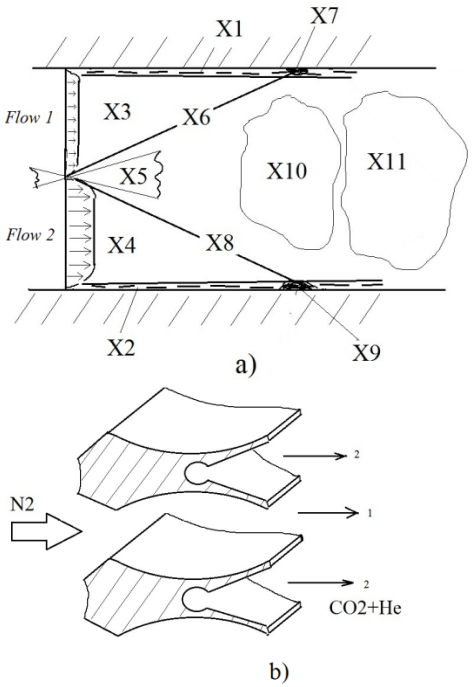

Fig. 2. *Longitudinal blowing of the reacting environments: a – scheme; b – constructive solution*
X1, X2 – turbulent boundary layers; X3,X4 – flows, entering the mixing chamber; X5 – boundary layer between the flows; X6, X8 – shock waves; X7, X9 – fields of the shock-wave interaction with turbulent boundary layers; X10 – field of the K-E turbulence model; X11 – field of A.N. Sekundov's model.

2. Hydrodynamic problem

As is well known, solution of the specified problems by means of purely grid methods is difficult. Indeed, the turbulent viscosity near the solid walls conveniently calculated using formulas of Prandtl and Spalding (1). Hereinafter A. Einstein's record of the field differential equations generally accepted:

1) index after the decimal point indicates the derivative on the corresponding coordinate.

2) the summation is performed according to the twice repeated index, although the sign of the sum is not specified.

(1) $$\nu_t = l^2\sqrt{(u_{1,2})^2 + (u_{2,1})^2}\,,$$

where l – the length of the mixing path

$$l = k \cdot L_{min},$$

$k=0,41$ – Karman constant, $Lmin$ – the shortest distance to the nearest hard surface.

It is also not difficult to find ν_t in the initial sections of flows.

But in the area of the mixing flows we have to use either the K-E model (2):

$$\begin{gathered}\frac{\partial K}{\partial t}+u_i K_{,j}=\left(\frac{\nu}{\sigma_K}K_{,j}\right)_{,j}+S_K,\\ \frac{\partial E}{\partial t}+u_i E_{,j}=\left(\frac{\nu}{\sigma_E}E_{,j}\right)_{,j}+S_E,\\ S_K=\nu_m D-E,\ S_E=(C_1 D-C_2 E)\cdot E/K,\\ D=(u_{i,j}(u_{i,j}+u_{j,i}))^{1/2},\\ \nu\ =\nu_t+\nu_m,\\ \nu_t=\mathrm{C}_\mu K^2/E\ ,\end{gathered} \tag{2}$$

here ν_m, ν_t, ν - molecular, turbulent and full kinematic viscosity, K, E – kinetic energy of fluctuations and their dissipation rate, σ_K, σ_E, C_1, C_2, C_μ - semi-empirical constants,

either model of G.N. Abramovich and A.N. Sekundov is applied (3):

$$\begin{gathered}\frac{\partial \nu_t}{\partial t}+u_j\nu_{t,j}=\left((\nu_m+\varkappa\nu_t)\nu_{t,j}\right)_{,j}+\nu_t\cdot f\left(\frac{\nu_t}{8\nu_m}\right)\cdot D-\\ -\gamma\cdot L_{min}^{-2}(\nu_m+\beta\nu_t)\cdot\nu_t\\ f(z)=0{,}2\,\frac{z^2+1{,}47z+0{,}2}{z^2-1{,}47z+1}\ ,\end{gathered} \tag{3}$$

$\varkappa, \gamma, \beta$ - empirical constants.

The most difficult area is the interaction of shock waves with turbulent boundary layers. It is proposed to present such a motley picture of flows in one piece with the following neural network technology.

3. Neural network synthesis

Let us select the areas Xs; S = 1,2,…,N, each of which is characterized by its apparent specificity (shock wave, turbulence of a certain type, flow boundary layer, shock wave and boundary layer interaction zone and so on). Each of the mentioned fields Xs is fixed by the plurality of signal points $\overrightarrow{X_{sr}}; r=1,2,\dots,N_s$; N_s - the number of signal points in the field Xs. For each field Xs the potential of Braverman is introduced (4):

$$\varphi_S(\vec{x})=\sum_{r=1}^{Ns}\left\|\vec{x}-\vec{X}_{sr}\right\|^{-m_s}\ , \tag{4}$$

m_s – positive value. $m_s>1$

If a random point $\vec{x}$ falls in the field Xs or approaches it, then the potential φ_S increases or takes the greatest value. We need to introduce some small constant protection from zero in the denominator in (4). Its neural function is introduced for each area Xs, for example, such (5):

$$g_S(\vec{x})=\frac{1-\exp(-\alpha_s\varphi_s(\vec{x}))}{1+\exp(-\alpha_s\varphi_s(\vec{x}))}\ , \tag{5}$$

which refers to the number closest to the one in the field Xs and tends to zero outside it.

The transition rapidity of $g_S(\vec{x})$ from one to zero is regulated by constants m_s, α_s .

Now turbulent viscosity can be calculated throughout the study area as (6):

$$\nu_t=\sum_{S=1}^{N}g_S\ \cdot\nu_{tS}\ . \tag{6}$$

Neural interpolation will fill ν_t value for those areas for which there is currently no corresponding semi-empirical model. A similar technique can easily be extended to determine the position of the shock waves, satisfying the boundary conditions, etc.

Since the solutions are presented in an analytical form, then there is no need to split the grid steps in the field of sharp transitions. This is achieved by using the properties of neural functions g_S. Let the equations of gas dynamics system be presented (7):

$$F\left(\vec{x},U(\vec{x})\right)=0\ , \tag{7}$$

F – some linear operators, $U(\vec{x})$ - the required fields, $\vec{x}$ - point of the real space field Ω, where the process is considered.

Equations (7) are joined by the boundary conditions (8):

$$f(U)|_\Gamma=f_\Gamma(x^/)\ , \tag{8}$$

Γ – boundary of Ω, $x^/\in\Gamma$, f – boundary operators.

We seek a solution in the form of constructions (9):

$$U(\vec{x})=\Phi(\vec{x},A\ )=\sum_{s\in\Omega}{}^{/}g_s(\vec{x},\alpha_s,m_s)U_s(\vec{x},\beta\ ), \tag{9}$$

here U_s - special solutions for different subdomains X_s, $A\equiv(\alpha_s,m_s,\beta)$ - free constant, from the selection of which depends the approximation quality. U_s can be both analytical and grid approximations. Желательно при конструировании $\Phi(\vec{x},A\ )$ удовлетворить граничные условия. It is desirable to satisfy the boundary conditions in the design of $\Phi(\vec{x},A\ )$. This can be done by means of already mentioned neural functions g_s. In the region Ω the system of control nodes is selected. Each node represents a basic point, surrounded by a system of additional. It is possible to calculate the values of the fields U_s in each node at random, but specific parameters A and find all the partial derivatives by means of the difference method.

It should be noted that one should not equate such a calculation of derivatives with the traditional grid method. The control nodes may be spaced apart by a substantial distance, and auxiliary points are concentrated near the main central, that provides high accuracy and does not affect the stability of the calculations. The discrepancies *F* for equations are calculated in control nodes for any *A* (10):

$$F_i\left(\vec{x}_j,A\ \right)=F\ \ \left(\vec{x}_j,\Phi(\vec{x}_j,\mathrm{A})\ \right),i=1,2,\dots,N. \tag{10}$$

Of these the proximity measure *Q(A)* of the approximate solution to the exact is constructed. For example, in such form(11):

$$Q(A\ )=\sum_{i=1}^{\bar{N}}\ \ \sum_{j=1}^{M}\ \ \gamma_i\left|F(\vec{x}_j,\mathrm{A})\right|^{p_i}\ , \tag{11}$$

where $p_i>0$; $\bar{N}$ – the number of discrepancies; γ_i – their weights; M – the number of control nodes, their number is considerably smaller than the number of nodes in the grid method.

Do other designs are possible to measure proximity. For optimal values *A* such values of *A** are taken, which deliver a minimum of the objective function *Q(A)*. As a tool of minimization the random search is used in the form, presented in [3]. An important advantage of random search in the ability to get out of local minimum and its insensitivity to non-smooth surface *Q(A)*.

Comment.

To determine the position of the shock waves and the zones of their interaction with turbulent boundary layers is convenient to take the continuity equation in the following form (12):

$$\dot{\rho}+(\rho u_i)_{,i}=\sigma\cdot\rho_{,ii}\ , \tag{12}$$

here σ - constant, allowing to change the shock wave, increasing its thickness. The resulting field ρ is then approximated by the surface *Y(x)* with the appropriate neuro constructions. During the iterations parameter σ decreases, and the shock wave sharpens. With such an arrangement is possible to model the zone of the shock wave interaction with a turbulent boundary layer. With

neuro approximation the boundary conditions are satisfied automatically. If this fails, then the measure of closeness Q is added by the penalty function.

4. Conclusion

The proposed method of solution of gas- and hydrodynamic problems, the foundation of which was laid in the works of Academician G.I. Petrov and his school, can be extended to a wide range of applications, where traditional grid approaches face major difficulties. Development of the system, consisting of neural network and dynamic model looks promising. Combination of dynamic model and neural network will allow to study physical processes more fully and quickly, to determine the unknown hidden parameters and initial conditions. These capabilities allow to apply the proposed method in many application areas.

5. References

[1] Golovko V.A. Neural networks: training, organization and use of, M., Publishing House of the magazine "Radiotehnica", 2001, 256 p.

[2] Solouhin R.I., Fomin N.A. Gas-dynamic lasers mixing, Publishing House «Nauka I tehnica», Minsk, 1984, 148 p.

[3] Yasinskiy I.F, Yasinskiy F.N. Improved random search// Vestnik IGEU-2004.-№3.-P.47-48.

[4] Kontorovic L.V., Krilov V.I. Approximate methods of higher analysis, State Publishing House of the phys.-math. Lit-re, M.:1962, 708 p.