INDUSTRIAL MATHEMATICS – PHASE TRANSITIONS, SCATERING, STRUCTURES

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Abstract: *Presents the basics of methodology to use the full knowledge of micro-producer (personality, micro-foundry) based on: methodology of mathematics, foundations of mathematics and mathematical physics. Example - phase transition of the first order of Stefan's problems, scattering connection with new structures.*

Key words: *phase transition of firs order, Stefan's problems and scattering connection with new structures.*

1. Introduction – Mathematics in Industrial Sub branch Machine building – Casting and Heat treatment

The processes of structures formation are: phase transitions of I^{-st} order (casting) and II^{-nd} order (heat treatment); elastic and plastic deformation. The type of these structures is polycrystalline grains with size of macro-scale to 1μm or micro-scale below 1μm under a lattice parameter in Å. The structure of all materials is a winner of its properties and the based interest of industries is: 1. improving the working properties of known materials; 2. creating new materials by structures design.

Mathematical description of the phase transitions [1÷8, 38 and 40] presented by: the theory of thermal conductivity with the tasks of Stefan and Stefan - Schwartz; the fundamental equation of the formation of new phases of Kashchiev.

On the Fig.1 we introduced castings technologies with different velocity of solidification and science support of micro-foundry

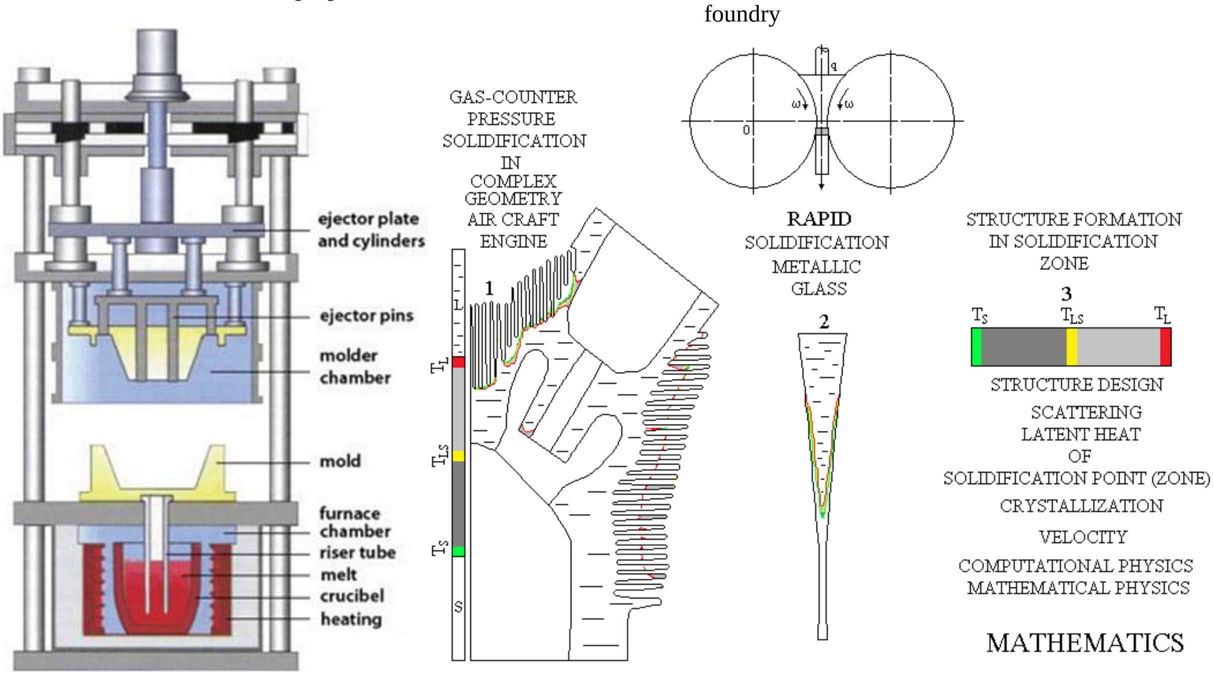

Fig. 1 *Industrial mathematics micro-foundry. Solidification process in temperature zone and character temperatures (T_S, T_{LS} and T_L, where solid S and liquid L): 1 air craft engine with complex geometry; 2 rapid solidification metallic glass; 3 scattering of latent heat of phase transition in solidification point (zone) – structure design, crystallization, velocity, computational physics, mathematical physics, fundamental and applied mathematics.*

Electronic structure is origin of all properties of the metals and its alloys. For description the structures and properties of the metals and alloys are approach mathematics and mathematical physics. It will not be wrong to say that we need a nearly full knowledge i.e. tasks Stefan and Stefan-Schwarz must include mathematics and mathematical physics. This is done through mathematical tasks bridges between mathematical fields and quantum mechanics.

An example of such approach is the use of the results of the theory of scattering. In paper [2] is developed classical Stefan's problem.

In Institute of Mathematics of Bulgarian academy of science are obtained great results about mathematical theory of scattering from Veselin Petkov [10] and Vladimir Georgiev [11].

The first base problem of scattering theory is proving existence of scattering operator. The second direction of development is inverse problem.

It is well known, that thermodynamics driving force of the nucleation at phase transitions of 1^{-st} order (crystallization: liquid (L) ↔ solid (S)) is $\Delta\mu = \mu_L - \mu_S = Q_L ln(T_m/T)$, where $\mu_{L(S)}$ are thermodynamics potentials of base (matter) and new phases; T_m and T are temperatures of transition and supercooling of the base (L) phase; Q_L – latent heat of melting. The supercooling is $ln(T_m/T) = \Delta T_r + \Delta T_k + \Delta T_S$, where ΔT_r – supercooling effect radius of curvature, ΔT_k – supercooling for the transfer of atoms in the interface of liquid-solid phase; ΔT_S – supercooled liquid phase, where **scattering** heat of the phase transition. Received development task of Stephen with scatter theory [2] and assessments of supercooled melt (ΔT_S = 0, 1, 2, ..., max K) in charge of literature [9]. Here mast many investigations – measurements and theoretically; but our opinion, that ΔT_S is a bridge to Quantum mechanics.

Aim of this papere is to show methodological need to use the full modern knowledge required by the industry examples of casting and heat treatment.

2. Mathematics – Philosophy and Foundations of Mathematics

Philosophy is a science with a systematic approach and rational argument, but it does not rely on the scientific method and not strictly defined and accepted subject [17]. Scientific method (generalized) [18]: from Greek origin is "research, teaching, way of knowledge"; today is a system of principles (in development and change) to reach the objective knowledge of reality.

2.1 Philosophy – mine areas and objects research

The origin of the term is of Greek philosophy: φιλοσοφια, from φιλεῖν - love and σοφία – wisdom; and definition: philosophy is the study of general and fundamental questions concerning man and the world [17], with mine areas and objects research.

Fundamentals of philosophy: **Metaphysic** – Nature and Origin of the existing and the world; **Ontology** – Being; **Epistemology** – Knowledge of nature and the possibility of cognitive process; **Ethics** – Morality – how to act human, correct behavior and "good life"; **Political philosophy** – Governance and respect for human and communities to the state; **Aesthetics** – beautiful, sublime, art, pleasure; **Logic** (mathematical and philosophical) – Forms and laws of thinking; valid forms of argumentation; **Philosophy of Language** – Beginning, development, use and attitudes towards thinking; **Scientific methodology** (academic disciplines) – Grounds and subject: science; history; mathematics; physics; psychology; anthropology; etc.

Philosophical achievements and results strongly influence [17]: the development of a given society; educational institutions and practices. The results of philosophy are a way of mediation in their application in academic and scientific disciplines [17]: as the importance of logic in mathematics, linguistics, psychology and computer science; generalized the key role of the philosophy of science through scientific methodology. The philosophy has many branches and one of them is the philosophy of science [17] with branches: philosophy of mathematics; philosophy of physics; philosophy of biology, etc. Philosophy of science is divided and developed very intensive branches in the 19th and 20th centuries, which continues to this day. It is well known that mathematics is separated from philosophy in 1600year after Christ.

2.2 Philosophy of Mathematics holds a special place in the philosophy of science

The philosophy of mathematics – **subject**: studying philosophical assumptions, foundations and implications of mathematics; **aim**: to present the nature and methodology of mathematics to understand the place of mathematics in people's lives [19, 20, 21, 22].

Reasons for the emergence of mathematics are the need for **description of reality** first appeared historically known mathematical knowledge is ~ 2500year before Christ (to new era) [26, 27, 28]. In antiquity it was gradually realized **the abstract nature of mathematical objects**. Mathematics differs from the humanities and natural sciences [19, 20, 21, 22] for example: 1. The objects of study in the natural sciences are located in space and in time, which is not at all clear that applies to objects in mathematics; 2. Methods of study of mathematics very different from the methods of the natural sciences. The use of mathematics in the humanities and natural sciences is due to: there are mathematical properties and regularities of reality, and any object has some mathematical properties. Therefore any theory in natural sciences is mathematized because it contained mathematical knowledge. For example, physics is heavily mathematized.

Explanatory notes on some results on the philosophy of mathematics we present according to works [19, 20, 21, 22]: *Mathematical realism*: there are mathematical structures, regardless of the human mind; *Mathematical anti-realism*: mathematical statements are true values, but they are not corresponding to a specific area of intangible or non-empirical objects; *Mathematical Platonism*: a form of realism, showing that mathematical structures are abstract, have no spatial and temporal or causal properties, and are eternal and immutable; *Platonism of Gödel* posits a special kind of mathematical intuition, allowing to perceive mathematical objects directly; *Full-blooded Platonism* is a modern response to the fact different sets (groups) of mathematical structures can be proven to exist according to the axioms and inference rules employed (for example, the right of excluded middle and the axiom of choice); *Empiricism* is a form of realism that denies that mathematics can be known *a priori* at all; *Mathematical monism* Max Tegmark's hypothesis mathematical universe if there are any mathematical objects, they exist physically. There all mathematical objects, they exist physically; *Logicism* is thesis boils down to mathematics, logic, and hence nothing but a part of logic; *Formalism* argues that mathematical statements may be thought of as statements about the consequences of certain rules of strings; *Conventionalism* convectional, conditional; *Psychologism* is the position that mathematical concepts and/or truths are based on materials derived from or explained by psychological facts (or laws); In mathematics, *intuitionism* is a program of methodological reform whose motto is that "there are no non-experienced mathematical truths" (L.E.J. Brouwer); *Constructivism* includes regulative principle that only mathematical structures that cannot be explicitly constructed in a way to be admitted to mathematical discourse; *Finitism* is an extreme constructivism, according to which a mathematical object does not exist, unless it can be made of natural numbers in a finite number of steps; *Ultrafinitism* is even more extreme version of finitism, who rejected not only infinities, but limited quantities that can actually be constructed with available resources. *Structuralism* is position, considering that mathematical theories describe structures and that mathematical objects are exhaustively defined by their places in such structures, therefore, has no *the intrinsic properties*; *Embodied mind theories* (perfectly realized mind theories) claim that mathematical thinking is a natural product of the cognitive apparatus of the man who is in our physical universe.

Mathematics and mathematical knowledge are used by the methodology of mathematics in a concrete science. This is achieved by applying the philosophy of mathematics in the philosophy of science for specific use of the methodology of mathematics.

Mathematical monism we believe is a heavy time for mathematicians. They require a huge amount of experimental data and described by mathematics. We think that should be used and develop multi-scales approach.

2.3 Foundations of Mathematics [23, 24 and 25]

Hallmark of mathematics is its *logical rigor*: it deals with precisely defined concepts and proven safe allegations. Mathematical definitions are comprehensive: they contain the *necessary* and *sufficient conditions* under which an object can be assigned to the volume of identifiable concept. This attribute *mathematical concept* is suitable for preparation of *mathematical statements* whose meaning is quite clear. About the veracity of these claims can be checked so that the resulting conclusions to be absolutely sure. Mathematical assertions, once established, *are outside any possible claim*. This distinguishes mathematics from both the natural and the humanities, whose allegations can be substantiated most beyond any reasonable doubt, but beyond any doubt at all. This explains *the fundamental role of proof in mathematics*. Like any science and mathematics is developed, which includes among other things, ***the existence of unresolved issues***. But mathematics is very clear distinction between *hypotheses* (i.e., statements which may sound plausible, but still not completely certain) and *theorems* (i.e. statements that are proven and rigorous about their authenticity longer doubt).

In addition to establishing the veracity of claims *proof* has another function: with its help investigate links between claims - an indispensable tool in mathematics since its object includes not only mathematical truths, but their connections. In addition to establishing the veracity of claims *the proof* has another function: with its help *investigate links between claims* – an indispensable tool in mathematics since its object includes not only *mathematical truths*, but *their connections*. However, mathematics is not a science of evidence such science is *logic* [35].

Causality or *causal connection* (more causalitet, causality and modality) is called the relationship between one event (cause) and another event called effect, where the second event is understood as a consequence of the first [29, 30, 31, 32, 33 and 37]. In ordinary use the concept of causality can refer to the connection of several factors (reasons) and a phenomenon. Anything that affects an effect is a factor that effect. Everything that influences on the given effect is a factor of this effect. Direct factor called factor that achieves on effect directly i.e. without the involvement of other factors ("intermediate variables"). The relationship between cause (causes) and effect is called casual or causal connection. Philosophical reflections on the issues of causality still more than a millennium. In the western philosophical tradition discussion goes back at least since Aristotle and the topic remains on the agenda of modern philosophers.

Explanation [33] causation 1. the relation of cause and effect. 2. The result is the same, however differently the causality is interpreted. causality 1. the action of causing or producing. 2. the relation of cause to effect; 3. anything that produces an effect; cause.

The most accurate science "mathematics" is experiencing methodological difficulties to define itself. The definition of Aristotle that mathematics is the science of the quantity in force until the 18th century. Mathematics has no modern definition, but it is easily recognizable because its shares exist between deep inner connections. It is not uncommon one mathematical reasoning contain knowledge of several mathematical disciplines. It is also important that the mathematics take care of itself. Anyone not solved a mathematical problem (task) is not actually "cancel" and is highly illuminated and attack (many time!) to obtain the result. Incompleteness of definitions cannot be filled by a list of mathematical disciplines, as this list is constantly changing, constantly arise in the mathematics new areas.

Definition: Mathematics contains mathematical knowledge, foundations of mathematics, methodology of mathematics and philosophy of mathematics in a complex interconnectivity and continuous development.

3. Mathematical Physics – Applied Mathematics

Principle of causality is a fundamental principle in physics, which states that any event that takes place in a physical system may influence this system in the future, but not to influence her behavior in the past. This means, that if the two events are separate, it neither can be no reason, no consequence to the other.

3.1 Phase transition of first order – thermodynamics, theory of heat conductivity

Mathematical fundamental theories are: Stefan and Stefan-Schwartz problems

-equations conductivity of liquid L and solid S phases

$$\frac{\partial T_L}{\partial t} = a_L \frac{\partial^2 T_L}{\partial x^2}, \ \frac{\partial T_S}{\partial t} = a_S \frac{\partial^2 T_S}{\partial x^2}, \qquad (1,1;\ 2)$$

-initial condition of liquid phase (L)

t=0: $T_L(x, t) = T^{/} = const_L$, (1, 3)

-boundary conditions

x = ∞ and t ≥ 0 $T_L(x, t) = T^{/} = const_L$, (1, 4)

x = 0 and t ≥ 0 $T_S(0, t) = T^{//} = const_S$, (1, 5)

-boundary conditions at the moving interfacial surface (or solidification (front))

x_{fr} и t ≥ 0 $T_L(x_{fr}, t) = T_S(x_{fr}, t) = T_m$, (1, 6)

-heat balance of x_{fr}

$$X_{fr}: \ -\lambda_L \frac{\partial T_L}{\partial x}\bigg|_{x_{fr}-0} = -\rho_S Q_{cr} \frac{\partial x}{\partial t} - \lambda_S \frac{\partial T_S}{\partial x}\bigg|_{x_{fr}+0}. \quad (1,\ 7)$$

The solution is equations for K, T_L, T_S and X_{fr}:

$$\rho_L Q_m \frac{\sqrt{\pi}}{2} K = b_S (T_m - T_{Ct}) \frac{e^{-\frac{K^2}{4a_S}}}{erf\left(\frac{K}{2\sqrt{a_S}}\right)} - b_L (T^{/} - T_m) \frac{e^{-\frac{K^2}{4a_L}}}{1 - erf\left(\frac{K}{2\sqrt{a_L}}\right)}$$

$$T_L = T^{/} - (T^{/} - T_m) \frac{1 - erf\left(\frac{x}{2\sqrt{a_L t}}\right)}{1 - erf\left(\frac{K}{2\sqrt{a_L}}\right)}, \ T_S = T^{//} + (T_m - T^{//}) \frac{erf\left(\frac{x}{2\sqrt{a_S t}}\right)}{erf\left(\frac{K}{2\sqrt{a_S}}\right)}, \ X_{fr} = K\sqrt{t}, \ (1,\ 8,\ 9,\ 10)$$

Stefan-Schwartz problem:

$$\frac{\partial T_M}{\partial t} = a_M \frac{\partial^2 T_M}{\partial x^2}, \quad \frac{\partial T_L}{\partial t} = a_L \frac{\partial^2 T_L}{\partial x^2}, \quad \frac{\partial T_S}{\partial t} = a_S \frac{\partial^2 T_S}{\partial x^2}, \qquad (2,\ 1;\ 2;\ 3)$$

-initials conditions liquid and mould

t=0: $T_L(x, t) = T^{/} = const_L$, $T_M(x, t) = T^{///}$, (2, 4)

-boundary conditions melt, a solid phase and contact (index Ct) (solid phase/mould)

x = ∞ and t ≥ 0 $T_L(x, t) = T^{/} = const_L$, (2, 5)

x_{fr} and t ≥ 0 $T_L(x_{fr}, t) = T_S(x_{fr}, t) = T_m$, (2, 6)

x = 0 and t ≥ 0 $T_S^{Ct}(x,t) = T_M^{Ct}(x,t) = T_{Ct}(0, t) = T^{//}$, (2, 7)

-mould

x = -∞, and t ≥ 0 $T_M^0(x,t) = T^{///} = const_M$, (2, 8)

-boundary conditions at moving (solidification (front)) x_{fr} and heat balance is

$$X_{fr}: \ -\lambda_L \frac{\partial T_L}{\partial x}\bigg|_{x_{fr}-0} = -\rho_S Q_m \frac{\partial x}{\partial t} - \lambda_S \frac{\partial T_S}{\partial x}\bigg|_{x_{fr}+0}. \quad (2,\ 9)$$

The solution is the equations for T_{Ct}, K, T_L, T_S, T_M and X_{fr}:

$$T_{Ct} = \frac{T_m}{1 + \frac{b_M}{b_S} erf\left(\frac{K}{2\sqrt{a_S}}\right)}, \qquad (2,\ 10)$$

$$\rho_L Q_m \frac{\sqrt{\pi}}{2} K = b_S (T_m - T_{Ct}) \frac{e^{-\frac{K^2}{4a_S}}}{erf\left(\frac{K}{2\sqrt{a_S}}\right)} - b_L (T^{/} - T_m) \frac{e^{-\frac{K^2}{4a_L}}}{1 - erf\left(\frac{K}{2\sqrt{a_L}}\right)}, \quad (2, 11)$$

$$T_L = T_m - (T^{/} - T_m) \frac{1 - erf\left(\frac{x}{2\sqrt{a_L t}}\right)}{1 - erf\left(\frac{K}{2\sqrt{a_L}}\right)}, \quad (2, 12)$$

$$T_S = T_{Ct} + (T_m - T_{Ct}) \frac{erf\left(\frac{x}{2\sqrt{a_S t}}\right)}{erf\left(\frac{K}{2\sqrt{a_S}}\right)}, \quad (2, 13)$$

$$T_M = T_{Ct} - (T_{Ct} - T^{///}) erf\left(\frac{x}{2\sqrt{a_M t}}\right), \quad (2, 14)$$

$$X_{fr} = K\sqrt{t}. \quad (2, 15)$$

In tasks (1) and (2) use the following symbols, a_S a_M a_L and $a=\lambda/c\rho$ are thermo conductivity numbers and λ, c, ρ are thermal conductivity, heat capacity and density of liquid (L), solid (S), mould (M) materials; $b = \sqrt{\lambda c \rho}$ and b_L, b_S b_M are capability accumulation material (L), (S) and (M); K is coefficient of solidification; $T_{L|S|M}$ is temperature of cast phases (L), (S) and (M); T_{Ct} is contact temperature between cast and mould, and the same, but T_S^{Ct}, T_M^{Ct} at x=0; t is time; Q_m – latent heat of melting; x is coordinate.

The tasks of Stefan's type are verified experimentally and used in the creation of mathematical models for technology phase transformations. At high cooling rates and small size of the studied open system these tasks provide huge opportunity for creating mathematical models. This methodological approach is: estimates of temperature fields and thermal balance.

This methodology based on the task of Stefan we can applied of process and the phase transition of the second order (heat treatment). It should consider the specific difference between the two phase transitions. The description of the temperature field in the phase transition of the first kind is more complicated in comparison with the temperature field of phase transition of the second order.

For example we have chosen the application of task Stephen-Schwartz for thermodynamics open system from aluminum melt and steel mould – phase transition of first order. The thickness of cast and mould each is with a size of 100 microns. To calculate the temperature field of the rapid solidification process of molten pure aluminum in quenching steel mould is on the Fig. 2 [38] – comparative analysis between the analytical solution and numerical solution with the Finite element method of Stephen-Schwartz problem

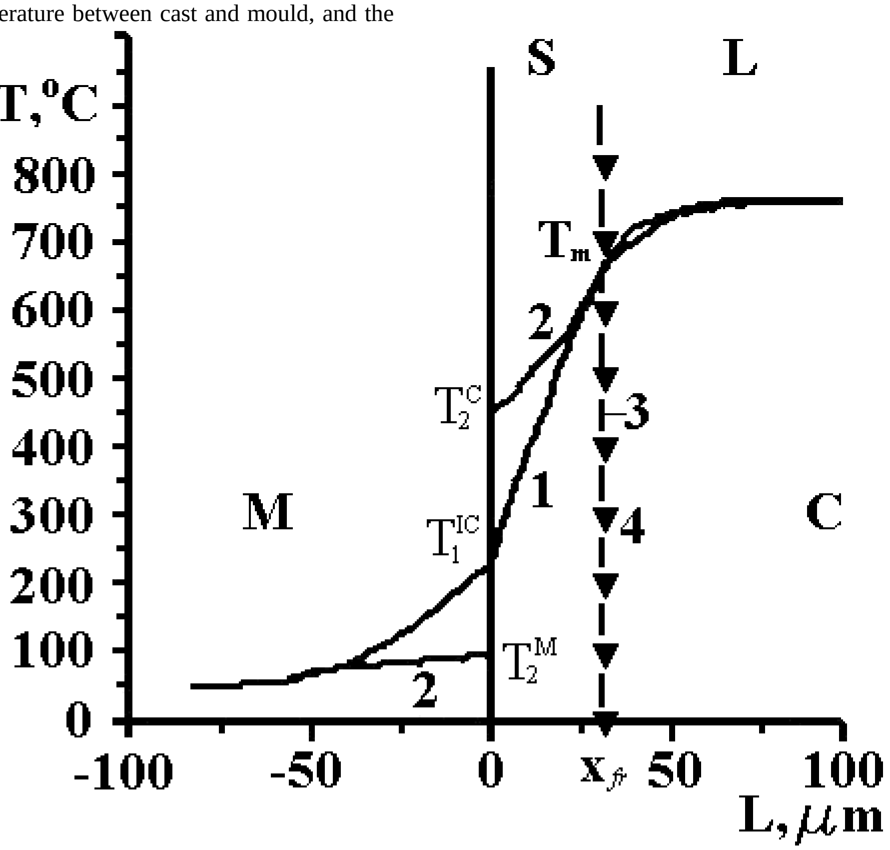

Fig. 2 *Stefan-Schwartz problem solutions of rapid solidification at high-speed quenching in open thermodynamics system of cast (C) with liquid (L) and solid (S) phases and steel mould (M) [38]. Analytical solution eq.(2): time-temperature curve 1; x_{fr} is calculated place at the front: $X_{fr}^{Analitical}$ = 32,55 μm line 3 (▬ ▬); ideal contact T_1^{IC} =229,15 °C. Numerical solution of Finite elements method: time-temperature curve 2; $X_{fr}^{Numerical}$ = 32,653 μm line 4 (▼); real contact T_2^M = 97,7 °C and T_2^C = 444,1 °C at coefficient of heat transfer α = 6x10[6] w/m[2] K.*

For the process of heat transfer from Figure 2 we compare the ideal and the real contact via the contact temperatures T_1^{IC} c $\overline{T}_{\mathrm{Re}\,al}^{Ct} = (T_2^M + T_2^S)/2$ and values of x_{fr} from the analytical $X_{fr}^{Analitical}$ and the numerical $X_{fr}^{Numerical}$ solutions.

Contact temperatures: T_1^{IC} =229,15 ^{o}C and $\overline{T}_{\mathrm{Re}\,al}^{Ct}$ =270,9 ^{o}C are different with 18,22 %.

Heat balance: $X_{fr}^{Analitical}$ = 32,55 μm and $X_{fr}^{Numerical}$ = 32,653 μm have differences 0,316 %.

Pure metals and part of the alloys solidified with one interface surface and other alloys solidified in temperature interval between two moving interface surfaces. *Foundry engineering methodology*: 1) Experimental measurement and numerical description of non-stationary temperature field with moving liquid/solid (L/S) interfacial surface(s) and constant temperature; 2) Microscopic measurements of the obtaining structure and a description of the crystallization (structure formation) processes.

3.2 Phase transition of first order – driving force of crystallization, scattering

Stefan's tasks do not give anything for crystallization processes, but the first step is the heat transfer at the interface surface [9]. *Growth of mono crystals*: Such a temperature gradient in the liquid and solid phases in interfacial boundary that ensures its sustainable equilibrium (i.e. interfacial boundary does not move); such subsequent modification of this gradient, which would provide relocation of interfacial surface with controllable speed. The heat balance (Stefan's boundary condition) we can write eq.(1, 7 or 2, 9) in the form:

$\lambda_S G_S - \lambda_L G_L = \rho_S Q_m R$ at $G_L \to 0$, (3, 1)

where $G_S = \nabla T_S = \partial T_S / \partial x$ and $G_L = \nabla T_L = \partial T_L / \partial x$ are temperature gradients in the respective phases; $R = \partial x / \partial t$ is growth rate. Condition for high quality mono crystal is

$$R_{MAX} = \lambda_S G_S / \rho_S Q_m \,. \quad (3, 2)$$

Simple example is of mono crystal growth by the method of floating zone [9] for home routines to temperature move element is left with constant and heat balance (per unit of time) can be saved in the following way

$$\begin{bmatrix}\text{Summary amending} \\ \text{heat at the expense} \\ \text{of conductivity}\end{bmatrix} + \begin{bmatrix}\text{Summary amending} \\ \text{the heat at the expense} \\ \text{of moving the boundary}\end{bmatrix} + \begin{bmatrix}\text{Summary amending} \\ \text{the heat on account of} \\ \text{losses to the environment}\end{bmatrix} = 0 \cdot \quad (3, 3)$$

The solidification (macro level of phase transition first order) has two typical cases on Fig. 3: 1. Equilibrium – without supercooled of metal melt; 2. Non-equilibrium – Low or High supercooled of metal melt. In both cases of solidification is released the latent heat of transition (melting) Q_m.

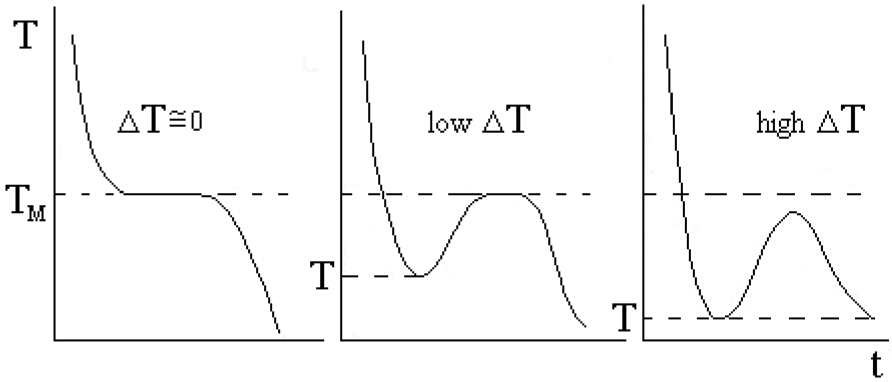

Fig. 3 *Time-temperature curves pure metals solidification (macro level of phase transition first order) at according to supercooled: 1. Equilibrium – without supercooled of metal melt; 2. Non-equilibrium – Low or High supercooled of metal melt.*

The numerical solution of 3D task of Stefan and Stefan-Schwarz is for equilibrium phase transition of the first order independently of the cooling rate. All numerical experiments on Figures 1 and 2 not accounted for supercooling of melt but in practice the phase transition first order is non-equilibrium. For reporting in particular local processes of structure formation is necessary to take into account local conditions: 1. the local cooling rates; 2. and local supercooling of the melt.

The structure of cast pure metal or alloy is polycrystalline grains mainly in the form of cells with simple geometry or dendrites – complex geometry. The character geometrical parameters are the middle radius of grains, and primary and secondary distances. The local conditions influence on character geometry are [9]: the product of the gradient of the temperature eq.(3, 1) and the speed of the growth eq.(3, 2) (rate of cooling GR), or the duration of the local time of solidification t_f, and temperature interval of non-equilibrium solidification ΔT_S and dimensionless supercooling with (supercooling ΔT)

$$t_f = \frac{\Delta T_S}{GR}, \; \Delta U = \frac{c_S \Delta T}{Q_m}. \quad (3, 4)$$

It should be noted in equations (3, 4) we only have thermodynamic variables and time. The supercooling is very important, but it many difficult is defined. Factors that affect supercooling are many and relate to the mechanism of structure formation of each material (metal, alloy, etc.). The supercooling of eq.34 as local conditions is similar to similar to supercooling $ln(T_m/T)$ (see the introduction).

Latent heat of phase transition is scattering in volume with supercooled melt [9] i.e. connectivity between the release of latent heat of melting and its scattering in supecooled melt is applied the theory of diffusion of works [10, 11]. In [2] is a developed Stefan task considering the supercooled area, which disperses and releases latent heat Q_m. On Fig. 4 is shown only temperature field [4] of supercooled zone of pure Al

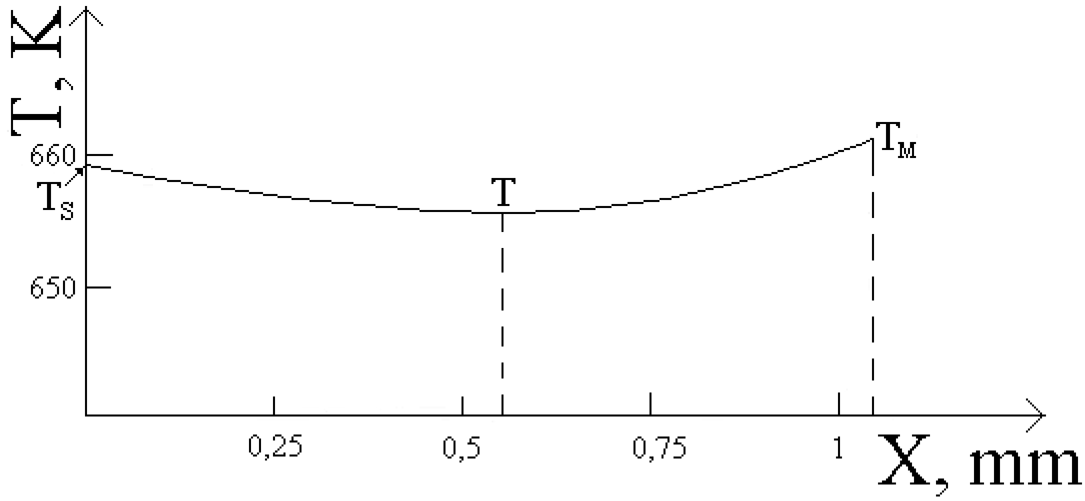

Fig. 4 *Temperature curve of supercooled zone [4]: T_m = 660,1 ^{o} C, T_S = 659,68 ^{o} C T = 655,873 ^{o} C, Q_m = 401819 J/kg.*

The maximum value of reported supercooling does not contradict the data from the literature. These data can be used for the thermodynamic driving force of crystallization *ln*(T_m/T). But we recall that there is a lot of work. For the modeling of the crystallization process (the structures formation) were used other theories, such as the classical theory Kossel–Stranski–Folmer–Kaishev (atomistic approach) and the fundamental equation of new phases formation of Kashchiev generalized for variable thermodynamic driving force [7 and 8]. Finally result of Fig. 4 suggests close nano-research areas i.e. using quantum mechanics or multiscale approach in one iteration (one step in time of Stefan's problem) use calculations of quantum mechanics.

4. Industry mathematics – Industry 4

The fundamental mathematics, mathematical physics and fundamental experimental physics lead to a revolution in physics [34]. Based on the results of fundamental research are create new technologies.

Industry 4 is a comprehensive revolutionary change of society by the advent of technology [16]: artificial intelligence, robotics, internet things, autonomous vehicle without a driver, 3D printing, nanotechnology, biotechnology, materials science, energy storage and quantum computing. These new technologies are the foundation for creating "smart factories". New technologies are related to the requirements of ecology; circulation of materials on the market - production - use - scrap for production; renewable energy and renewable energetics. Knowledge becomes a commodity and opens a permanent market for it.

Industry 4 requires a complete mathematization of everything [12, 13 and 14].

The change is exponential rate, close to many branches and released millions of jobs,
which are often called subversive effect. For these reasons, society appears conditions for chaos, a huge challenge for man.

Appears acute need for training and retraining of large groups of people, and of course we have education for all life.

5. Mathematical Education – Complete orientation (talented and others)

Mathematics has become the language of science and technology based on research results. Part of the Industry 4 and math education and retraining that obliges the development of education in mathematics in the direction of maximum (almost full) orientation of the student (child or adult). In this direction, working in many educational systems scientists' mathematicians develops new educational programs. An example is the educational system, created by the famous Bulgarian mathematician Blagovest Sendov called "Sendov's system" [36].

Program for lifelong learning is developed in Europe. Universität Wien, Dipartamento di Matematica, Università di Pisa, VIA University College – Læreruddannelsen i Århus and Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences was held seminar [39]. The importance of this workshop is wider than the specific reports. This is to answer the question "how methodologically to help in the economy by math (single person or micro-business?)".

6. Conclusions

The assessment of overcooling is a quantitative relationship and driving force of crystallization processes and structures.

Mathematics needed to describe the process of phase transition and the formation of structures corresponds to the basics (pure) mathematics.

Modern technologies require mathematical physics.

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