POWER GENERATION OPTIMAL DISPATCH MODEL WITH CROSS-BORDER EXCHANGE MODEL FORMULATION

M.Sc. Trashlieva V.[1], M.Sc. Radeva T. PhD.[1]
Department of Electrical Power Engineering – Technical University of Sofia, Bulgaria

vesselina.trashlieva@gmail.com

Abstract: *In this paper a model for optimal hydro-thermal coordination of a power system is proposed when contracts for power import and export power are available. The power flows outside the localized area of the local power system are divided into planned and planned quantities in order to assign different penalties for them in the cost criterion. The model also introduces the work of thermal and hydro power plants by separating each unit with a linear approximation. The resulting model is a mixed-integer linear programming problem. .*

Keywords: HYDRO-THERMAL COORDINATION, OPTIMIZATION MODELING, MILP

1. Introduction

The general formulation of the Unit Commitment (UC) problem requires the power generating mix units determination for each single interval in the planning horizon[1-2]. It is the process where the working power generating units are defined (Power Plants - PPs) as well as those providing a reserve required to satisfy the demand (i.e. the sum of load and losses) taking account of constraints there included network transmission capacity. The actual generating levels for each unit is a subject to the economic dispatch ED problem[3]. Generating units are also considered as base and peak (depending on their capabilities) according to their capabilities to rapidly change their output power levels according the demand fluctuations[2]. In an electric power system (EPS) the set of power generation plants generally includes hydropower (HPP), pump hydropower (PHPP), nuclear and thermal power plants, the latter two are considered base PPs. The general role of a base PP into active power balance is to cover the base load while peak PPs are HPPs. In the current model, the network transmission capacity is limited not to the its physical capacity but to distinct hourly limitations to the PPs production and the system import and/or export. As such assumptions the model proposed is not intended to address a trivial optimal power flow (OPF) problem (where power flows are optimized [2]) but to be used as a benchmark for modeling a contract power exchange of an EPS with surrounding EPS.

OPF problems combine the goal of minimal production costs as the UC and ED with the goal of minimal losses and optimal voltage levels with regard to the conditions of the transmission network. From the first publications of Carpentier in 1962 [5,6] to present days, the formulations and the methods for solving problems of this group are become more and more complicated but refined[4,7]. Meanwhile, the other two groups, those of UC and ED are being developed too with the addition of contemporary optimization [8] goals for emission costs, optimal load limitation [energy demand management, demand side management] in moment of power shortage, uncertainty and variability of input data, including the additional uncertainty of renewable energy resources (RES) power plants. So the cost function and the constraints in the three groups of optimization problems for active power optimization (OPF, UC and ED) have similar objectives but differ in their content and form, as well as different applications reflecting mainly the different points of vu when building the models, i.e. the modeling purpose. Here we try to combine the three objectives of the three groups of optimization problems mentioned above at a different degree and interpretation. OPFs aim to minimize a cost function constrained from certain conditions: power balance, power plants' physical production, physical capabilities of the branches in the transmission network, allowed tolerance for the mode voltages and angles, etc. A cost function can also minimize production costs or transmission network losses or monetary losses deriving from undelivered energy, and also a combination of them. But considering power flows in any case, the value of the cost function in an OPF problem will always exceed the value of the same cost function for an economic dispatch problem. When to these conditions add up also the occurrence of probable contingency, the OPF is a "*security-constrained optimal power flow* (SCOPF)"[9-11]. While risk is included in the cost criterion its value will be even bigger that the one obtained from a OPF optimal solution. That is because the values of power injections assuring the "*N*-1" criterion are evaluated.

2. Nomenclature

The imported and exported electrical power can be divided into two categories according the contracts that is planned (contracted) and unplanned (resulting as unwanted power deviations in the localized EPS due to frequency control actions). In order to achieve a better distinction between different events, the unwanted power is further divided into two additional groups modeling both the undesirable deviation in the local and in the neighboring EPS and power aiming to alter the unplanned values. So the following nomenclature may be presented:

$P_{impC,j}$ - planned (contracted) import in a unit interval *j*, as a forecast or a contracted determined value

$P_{impL,j}$ - imported power in a unit interval *j* because there is a shortage of power in the local EPS

$P_{impF,j}$ - imported power in a unit interval *j* because of additional load necessity in a neighbor ESP, as forecasted values

$P_{impLev,j}$ - imported power in a unit interval *j* that aims to balance any unplanned export

$c_{impC,j}$ - costs for contracted import in a unit interval j

$c_{impL,j}$ - costs for undesired import in a unit interval *j* when the local EPS needs additional generation

$c_{impF,j}$ - costs for import in a unit interval *j* when a neighbor EPS needs load

$c_{impLev,j}$ - costs for unplanned import in a unit interval *j* for balancing any unplanned previous exports

$P_{ExpC,j}$ - planned export in a unit interval *j* as a forecasted value

$P_{ExpL,j}$ - unplanned export in a unit interval *j* caused by a insufficient generation in a neighbor power system as forecast

$P_{ExpF,j}$ - unplanned export in a unit interval *j* caused by a necessity of additional load in the local power system

$P_{ExpLev,j}$ - unplanned export in a unit interval *j* for returning all othe unplanned deliveries

$c_{ExpC,j}$ - costs for planned export for a unit interval *j*

$c_{ExpL,j}$ - costs for unplanned export in a unit interval *j* because power is requested from outside the local power system

$c_{ExpF,j}$ - costs for unplanned export in a unit interval *j* because additional load is needed in the local system

$c_{ExpLev,j}$ - costs for unplanned power in a unit interval j to equalize unplanned import

$P_{imp,j}^{\max}$ and $P_{Exp,j}^{\max}$ - maximal admissible values for bothe export and import

$P_{W,j}$ and $P_{PV,j}$ - generation from wind power plants and solar power plants in a unit interval j as forecasted values

$c_{W,j}$ and $c_{PV,j}$ - costs for generation from wind and solar PPr in a unit interval j

α_j and β_j - the percentage of renewable power generation form both RES that is actually delivered to the power system, 1 minus these α_j and β_j shows the renewable power rejection in a unit interval j

The model adopts a linear approximation approach to the work of all generation units under consideration, but considers separately the different energy resources (water and fuels).

$C_{T,ij}(P_{T,ij})$ - performance of a thermal unit i as a function of its outpur power

$P_{T,ij}$ - output power of thermal unit i in unit interval j

$P_{T,lij}$, $c_{T,lij}$ - output power and costs growth of a thermal unit i in a linear approximation interval $l \in L_T$; the costs growth represent the fuel consumption growth multiplied by the cost for the specific fuel used by a thermal unit i

L_T - number of the linear intervals in the linear approximation of the performance function of a thermal unit i

$P_{T,l}^{\min}$ and $P_{T,l}^{\max}$ - minimal and maximal output power levels of a thermal unit i in a linear approximation interval $l \in L_T$

δ_i - maximal admissible value of output power change of a thermal unit i

$C_{H,mj}(P_{H,mj})$ and $\Phi_{H,mj}(P_{H,mj})$ - performance a turbine m as a function of its output power level and water processing performance in a function of the same optimization variable

$P_{H,mj}$ - output power of a turbine m a unit interval j

$P_{H,mlj}$ - output power of a turbine m in a linear approximation interval $l \in L_H$

$c_{H,mlj}$ - costs growth of a turbine m in a linear approximation interval $l \in L_H$; since the cost of the water is negligible to this of a thermal power plant, this coefficient here implies costs that are induced because of a different financial matter;

$\varphi_{H,mlj}$ - growth of the water expenses of a turbine m in a linear approximation interval $l \in L_H$

L_H - number ot the intervals in the linear approximation of the performance finction of a turbine m

$P_{H,lm}^{\max}$ - maximal output power of a turbine m in a linear approximation interval $l \in L_H$

$C_{P,nj}(P_{P,nj})$ and $\Phi_{P,nj}(P_{P,nj})$ - costs and processed waters of a pump n in a function of the active power consumed

$P_{P,nj}$ - consumed power by a pump n in a unit interval j

$c_{P,nlj}$ - growth of costs for a pump n in a linear approximation interval $l \in L_P$, same presumptions as the turbines apply to pumps

$\varphi_{P,nlj}$ - growth of water consumption for a pump n in a linear interval $l \in L_P$ of the approximation

$P_{P,nlj}$ - power consumed by a pump n in a linear approximation interval $l \in L_P$

L_P - number of the intervals in the linear approximation of the work of a pump n

$P_{P,nl}^{\min}$ and $P_{P,nl}^{\max}$ - minimal and maximal power of a pump n in a linear approximation interval $l \in L_P$

v_{mj} - a binary variable that is 1 (true) if a pump m is working in a unit interval j

$V_{r,j}$ - water volume in reservoir r at the end of a unit interval j

V_r^{max} and V_r^{min} - maximal and minimal volumes for a reservoir r

$Q_{F,rj}$ and $Q_{C,rj}$ - uncontrollable (free) and controllable water inflows in a reservoir r in a unit interval j; generally controllable quantities are associated with processed and unprocessed water quantities from reservoirs feeding r, while the free inflows are of a natural origin for example snow melting and rains

$R_{F,rj}$ and $R_{C,rj}$ - uncontrollable (free) and controllable water outflows from a reservoir r in a unit interval j; uncontrollable outflow from a reservoir is associated with natural event such as evaporation while a controllable outflow is a water quantity that is not fed to a HPP but to another reservoir for different purposes or for irrigation

L_j - system load in a unit interval j as a forecast

σ - reserve to be assured as a percentage over the system load

The linear approximation of the thermal plants performance function is: $C_{T,ij}(P_{T,ij}) = \sum_{l=1}^{L_T} c_{T,lj} P_{T,lj}$ where $P_{T,j} = \sum_{l=1}^{L_{T,i}} P_{T,lj}$ for every unit interval j and thermal unit i as $0 \le P_{T,lj} \le P_{T,l}^{\max}$ for every j and interval l of a unit i.

The linear approximations of the performance functions of the hydro power plants are: $C_{H,mj}(P_{H,mj}) = \sum_{l=1}^{L_H} c_{H,mlj} P_{H,mlj}$ and $\Phi_{H,mj}(P_{H,mj}) = \sum_{l=1}^{L_H} \varphi_{H,mlj} P_{H,mlj}$ for every j, where $P_{H,mj} = \sum_{l=1}^{L_H} P_{H,mlj}$ and the bounds are respected $0 \le P_{H,mlj} \le P_{H,ml}^{\max}$. The approximations for the pumps are introduced in the same manner: $C_{P,nj}(P_{P,nj}) = \sum_{l=1}^{L_P} c_{P,nlj} P_{P,nlj}$ and $\Phi_{P,nj}(P_{P,nj}) = \sum_{l=1}^{L_P} \varphi_{P,nlj} P_{P,nlj}$ for each j, where $P_{P,nj} = \sum_{l=1}^{L_P} P_{P,nlj}$ for each j and pump n with regard to $0 \le P_{P,nlj} \le P_{P,nl}^{\max}$.

3. Model formulation

With the introduced performance characteristics of the different power plants and variables, the cost function represents the total electricity generation costs:

$$\begin{aligned}\min J = & \sum_{i,j} C_{T,ij}(P_{T,ij}) + \sum_{m,j} C_{H,mj}(P_{H,mj}) + \sum_{n,j} C_{P,nj}(P_{P,nj}) + \\ & + c_{W,j} P_{W,j} + c_{PV,j} P_{PV,j} + c_{impC,j} P_{impC,j} + c_{impL,j} P_{impL,j} + \\ & + c_{impLev,j} P_{impLev,j} + c_{impF,j} P_{impF,j} + c_{ExpC,j} P_{ExpC,j} + \\ & + c_{ExpL,j} P_{ExpL,j} + c_{ExpLev,j} P_{ExpLev,j} + c_{ExpF,j} P_{ExpF,j}\end{aligned} \quad (1)$$

System balance constraint:

$$\begin{aligned}L_j = & \sum_i P_{T,ij} + \sum_m P_{H,mj} - \sum_n P_{P,nj} + \alpha_j P_{W,j} + \beta_j P_{PV,j} + \\ & + P_{impC,j} + P_{impL,j} + P_{impLev,j} + P_{impF,j} - \\ & - P_{ExpC,j} - P_{ExpL,j} - P_{ExpLev,j} - P_{ExpF,j}\end{aligned} \quad (2)$$

Reliability constraint for increased load - the system has to maintain power balance in the case of an increase in the load relative to the forecast:

(3) $$\sum_i \left(P_{T,ij} + \Delta P_{T,ij}\right) + \sum_m P^{max}_{H,m} + +P_{W,j} + P_{PV,j} + P_{impL,j} \geq \sigma L_j$$

Constraints ensuring that there is no simultaneous operation in the pump and generator mode of the reversible units:

(4) $$P_{H,mj} - \left(1 - v_{nj}\right) P^{max}_{H,m} \leq 0$$

(5) $$P_{H,mj} \leq v_{nj} P^{max}_{H,m}$$

The current model can treat every turbine and/or pump in a HPP as a separate unit with a separate set of variables. Such approach provides for optimizing a whole power plant respecting the nomenclature. The structure of the latter constraints implies that all available units of the HPP composition are capable of working in reversible mode, so all available pump and generator capacities need to be aligned in number and composition. That is for each turbine with a pump mode available there is a variables set v_{mj}. For each turbine that is not reversible there must be also a set of variables v_{mj} with zero lower and upper bounds (that is $v_{mj} = 0$ for each *j*). For each pump that is a single accumulation unit but not a reversible one there is a set of variable v_{mj}. but.(4) will not be included. With such approach for the binary variables, the model allows the optimization of the operation of each turbine and pump individually or all units in a given HPP and / or PSHP. When the model is written in the proposed matter this poses some redundancy but most of the optimization software environments deal appropriately with a such artificial redundancy on the stage of pre-solving when an analysis of the model structure is performed and some of the variables and constraints are ignored or fixed. It is also important to notice that an additional constraint must be applied

The base conventional thermal power plants are not flexible enough and this results in the introduction of an additional constraint related to the technological limitations for charging and discharging a thermal unit, that is the maximal allowable growth or increase of output power level that is possible for a certain thermal unit δ_i: $\left|\Delta P_{T,ij}\right| \leq \delta_i$ - also named load speed change constraint. This constraint is mainly sensitive to detailed separation of thermal plants and the duration of the unit interval. These constraints generally are of the following form:

- $\Delta P_{ij} \leq \Delta^{Up}_i$ when increasing the output level, that is when $P_{ij} > P_{i,j-1}$ and
- $\Delta^{Dn}_i \leq \Delta P_{ij}$ when decreasing the thermal unit when $P_{ij} < P_{i,j-1}$, so

$$\Delta P_{ij} \begin{cases} \leq \Delta^{Up}_i, \Delta P_{ij} > 0 \\ \leq \Delta^{Dn}_i, \Delta P_{ij} < 0 \end{cases}$$

Such nonlinearity may be dealt with the introduction of additional binary variables sets that will model the sign of the difference $\Delta P_{ij} = P_{ij} - P_{i,j-1}$. According the values of these binary variables the proper bound will be respected. Generally, the norms for speeding in both directions for thermal plants are given in percentage of their nominal power that is often 1,5 % for up direction and 2% in down direction. So if the duration of a unit interval is 60 minutes a maximal output power level of a unit with a working range for example between 140 MW and 210 MW (or 230 MW) will be achieved in 20-25 minutes. These fact provide for the opportunity to skip speed constraints in the case when thermal units are considered separately in an optimization problem and the duration of an interval is more than 30 minutes. However, the formulation presented here gives chance to this specifics in the work of thermal power plants but considers the simplification that both up and down constraints are equal, that is $\Delta^{Dn}_i = \Delta^{UP}_i$. Because the model is linear the latter leads to two expressions if δ_i is considered the value of maximum allowed charge or discharge of a thermal power plant:

(6) $P_{T,ij} - P_{T,ij-1} \leq \delta_i$ and $-P_{T,ij} + P_{T,ij-1} \leq -\delta_i$ for each *i* и *j*

Water balance constraint is needed in this problem to properly model the water stocks feeding the respective power stations for each reservoir *r* and time interval *j*:

(7) $$V_{r,j} = V_{r,j-1} - \sum_{m\in\Gamma from} \Phi_{H,mj}\left(P_{Hmj}\right) + \sum_{n\in\Pi in} \Phi_{Pnj}\left(P_{Pnj}\right) + + \sum_{m\in\Pi from} \Phi_{H,mj}\left(P_{Hmj}\right) - \sum_{n\in\Gamma in} \Phi_{Pnj}\left(P_{Pnj}\right) + +Q_{Fr,j} + \sum_{w\in P in} R_{C,wj} - R_{Fr,j} - R_{Cr,j}$$

Γ_{from} and Π_{from} model the set of pumps and turbines that fed to and are fed from the current reservoir *r*. Γ_{in} and Π_{in} model the sets of turbines and pumps that feed in and out water from *r*. Π_{in} are the controllable outflows for all reservoirs that are connected to *r*.

The latter constraint guarantees that the change in the water quantity of a given reservoir will exactly correspond to the changes caused by the hydro power stations operation, the forecasted and controllable inflows and also the outflows. Often in hydro-thermal coordination problems a simplification is performed do that the water head is neglected. The water head influence on to the hydro plants performance may be conveniently modeled with the performance characteristics. The water balance constraint also can be reviewed as an extended formulation of a system constraint stating the efficiency of the cycle accumulation-generation:

(7) $$\sum_j P_{H,mj} = \eta \sum_j P_{P,nj} .$$

In the formulation of the water balance constraint the efficiency coefficient η of the cycle is somehow implicitly included via the water outgo growth constraints φ_H and $\varphi_{P.}$ This formulation also provides for the evaluation of real-time aspects as larger water quantities in the high moisture periods of the transitional seasons, decrease in the water inflows during hot days and also the unprocessed (by the power stations) water volumes that might be further divided into stochastic (but predictable as evaporation) and controllable (intentional) release of water for irrigation, other reservoirs feeding and flood control. Since rainfall and climate forecast are typically given over periods exceeding the model period (e.g. 1 hour), this allows uncontrollable water quantities to be included in the water balance constraints for each interval as a constant value (average predicted/expected value for an hour):

(8) $$V_{r,j} = V_{r,j-1} - \sum_{m\in\Gamma from} \Phi_{H,mj}\left(P_{Hmj}\right) + \sum_{n\in\Pi in} \Phi_{Pnj}\left(P_{Pnj}\right) + + \sum_{m\in\Pi from} \Phi_{H,mj}\left(P_{Hmj}\right) - \sum_{n\in\Gamma in} \Phi_{Pnj}\left(P_{Pnj}\right) + +\sum_{w\in P in} R_{C,wj} - R_{Cr,j} + \left(Q^{Day}_{F,r} - R^{Day}_{F,r}\right)\Big/ j$$

The daily management of the water levels ($V_r^{min} \leq V_{r,j} \leq V_r^{max}$) is modeled by fixing the appropriate values for the volumes in the first and the last period of the optimization horizon.

(10) $$\sum_j P_{impL,j} + \sum_j P_{impF,j} = \sum_j P_{ExpLev,j}$$

$$\sum_j P_{ExpL,j} + \sum_j P_{ExpF,j} = \sum_j P_{impLev,j} \qquad (11)$$

The latter two constraints state that any unplanned import and export should be equalized (returned back) in the same optimization horizon. Generally, unplanned equalization is performed in the day following the unplanned activities. So in this case, the *j* indices must be properly divided.

Also for every time interval the sum of the import and export must respect the physical conditions posed by the transmission network ($P_{imp,j}^{max}$ and $P_{Exp,j}^{max}$):

$$P_{impL,j} + P_{impF,j} + P_{impLev,j} + P_{impC,j} \le P_{imp,j}^{max} \qquad (12)$$

$$P_{ExpL,j} + P_{ExpF,j} + P_{ExpLev,j} + P_{ExpC,j} \le P_{Exp,j}^{max} \qquad (13)$$

4. Conclusion

The nomenclature and model formulation depicts two separate groups of powers that are exported and imported that is contracted and unplanned. Actually the "*unplanned*" are subject of another contract that specifically states the ideas of mutual helping in keeping the systems balances in a interconnected area. This gives the opportunity of the participants in such interconnected are to rely onto their neighbors in moments of shortages which is actually providing ancillary services from whole power systems to other power systems (or localized areas). The convention is that any unplanned deliveries of load or power (i.e. balance services) should be returned back, that is balanced over time with constraints (10) and (11).

Here two questions arise and should be observed in detail. First of all is that import and export actually represent power flows with variable direction and may physically happen via same transmission network so in a certain topology it might be useful to use either binary variables to manage the directions either consider import and export with a single unbounded by sign set of variables. Indices "*import*" and "*export*" also show the direction of the power flows. The latter might make the post optimal analysis a little complicated and also will pose some difficulties in the cost function if costs and profits have to be implied with a different coefficient. Then the introduction of binary variables will be imminent in order to apply the right quantity of increase or decrease of profits. Which leads to the second question.

Contracts for planned delivery often imply separate financial relations between the contracted subjects so that is convenient to separate planned and unplanned services one from another when costs are implied. In the nomenclature presented all coefficients included in the cost function concerning the variables sets of import and export powers are implied as costs. If the cost criterion is purely economic, it should be considered which of the amounts actually imply costs and which lead to profits and their monetary growth coefficient in the cost function modeling minimal costs goal should be invoked with a negative sign. In the presumptions presented in the current paper the contracted export, the export for providing unplanned power supplies and the import of power for helping a neighbor power system to deal with shortage of its own load are actually profitable (see the respective column in Table 1). The costs derive from planned import, unplanned export when there is a higher production than the controllable and uncontrollable leads can manage, and when power is imported outside of the planned volumes for keeping the balance when power production is not enough to cover the load (see Table 1).

Two sets of variables are left outside of the contents in Table 1, that are the equalization flows P_{impLev} and P_{ExpLev} stated with (10) and (11). The reason for not applying the cost/profit idea to these two sets is that one of the two variables set in (10) and (11) is in the profits column and the other is in the costs column of Table 1 so equalizing flows should be handled separately and this is provided with separate coefficients in the cost function that may be taken zero.

Table 1: *Variables that imply profits and costs in the cost function*

Profit	Costs
P_{ExpC}	P_{impC}
P_{ExpL}	P_{ExpF}
P_{impF}	P_{impL}

Two sets of variables are left outside of the contents in Table 1, that are the equalization flows P_{impLev} and P_{ExpLev} stated with (10) and (11). The reason for not applying the cost/profit idea to these two sets is that one of the two variables set in (10) and (11) is in the profits column and the other is in the costs column of Table 1 so equalizing flows should be handled separately and this is provided with separate coefficients in the cost function that may be taken zero.

The model presented here is linear and mixed integer. Input data with fixed values are: σ, $P_{impC,j}$ and $P_{ExpC,j}$ with respect to the constraints (10) and (11). Input data with forecasted values are the load and renewable generation forecats (L_j, P_{Wj}, $P_{PV,j}$), the forecasts / expectations for unplanned import and export ($P_{impF,j}$ and $P_{ExpL,j}$) and the uncontrollable forecasted water flows ($Q_{F,r}^{Day}$, $R_{F,r}^{Day}$). Optimization variables are the values of the following sets: $R_{C,rj}$, $P_{H,mj}$, $P_{P,nj}$, v_{nj}, α_j, β_j, $P_{impL,j}$, $P_{impLev,j}$, $P_{ExpF,j}$, $P_{ExpLev,j}$ and $V_{r,j}$.

5. Bibliography

[1] Carrion M., Arroyo J. M., A Computationally Efficient Mixed-Integer Linear Formulation for the Thermal Unit Commitment Problem, IEEE Transaction on Power Systems, Vol. 21, No. 3, August 2006, p. 1371 - 1377

[2] Wood A. J., Wollenberg B. F., Power Generation, Operation, and Control, 2nd ed. New York, Wiley, 1996, ISBN 978-0-471-79055-6

[3] Squires R.B., Economic Dispatch of Generation Directly from Power System Voltages and Admittances, AIEE Trans. vol. 79, pt. III, pp. 1235-1244, 1961.

[4] Cain M. B., O'Neill R. P., Castillo A., History of Optimal Power Flow and Formulations, Optimal Power Flow Paper I, December 2012, 36 pages

[5] Carpentier J., Contribution á l'étude du dispatching économique, Bulletin de la Société Française des Électriciens, ser. 8, vol. 3, pp. 431-447, 1962.

[6] Carpentier J., Optimal power flows, International Journal of Electrical Power and Energy Systems, Vol. 1, Issue 1, pp. 3-15, Apr. 1979.

[7] Frank S., Rebennack S., An Introduction to Optimal Power Flow: Theory, Formulation and Examples, IIE Transactions, Vol. 42, 2016, Issue 12 - Operations Engineering and Analysis, p. 1172-1197, doi: https://doi.org/10.1080/0740817X.2016.1189626

[8] Shahidehpour M., Yamin H., Li Z., Market Operations in Electric Power Systems: Forecasting, Scheduling, and Risk Management. Piscataway, NJ: IEEE-Wiley-Interscience, 2002, ISBN 0-471-44337-9

[9] Qiu W., Flueck A, Tu F., A new parallel algorithm for security constrained optimal power flow with a nonlinear interior point method. In: Power Engineering Society General Meeting, 2005. IEEE, pp 447 – 453 Vol. 1, DOI 10.1109/PES. 2005.1489574

[10] Yumbla P. E. O., Ramirez J. M. C., Coello A., Optimal power flow subject to security constraints solved with a particle swarm optimizer, IEEE Transactions on Power Systems, vol. 23, no. 1, pp. 33-40, Feb. 2008.

[11] Monticelli A., Pereira M. V. F., Granville S., Security-constrained optimal power flow with post contingency corrective rescheduling, IEEE Transactions on Power Systems, vol. 2, no. 1, pp. 175-181, Feb. 1987.