MATHEMATICAL MODELLING OF THE OSCILLATING BODY SUBMERGED IN THE LIQUID AT THE PRESENCE OF THE INITIAL AND CONSTANTLY ACTING DISTURBANCES

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Abstract: *The problem of the rigid body oscillations, submerged in the liquid of the uniform depth, as the problem of the potential theory is concerned. Small oscillations of the body and disturbances caused by the presence of a body in a liquid are studied. The problem is solved by the Green's-function method. The solution for the velocity potential is obtained in the form of a system of integral equations with a real kernel.*

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1. Introduction

Of great practical interest is the study of oscillations of hydroacoustic transducers and antennas placed in a layer of liquid with boundaries. We consider this problem as the initial-boundary value problem of potential theory.

Let a solid body be immersed in a liquid bounded from above by a free surface, from below - by a horizontal bottom. The liquid is assumed to be perfectly incompressible, and its motion is irrotational. We choose the Cartesian coordinate system *Oxyz* as follows: the axes *Ox, Oy* lie in the horizontal plane coinciding with the free surface of the liquid in the unperturbed state, the axis *Oz* is directed vertically upwards. The problem of the oscillations of a body in a liquid under these assumptions is nonlinear [1]. In this paper we consider the case of small oscillations of a liquid and a body.

2. Formulation of the problem

In the linear case [2], the perturbed motion of the liquid is described by means of a harmonic function $\Phi = \Phi(x, y, z, t)$, satisfying the following boundary and initial conditions:

$$\frac{1}{g}\frac{\partial^2\Phi}{\partial t^2} + \frac{\partial\Phi}{\partial z} = -\frac{1}{\rho g}\frac{\partial p_0}{\partial t} = P(x, y, t), \qquad z = 0,$$

$$\frac{\partial\Phi}{\partial z} = 0, \qquad z = -H,$$

$$\frac{\partial\Phi}{\partial n} = V_n - \frac{\partial\varphi_\infty}{\partial n}, \qquad (x, y, z) \in S,$$

$$\lim_{R\to\infty}\Phi = 0, \qquad \lim_{R\to\infty}\nabla\Phi = 0, \qquad R = \sqrt{x^2 + y^2},$$

$$\Phi = F_1(x, y, t), \qquad z = 0, \qquad t = t_0,$$

$$\frac{\partial\Phi}{\partial t} = F_2(x, y, t), \qquad z = 0, \qquad t = t_0.$$

Here we have adopted the following notation: Φ is the velocity potential of the disturbed motion of the liquid; φ_∞ is potential of the system of incoming waves; p_0 is pressure applied to the free surface; ρ is the density of the liquid; *g* is the acceleration due to gravity; *H* is the depth of the liquid layer; *S* - position of the body surface in the static state; *n* is the normal to *S*, directed into the interior of the liquid; V_n is the projection of the velocity of points on the surface of the body on the normal; F_j, *j = 1,2* are known functions.

3. Construction of the Green's function

Bearing in mind the use of the formula for the theory of harmonic functions [3], we construct a function *G* corresponding to the wave motion of a liquid caused by the action of a source placed at a point $N = (\xi, \eta, \zeta)$ from $t = \tau$. The function *G* everywhere, except the point $M = (x, y, z)$, is harmonic and satisfies the following conditions:

$$\frac{\partial^2 G}{\partial t^2} + \frac{\partial G}{\partial z} = 0, \qquad z = 0,$$

$$\frac{\partial G}{\partial z} = 0, \qquad z = -H,$$

$$\lim_{R\to\infty} G = 0, \qquad \lim_{R\to\infty}\nabla G = 0,$$

$$G = \frac{\partial G}{\partial t} = 0, \quad z = 0, \quad t = \tau.$$

Let us construct the function *G*. Let

$$G = \frac{1}{r} + \frac{1}{r_1} + B(M, t; N, \tau),$$

$$r = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2}, \quad r_1 = \sqrt{R_0^2 + (z-\zeta')^2},$$

$$R_0 = \sqrt{(x-\xi)^2 + (y-\eta)^2}, \quad \zeta' = -(2H + \zeta),$$

here $B = B(M, t; N, \tau)$ is a function to be determined.

By this representation we have

$$\frac{\partial B}{\partial z} = 0, \qquad z = -H.$$

The function *B* satisfies the condition at $z = 0$

$$\frac{\partial^2 B}{\partial t^2} + g\frac{\partial B}{\partial z} = -\frac{\partial}{\partial z}\left(\frac{1}{r} + \frac{1}{r_1}\right) = \frac{\partial}{\partial\zeta}\left(\frac{1}{r_1} - \frac{1}{r}\right).$$

By the equality of [3]

$$\left.\left(\frac{1}{r_1} - \frac{1}{r}\right)\right|_{z=0} = -\int_0^\infty (\exp(\zeta s) - \exp(\zeta' s))J_0(sR_0)ds,$$

where $J_0(x)$ is the Bessel function of zero order, the condition at $z = 0$ becomes:

$$\frac{\partial^2 B}{\partial t^2} + g\frac{\partial B}{\partial z} = 2g\int_0^\infty s\exp(-Hs)\operatorname{ch}s(H+\zeta)J_0(sR_0)ds.$$

Now for the harmonic function B we obtain the following representation:

$$B = \int_0^\infty sT(t,s)\exp(-Hs)\,\mathrm{ch}\, s(H+\zeta)\frac{\mathrm{ch}\, s(H+z)}{\mathrm{ch}\, sH}J_0(sR_0)ds,$$

in which the function $T = T(t,s)$ satisfies equation

$$\frac{\partial^2 T}{\partial t^2} + \sigma^2 T = 2g, \quad \sigma^2 = gs\,\mathrm{th}\, sH$$

and the initial conditions

$$2 + sT(\tau,s) = 0, \qquad \frac{\partial T(\tau,s)}{\partial t} = 0.$$

The solution of this problem has the form:

$$T(t,s) = -\frac{2}{s}(1+\mathrm{cth}\, sH)\cos\sigma(t-\tau) + \frac{2}{s}\mathrm{cth}\, sH.$$

Consequently, for the function B we obtain the following representation:

$$B = 2\int_0^\infty [-(1+\mathrm{cth}\, sH)\cos\sigma(t-\tau) + \mathrm{cth}\, sH]\cdot$$
$$\cdot\exp(-Hs)\,\mathrm{ch}\, s(H+\zeta)\frac{\mathrm{ch}\, s(H+z)}{\mathrm{ch}\, sH}J_0(sR_0)ds.$$

4. The solution of the problem

Now we turn to the solution of the main problem. Let us consider an area τ_Σ, that is bounded by a surface Σ consisting of the following parts: the surface of the body *S*, the surface S_R of the cylinder of radius *R* with the *z*-axis, the circle S_1 in the plane $z = 0$, and the circle S_2 in the plane $z = -H$.

In the domain τ_Σ we apply to the functions $\frac{\partial \Phi}{\partial t}$ and *G* the formula for the theory of harmonic functions:

$$\frac{1}{4\pi}\iint_\Sigma\left(G\frac{\partial^2\Phi}{\partial t\partial n} - \frac{\partial\Phi}{\partial t}\frac{\partial G}{\partial n}\right)d\sigma = \begin{cases}\frac{1}{2}\frac{\partial\Phi}{\partial t}, M\in\Sigma\\ \frac{\partial\Phi}{\partial t}, M\in\tau_\Sigma\end{cases},$$

Where $\bar{n}$ is the unit vector of the outer normal to the surface Σ at the point *N*, over which the integration takes place.

Using the boundary conditions at the bottom and at infinity ($R\to\infty$), we obtain:

$$\frac{1}{4\pi}\iint_{\zeta=0}\left(G\frac{\partial^2\Phi}{\partial t\partial\xi} - \frac{\partial\Phi}{\partial t}\frac{\partial G}{\partial\zeta}\right)d\xi d\eta +$$
$$+\frac{1}{4\pi}\iint_S\left(\frac{\partial\Phi}{\partial t}\frac{\partial G}{\partial n} - G\frac{\partial^2\Phi}{\partial t\partial n}\right)d\sigma = \begin{cases}\frac{1}{2}\frac{\partial\Phi}{\partial t}, M\in S\\ \frac{\partial\Phi}{\partial t}, M\in\tau_\infty\end{cases},$$

where τ_∞ is the region outside *S* between the planes $z = 0$ and $z = -H$, $\bar{n}$ is the outer normal to *S*.

Integrating the last equation with respect to *t*, we obtain:

$$\frac{1}{4\pi}\iint_{\zeta=0}\int_{t_0}^{t}\left(G\frac{\partial^2\Phi}{\partial t\partial\xi} - \frac{\partial\Phi}{\partial t}\frac{\partial G}{\partial\zeta}\right)dtd\xi d\eta +$$
$$+\frac{1}{4\pi}\int_{t_0}^{t}\iint_S\left(\frac{\partial\Phi}{\partial t}\frac{\partial G}{\partial n} - G\frac{\partial^2\Phi}{\partial t\partial n}\right)d\sigma dt = \begin{cases}\frac{1}{2}\Phi\big|_{t_0}^{t}, M\in S\\ \Phi\big|_{t_0}^{t}, M\in\tau_\infty\end{cases}.$$

Let us consider the integral

$$\int_{t_0}^{t}\left(G\frac{\partial^2\Phi}{\partial t\partial\zeta} - \frac{\partial\Phi}{\partial t}\frac{\partial G}{\partial\zeta}\right)dt = \int_{t_0}^{t}\left(G\frac{\partial^2\Phi}{\partial t\partial\zeta} + \frac{\partial\Phi}{\partial t}\frac{\partial^2 G}{\partial t^2}\right)dt.$$

As

$$\frac{d}{dt}\left(G\frac{\partial\Phi}{\partial\zeta} + \frac{1}{g}\frac{\partial\Phi}{\partial t}\frac{\partial G}{\partial t}\right) = G\frac{\partial^2\Phi}{\partial t^2} + \frac{1}{g}\frac{\partial\Phi}{\partial t}\frac{\partial^2 G}{\partial t\partial\zeta} + \frac{\partial G}{\partial t}P,$$

we get:

$$\int_{t_0}^{t}\left(G\frac{\partial^2\Phi}{\partial t\partial\zeta} - \frac{\partial\Phi}{\partial t}\frac{\partial G}{\partial t}\right)dt = \left(G\frac{\partial\Phi}{\partial\zeta} + \frac{1}{g}\frac{\partial\Phi}{\partial t}\frac{\partial G}{\partial t}\right)\Bigg|_{t_0}^{t} - \int_{t_0}^{t}P\frac{\partial G}{\partial t}dt.$$

Setting now in the last equation $t = \tau$ and taking into account the initial conditions imposed on *G* and $\frac{\partial G}{\partial t}$, we have the equalities:

$$-\frac{1}{4\pi}\iint_{\zeta=0}\left\{\left(\mathrm{G}\frac{\partial\Phi}{\partial\zeta} + \frac{1}{g}\mathrm{F}_2\frac{\partial G}{\partial t}\right)\Bigg|_{t=t_0} + \int_{t_0}^{t}P\frac{\partial G}{\partial t}dt\right\}d\xi d\eta +$$
$$+\frac{1}{4\pi}\int_{t_0}^{\tau}\iint_S\left(\frac{\partial\Phi}{\partial t}\frac{\partial G}{\partial n} - G\frac{\partial^2\Phi}{\partial t\partial n}\right)d\sigma dt = \begin{cases}\frac{1}{2}\Phi\big|_{t_0}^{\tau}, M\in S\\ \Phi\big|_{t_0}^{\tau}, M\in\tau_\infty\end{cases}. \tag{1}$$

The potential of the velocity at the initial instant of time $t = t_0$, $\Phi(\mathrm{M}, t_0)$ is the solution of the problem:

$$\left.\begin{array}{c}\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0\\ \Phi(x,y,0,t_0) = F_1(x,y)\\ \frac{\partial\Phi}{\partial z} = 0, z = -H\\ \Phi\to 0, \nabla\Phi\to 0, R\to\infty\end{array}\right\}.$$

Let us represent the function $F_1(x,y)$ in the form of a Fourier integral:

$$F_1(x,y) = \frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_0^\infty sA(s,\vartheta)\exp(-is(x\cos\vartheta + y\sin\vartheta)dsd\vartheta$$

$$A(s,\vartheta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F_1(x,y)\exp(is(x\cos\vartheta + y\sin\vartheta)dxdy,$$

by means of which the solution of the problem for the potential Φ would be:

$$\Phi(\mathrm{M},\mathrm{t}_0) = \frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_0^\infty sA(s,\vartheta)\frac{\mathrm{ch}\, s(H+z)}{\mathrm{ch}\, sH}\exp(is(x\cos\vartheta + y\sin\vartheta))dsd\vartheta.$$

Let us find the derivative of the potential:

$$\frac{\partial\Phi}{\partial z}\bigg|_{z=0} = \frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_0^\infty sA(s,\vartheta)\,\mathrm{th}\, sH\exp(-is(x\cos\vartheta + y\sin\vartheta))dsd\vartheta.$$

The lower row in equation (1) determines the form of the potential $\Phi(\mathrm{M},\tau), \mathrm{M}\in\mathrm{S}$ at each point of the region occupied by the liquid, and the upper row leads to the equation for finding the velocity potential of the disturbed liquid motion on the surface *S*. This equation has the form:

$$\Phi(\mathrm{M},\tau) = \frac{1}{2\pi}\int_{t_0}^{\tau}\iint_S\frac{\partial\Phi}{\partial t}\frac{\partial G}{\partial n}d\sigma dt - \frac{1}{2\pi}\int_{t_0}^{\tau}\iint_S\frac{\partial^2\Phi}{\partial t\partial n}Gd\sigma dt + \beta(M,\tau),$$

$$\beta(\mathrm{M},\tau)=-\frac{1}{2\pi}\iint_{\zeta=0}\left\{\left(\mathrm{G}\frac{\partial\Phi}{\partial\zeta}+\frac{1}{g}\mathrm{F}_2\frac{\partial G}{\partial t}\right)\bigg|_{t=t_0}+\int_{t_0}^{\tau}P\frac{\partial G}{\partial t}dt\right\}d\xi d\eta+\Phi(M,t_0).$$

If we take the boundary condition on the surface *S* of the body into account explicitly, then we can obtain some consequences from this equation. This condition can be written as:

$$\frac{\partial^2\Phi}{\partial t\partial n}=\sum_{j=1}^{6}\gamma_j\frac{\partial u_j}{\partial t}-\frac{\partial^2\varphi_\infty}{\partial t\partial n},$$

where $(u_1,u_2,u_3)=\overline{\mathrm{V}}$ is the linear velocity of the body, $(u_4,u_5,u_6)=\overline{\omega}$ is the angular velocity of the body, $(\gamma_1,\gamma_2,\gamma_3)=\overline{n}$ is the normal to its surface *S*, $(\gamma_4,\gamma_5,\gamma_6)=(\overline{r}-\overline{r}_0)\times\overline{n}$, $\overline{r}=(x,y,z)$ is the radius vector of the point *M*, $\overline{r_0}$ is the radius vector of the center of gravity of the body in the static state.

Now we can put an additive expression for the velocity potential:

$$\Phi=\sum_{j=0}^{7}\Phi_j(M,\tau).$$

The functions Φ_j, $j=0\div 7$, are solutions of the following integral equations:

$$\Phi_0(M,\tau)=\frac{1}{2\pi}\int_{t_0}^{\tau}\iint_S\frac{\partial\Phi_0}{\partial t}\frac{\partial G}{\partial n}d\sigma dt+\frac{1}{2\pi}\int_{t_0}^{\tau}\iint_S G\frac{\partial^2\varphi_\infty}{\partial n\partial t}d\sigma dt,$$

$$\Phi_j(M,\tau)=\frac{1}{2\pi}\int_{t_0}^{\tau}\iint_S\frac{\partial\Phi_j}{\partial t}\frac{\partial G}{\partial n}d\sigma dt-\frac{1}{2\pi}\int_{t_0}^{\tau}\iint_S G\gamma_j\frac{\partial u_j}{\partial t}d\sigma dt,$$

$$j=1\div 6;$$

$$\Phi_7(M,\tau)=\frac{1}{2\pi}\int_{t_0}^{\tau}\iint_S\frac{\partial\Phi_7}{\partial t}\frac{\partial G}{\partial n}d\sigma dt+\beta(M,\tau).$$

Let us represent the function *G* as follows:

$$G=\frac{1}{r}+\frac{1}{2\pi}\int_{-\pi}^{\pi}\int_0^{\infty}\exp(-(z+\zeta+2H)s-is[(\mathrm{x}-\xi)\cos\vartheta+(\mathrm{y}-\eta)\sin\vartheta])dsd\vartheta+$$
$$+\frac{1}{\pi}\int_{-\pi}^{\pi}\int_0^{\infty}[\mathrm{cth\,sH}-(1+\mathrm{cth\,sH})\cos\sigma(\mathrm{t}-\tau)]\times$$
$$\times\exp(-Hs)\,\mathrm{ch}\,s(H+\zeta)\frac{\mathrm{ch}\,s(H+z)}{\mathrm{ch}\,sH}\exp(-is[(\mathrm{x}-\xi)\cos\vartheta+(\mathrm{y}-\eta)\sin\vartheta])\,\mathrm{dsd}\,\vartheta.$$

We transform with the help of this expression the resulting integral equations. As a result, we will have:

$$\Phi_0(M,\tau)=\frac{1}{2\pi}\iint_S\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\Phi_0(N,\tau)d\sigma-$$
$$-\frac{1}{2\pi}\iint_S\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\Phi_0(N,t_0)d\sigma+\frac{1}{2\pi}\iint_S\frac{1}{r}\frac{\partial\varphi_\infty}{\partial n}\bigg|_{t_0}^{\tau}d\sigma+N_0(M,\tau),$$
$$j=1\div 6,$$

$$\Phi_j(M,\tau)=\frac{1}{2\pi}\iint_S\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\Phi_j(N,\tau)d\sigma-$$
$$-\frac{1}{2\pi}\iint_S\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\Phi_j(N,t_0)d\sigma-\frac{1}{2\pi}[\mathrm{u_j}(\tau)-u_j(t_0)]\iint_S\frac{1}{r}\frac{\partial\varphi_\infty}{\partial n}\bigg|_{t_0}^{\tau}d\sigma+N_j(M,\tau),$$

$$\Phi_7(M,\tau)=\frac{1}{2\pi}\iint_S\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\Phi_7(N,\tau)d\sigma-$$
$$-\frac{1}{2\pi}\iint_S\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\Phi_7(N,t_0)d\sigma+N_7(M,\tau).$$

Here the following notation is adopted:

$$N_j(M,\tau)=-\frac{1}{4\pi^2}\int_{t_0}^{\tau}\int_{-\pi}^{\pi}\int_0^{\infty}\{\exp(-(z+H)s)\tilde{H}_j(s,\vartheta,t)+$$
$$+2\exp(-Hs)[1-(1+\mathrm{th}\,sH)\cos\sigma(t-\tau)]\times$$
$$\times\mathrm{ch}\,s(H+z)H_{\mathrm{j}}(\mathrm{s},\vartheta,\mathrm{t})\}\exp(-\mathrm{is}(\mathrm{x}\cos\vartheta+\mathrm{y}\sin\vartheta))\,\mathrm{dsd}\,\vartheta\,\mathrm{dt},$$
$$j=0\div 7;$$

$$\tilde{H}_0(s,\vartheta,t)=-\iint_S\exp(-(\zeta+H)s+is(\xi\cos\vartheta+\eta\sin\vartheta))\times$$
$$\times\left\{\begin{matrix}\frac{\partial\varphi_\infty}{\partial n\partial t}+s\frac{\partial\Phi_0}{\partial t}[i\cos\vartheta\cos(n,\xi)+\\ +i\sin\vartheta\cos(n,\eta)-\cos(n,\zeta)]\end{matrix}\right\}d\sigma,$$

$$H_0(s,\vartheta,t)=-\iint_S\exp(is(\xi\cos\vartheta+\eta\sin\vartheta))\frac{\mathrm{ch}\,s(H+\zeta)}{\mathrm{sh}\,sH}\times$$
$$\times\left\{\begin{matrix}\frac{\partial^2\varphi_\infty}{\partial n\partial t}+s\frac{\partial\Phi_0}{\partial t}[i\cos\vartheta\cos(n,\xi)+\\ +i\sin\vartheta\cos(n,\eta)+\mathrm{th}\,s(\zeta+H)\cos(n,\zeta)]\end{matrix}\right\}d\sigma,$$

$$\tilde{H}_j(s,\vartheta,t)=-\iint_S\exp(-(\zeta+H)s+is(\xi\cos\vartheta+\eta\sin\vartheta))\times$$
$$\times\left\{\begin{matrix}\gamma_j(N)\frac{\partial u_j}{\partial t}-s\frac{\partial\Phi_j}{\partial t}[i\cos\vartheta\cos(n,\xi)+\\ +i\sin\vartheta\cos(n,\eta)-\cos(n,\zeta)]\end{matrix}\right\}d\sigma,$$

$$H_j(s,\vartheta,t)=-\iint_S\exp(is(\xi\cos\vartheta+\eta\sin\vartheta))\frac{\mathrm{ch}\,s(H+\zeta)}{\mathrm{sh}\,sH}\times$$
$$\times\left\{\begin{matrix}\gamma_j(N)\frac{\partial u_j}{\partial t}-s\frac{\partial\Phi_j}{\partial t}[i\cos\vartheta\cos(n,\xi)+\\ +i\sin\vartheta\cos(n,\eta)+\mathrm{th}\,s(\zeta+H)\cos(n,\zeta)]\end{matrix}\right\}d\sigma,$$

$$\tilde{H}_7(s,\vartheta,t)=-s\iint_S\exp(-(\zeta+H)s+is(\xi\cos\vartheta+\eta\sin\vartheta))(\Phi_7)_{\mathrm{t}}\times$$
$$\times[i\cos\vartheta\cos(n,\xi)+$$
$$+i\sin\vartheta\cos(n,\eta)-\cos(n,\zeta)]d\sigma,$$

$$H_7(s,\vartheta,t)=-s\iint_S\exp(is(\xi\cos\vartheta+\eta\sin\vartheta))\frac{\mathrm{ch}\,s(H+\zeta)}{\mathrm{sh}\,sH}\times$$
$$\times\frac{\partial\Phi_7}{\partial t}[i\cos\vartheta\cos(n,\xi)+$$
$$+i\sin\vartheta\cos(n,\eta)+\mathrm{th}\,s(\zeta+H)\cos(n,\zeta)]d\sigma.$$

Functions $\tilde{H}_j(s,\vartheta,t)$ and $H_j(s,\vartheta,t)$ are functions of Kochin [1]. If the body is completely immersed in a liquid, these functions can be calculated using the values of the potential corresponding to the motion of the body in an unbounded liquid. All the resulting integral equations have the general form:

$$\Phi_j(M,\tau)=\frac{1}{2\pi}\iint_S\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\Phi_j(N,\tau)\,\mathrm{d}\sigma+f_j(M,\tau),\ j=0\div 7.$$

The kernel of this equation $K(M,N)=\frac{1}{2\pi}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)$ is real, and τ plays the role of a parameter. As its approximate solution, we can take that solution which is obtained by substituting the value of the potential corresponding to the motion of a body in an unbounded liquid under the integral sign.

Conclusion

The above solution allows to determine the velocity and pressure at any point occupied by liquid, both in the near and far zones.

The solution of the obtained system of integral equations (with the known functions of Kochin) can be carried out numerically using the known standard procedures [4, 5]. In this case, quantitative assessments of the transformation of the initial wave can be obtained by introducing a solid body immersed in a liquid with an invariable geometry into it.

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