

# A INVERSE PROBLEM IN ULTRASONIC TESTING AND MECHANICAL PROPERTIES OF POLYCRYSTALLINE MATERIALS

## ОБРАТНАЯ ЗАДАЧА В УЛЬТРАЗВУКОВОМ ТЕСТИРОВАНИИ И МЕХАНИЧЕСКИЕ СВОЙСТВА ПОЛИКРИСТАЛЛИЧЕСКИХ МАТЕРИАЛОВ

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**Abstract:** The direct problem in ultrasonic testing (UT) is: "Evaluation of attenuation coefficient by means velocity of ultrasonic wave propagation, frequency and grain size in polycrystalline materials". The inverse problem in UT is formulate as "Non-destructive evaluation of grain size by measurement of acoustical characteristics". The values of acoustical characteristics  $(V_L; V_T; \alpha_L; f)$  are measured, according ASTM E 494:2015. In this article a equation for grain size  $(\bar{D})$  is derived.

**Key words:**, ultrasonic testing, acoustical characteristics, grain size, yield stress, ultimate tensile stress, fatigue limit.

### 1. Introduction

In UT investigation of polycrystalline materials the relationship  $\alpha_L = \alpha_L(V_L; V_T; f; \bar{D})$  is obtain. This is Lifshic-Parhomenko-Merkulov's formula [1]

$$(1) \quad \alpha_L = \frac{8\pi^3}{375} \cdot \frac{\mu}{\rho^2} \left[ \frac{4\pi}{3} \left( \frac{\bar{D}}{2} \right)^3 \right] \cdot f^4 \cdot W(V)$$

where  $\rho$  - density;  $(\bar{D})$  - grain size;  $f$  - frequency;  $(V_L; V_T; \alpha_L; f)$  - velocity of longitudinal  $V_L$  and transversal  $V_T$  - ultrasonic waves,  $\alpha_L$  - attenuation in ultrasonic wave propagation,  $\mu = \rho \cdot (V_T)^2$ ,

$$W(V) = \left( \frac{1}{V_L^3} \right) \left( \frac{2}{V_L^5} + \frac{3}{V_T^5} \right).$$

This is direct problem in ultrasonic testing (UT).

### 2. Inverse problem in ultrasonic testing

The inverse problem in UT is formulate as "Non -destructive evaluation (NDE) of grain size  $(\bar{D})$  by measurement of acoustical characteristics  $(V_L; V_T; \alpha_L; f)$  of the material".

After reversing the direct problem (1), a cubic equation (with a single real root) to determine the grain size is obtained [2]

$$(2) \quad \left[ \frac{4\pi^4}{1125} \left( \frac{V_T^4}{V_L^3} \right) \left( \frac{2}{V_L^5} + \frac{3}{V_T^5} \right) f^4 \right] \cdot (\bar{D})^3 - \alpha_L = 0$$

This is equation for evaluation of the grain size  $(\bar{D})$  by means measured acoustical characteristics  $(V_L; V_T; \alpha_L; f)$ , according ASTM E 494:2015.

### 3. Mechanical properties

#### 3.1. Yield stress - $\sigma_S$

This value is evaluated by means Hall-Petch's relationship [3]

$$(3) \quad \sigma_S = \sigma_0 + K_y (\bar{D})^{-1/2}$$

It is accepted that the constant is called "Pierre-Nabarro's tension" [3]. It takes into account the frictional tension - which compensates for the forces overcome by the dislocations when they move into the grain, [3]:

$$(4) \quad \tau_{PN} = \frac{2G}{1-\nu} \exp \left[ - \frac{2\pi}{(1-\nu)} \left( \frac{c}{a} \right) \right]$$

where  $\nu$  - Poisson coefficient,  $G$  - sliding module,  $a$  and  $c$  - crystalline grid parameters in a plane {100}. For  $c \sim 0.2\%$  i.e. low - carbon steels. The approximation of parameter  $\sigma_0$  is

$$(5) \quad \sigma_0 \approx \frac{2}{3} \cdot \frac{G}{1-\nu}$$

where  $[\sigma_0] = MPa$ ;  $[G] = GPa$ .

The constant  $K_y$  is called Petch's coefficient. It takes into account the passage of dislocations from one grain to another [4]:

$$(6) \quad \frac{K_y}{G} = \frac{2}{\cos \vartheta} \left\{ \frac{|b|}{25\pi(1-\nu)} \right\}^{1/2}$$

where  $\vartheta$  - angle of maximum deviation of the slip directions,  $|b|$  - module of the Burger,s vector [3].

After regrouping the values in (6), for the Petch's coefficient  $K_y$  is obtain

$$(7) \quad K_y \approx \frac{1}{4} \cdot \frac{G}{\sqrt{1-\nu}}$$

where  $[K_y] = MPa / mm^{1/2}$ ,  $[G] = GPa$ . As

$$(8) \quad G = \rho \cdot (V_T)^2; \nu = \frac{0.5 - (V_T/V_L)^2}{1 - (V_T/V_L)^2},$$

then from relationships (5) and (7) are obtain

$$(9) \quad \sigma_0 \approx \frac{2}{3} \cdot \frac{G(V_L; V_T)}{1 - \nu(V_L; V_T)};$$

$$(10) \quad K_y \approx \frac{1}{4} \cdot \frac{G(V_L; V_T)}{\sqrt{1 - \nu(V_L; V_T)}}.$$

This is the obvious look of coefficients  $(\sigma_0; K_y)$  in Hall-Petch's relationship (3).

NDE of yield stress -  $\sigma_S$  is by (9), (10) and the solution  $\bar{D} = \bar{D}(V_L; V_T; \alpha_L; f)$  of equation (2).

### 3.2. Ultimate tensile stress - $\sigma_B$

Is being considered the Busibesq's solution [6,7]

$$(11) \quad \sigma_S = \varphi(\nu) \cdot HB$$

where  $HB$  - Brinel's hardness,  $\nu$  - Poisson ratio,

$$\varphi(\nu) = \left\{ \frac{1}{2}(1 - 2\nu) + \frac{2}{9}(1 + \nu) \cdot [2 \cdot (1 + \nu)]^{1/2} \right\};$$

$$\nu = \frac{0.5 - (V_T / V_L)^2}{1 - (V_T / V_L)^2}.$$

Putting (11) in Hall-Petch's relationship (3) we have

$$(12) \quad \sigma_S = \sigma_0 + K_y (\bar{D})^{-1/2} = \varphi(\nu) \cdot HB$$

This is Stroh's relationship [5] in type

$$(13) \quad HB = \sigma_0^{(HB)} + K_y^{(HB)} (\bar{D})^{-1/2}$$

where

$$(14) \quad \sigma_0^{(HB)} = \frac{\sigma_0}{\varphi(\nu)};$$

$$(15) \quad K_y^{(HB)} = \frac{K_y}{\varphi(\nu)}.$$

If  $\varphi(\nu) \equiv \varphi(V_L; V_T)$ , then according (5), (7), and (8), the relationships (14) and (15) are recorded in the form

$$(16) \quad \sigma_0^{(HB)} = \frac{2}{3} \cdot \frac{G(V_L; V_T)}{1 - \nu(V_L; V_T)} \cdot \frac{1}{\varphi(V_L; V_T)};$$

$$(17) \quad K_y^{(HB)} = \frac{1}{4} \cdot \frac{G(V_L; V_T)}{\sqrt{1 - \nu(V_L; V_T)}} \cdot \frac{1}{\varphi(V_L; V_T)}$$

The value of  $\sigma_B$ , by Markovic's relationship,  $\sigma_B \approx 0.34HB^{0.989}$  [7], is obtain.

NDE of ultimate tensile stress -  $\sigma_B$  is by (16), (17), the solution  $\bar{D} = \bar{D}(V_L; V_T; \alpha_L; f)$  of equation (2) and Markovic's relationship.

### 3.3. Fatigue limit - $\sigma_{-1}$

This value is evaluated by Terentiev's relationship [9]

$$(18) \quad \sigma_{-1} = \sigma_0^{(-1)} + K_y^{(-1)} \left\{ \bar{D} \right\}^{-1/2}$$

To derivate the coefficients  $(\sigma_0^{(-1)}; K_y^{(-1)})$  in (18) is being considered the ratio [8]

$$\frac{\sigma_{-1}}{\sigma_S} = \frac{0.165HB}{\varphi(\nu)HB} = 0.165\varphi^{-1}(\nu)$$

We have the recording

$$(19) \quad \frac{\sigma_{-1}}{0.165\varphi^{-1}(\nu)} = \sigma_0 + K_y (\bar{D})^{-1/2}.$$

Therefore

$$(20) \quad \sigma_0^{(-1)} = \frac{0.165}{\varphi(\nu)} \sigma_0;$$

$$(21) \quad K_y^{(-1)} = \frac{0.165}{\varphi(\nu)} K_y.$$

According (5) and (7) for the coefficients (20) and (21) are obtain

$$(22) \quad \sigma_0^{(-1)} \approx \frac{1}{10} \cdot \frac{G}{\varphi(\nu)(1 - \nu)};$$

$$(23) \quad K_y^{(-1)} \approx \frac{2}{5} \cdot \frac{G}{\varphi(\nu)\sqrt{1 - \nu}}.$$

According (8) the coefficients  $(\sigma_0^{(-1)}; K_y^{(-1)})$  are

$$(24) \quad \sigma_0^{(-1)} \approx \frac{1}{10} \cdot \frac{G(V_L; V_T)}{(1 - \nu(V_L; V_T))} \cdot \frac{1}{\varphi(V_L; V_T)};$$

$$(25) \quad K_y^{(-1)} \approx \frac{2}{5} \cdot \frac{G(V_L; V_T)}{\sqrt{1 - \nu(V_L; V_T)}} \cdot \frac{1}{\varphi(V_L; V_T)}.$$

This is the obvious look of coefficients  $(\sigma_0^{(-1)}; K_y^{(-1)})$  in Terentiev's relationship (18).

NDE of Fatigue limit -  $\sigma_{-1}$  is by (24), (25) and the solution  $\bar{D} = \bar{D}(V_L; V_T; \alpha_L; f)$  of equation (2).

### 4. Conclusion

It's resolved the inverse problem: Non destructive evaluation of  $(\bar{D})$  by means measure of acoustical characteristics of the material i.e.

$$(26) \quad \bar{D} = \bar{D}(V_L; V_T; \alpha_L; f).$$

The values measurement of  $(V_L; V_T; \alpha_L)$  are according ASTM E 494:2015.

For the non-destructive evaluation of mechanical properties:

$\sigma_S$ ,  $\sigma_B$  and  $\sigma_{-1}$  by means

$$(27) \quad \sigma_S = \sigma_S(V_L, V_T; \alpha_L; f);$$

$$(28) \quad \sigma_B = \sigma_B(V_L, V_T; \alpha_L; f);$$

$$(29) \quad \sigma_{-1} = \sigma_{-1}(V_L; V_T; \alpha_L; f).$$

are could be used.

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